THE LOGICAL STRUCTURE OF LINGUISTIC THEORY

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Introduction

This study of linguistic structure had its origin in certain problems that arose in attempting to extend linguistic techniques to the analysis of discourse. (1) This extension naturally presupposed standard linguistic analysis, and in the attempt to develop effective techniques of discourse analysis it was found necessary to assume certain knowledge about linguistic structure which was not in fact provided by existing methods, though it seemed within the range of distributional studies. In particular, these methods failed to account for such obvious relations between sentences as the active-passive variation. Systematic investigation of this problem exposed the framework of a syntactic theory, and led finally to this attempt at developing a unified approach to syntactic theory as a whole, including a theory of relations among sentences as an integral part. This latter subtheory does not appear here merely as an extension of syntactic theory. On the contrary, it turns out to play a central role in the establishment of the fundamental notion of syntax, i.e., in the procedures of constituent analysis. For these reasons, this study is heavily oriented towards the investigation of the formal relations among sentences. Finally, we will explore the general nature of linguistic theory, emphasizing the problem of validating grammars, the notion of a linguistic level, the problem of determining the substrate of a grammatical description, and similar topics. We then attempt to construct in an abstract (but sketchy) manner the standard linguistic levels, and to apply this abstract formulation to linguistic material. The central conclusion is that a new level of transformational analysis is needed, for the same reasons that led to the construction of the standard level.
We suspect a formulation of this level (i.e., a chiefly transformational analysis), and investigate, in some detail, the transformational structure of English.

This is basically a study of the arrangement of words and morphemes in sentences, hence a study of linguistic form. Thus it is syntactic study in both the narrow sense (as opposed to phonology) and in the broader sense (as opposed to semantics). In particular, no reference is placed on the meaning of linguistic expressions in this study, in part, because it is felt that the theory of meaning fails to meet certain minimum requirements of objectivity and operational verifiability, but more importantly, because semantic notions, if taken seriously, appear to be quite irrelevant to the problems being investigated here.

There can be no definitive formulation of syntactic theory at this point, and in the study which follows many more questions are asked than answered. Lack of data is the fundamental reason for this. There simply is not enough detailed syntactic work available, in the proper form, for theoretical conclusions to be able to receive empirical confirmation. On the other hand, the scarcity of adequate syntactic material is no doubt a result of the overwhelming complexity of the syntactic structure of natural languages. I will attempt to show that a good deal of this complexity can be eliminated by transformational analysis, i.e., by a more adequate syntactic theory. It is elementary that theoretical investigation and collection of data are interdependent activities. One cannot describe a linguistic system in any meaningful way without some conception of what is the nature of such a system, and what are the properties
and purposes of a grammatical description. For this reason, it is important to develop a precisely formulated and conceptually complete construction of linguistic theory on the clearest possible elementary notions, even if more elaborate constructions based upon these notions cannot, because of insufficient evidence, be empirically supported.

The establishment of such a theory may be an essential step towards obtaining this evidence. Nor can we demand that all lower levels of linguistic theory be thoroughly and finally established and empirically validated before higher level theoretical studies are undertaken. It is true that the higher levels of linguistic description depend on results obtained at the lower levels. But there is a good sense in which the converse is also the case. It would be absurd to attempt to give principles of sentence construction in terms of phonemes, but only the development of such higher levels as phrase structure indicates that this futile task need not be undertaken on the phonemic level. Thus even though higher level constituent analysis cannot be well-grounded unless the principles of phonemic analysis are firmly established, the scope of phonemic analysis cannot be delimited unless such higher levels are constructed at least in outline. Similarly, we will argue below that an attempt to describe sentence construction fully in terms of phrase structure will also fail, because it is attempting to do too much. But only the development of the still higher level of transformational analysis, and the examination of its potentialities, will give content to this assertion, and will thus prepare the way for the development of a more successful technique of constituent analysis with narrowed
limits. The grammar of a language is a complex system with many interconnections between its parts. In order to develop a thorough characterization of some part of grammar, it is often useful (or even absolutely necessary) to have at least some picture of the character of a completed system of grammar.

At the same time, it is important to formulate clear and precise criteria, and to apply these with complete rigor and consistency, even when it appears likely that they are only partially adequate. In this way we may hope to expose the source and exact location of this inadequacy. Pushing a precise, but inadequate formulation to an absurd conclusion may be an important method of discovery. Below, we will see that careful pursuit of this course exposes a gap in linguistic theory, and leads to the construction of a theory of transformations. Obscure and intuition-bound conceptions can of course never be pushed to absurd conclusions, but this can scarcely be regarded as a point in their favor.

Below, we will suggest definitions for linguistic elements and criteria for the validation of grammatical description within the framework of what has come to be known as 'distribution analysis.' The resulting sketch of a theory should be understood, in the sense of the preceding paragraphs, as suggesting a program of research, i.e., a specific model for syntactic description to be tested and elaborated. The following investigations are divided approximately equally between theory construction and application of the theory to linguistic material. The exact point where formal construction should stop and application should begin is a matter of personal preference. At several places below I have
indicated that I can see no reason for preferring any one of several alternative ways of developing the theory, and have left it at that. It would be misleading, then, to describe this as a proposed theory of linguistic structure. Rather it is an attempt to sum up and organize a certain amount of theoretical investigations into linguistic structure, and to examine the implications of these constructions for syntactic description of actual linguistic materials. Since these constructions are, necessarily, so tentative and incomplete, the motivation for the constructions is often more important than the actual construction. For this reason I will often give the general requirements that a construction must apparently meet in some detail, then giving what seems to be the natural way of meeting these requirements instead of collapsing this into one step. Similarly, in the grammatical examples of chapters 7 and 9, many instances will be discussed in some detail, even if ultimately to be rejected.

Footnotes — Introduction.
(?). As presented, for instance, in Harris, Methods of Structural Linguistics.
Chapter I: The Nature of Linguistic Theory

1. Descriptive linguistics is concerned with three fundamental problems. On the one hand, the descriptive linguist is interested in constructing grammars for particular languages. At the same time, he is interested in giving a general theory of linguistic structure of which each of these grammars is an exemplification. Finally, he must be concerned with the problem of justifying and validating the results of his inquiries, and demonstrating that the grammars that he constructs are in some sense the correct ones. All three of these problems will occupy us in this investigation of linguistic structure. Before proceeding with proposals for the construction of linguistic theory, we must determine quite clearly just what is the nature of each of these three projects, and how they are interrelated.

2.1. A grammar of a particular language can be considered, in what seems to me a perfectly good sense, to be a complete scientific theory of a particular subject matter, and if given in precise enough form, a formalized theory. Any interesting scientific theory will seek to relate observable events by formulating general laws in terms of hypothetical constructs, and providing a demonstration that certain observable events follow as consequences of these laws. In a particular grammar, the observable events are that such and such is an utterance of the language, and the demonstration that this observable event is a consequence of the theory consists in stating the structure of this predicted utterance on each linguistic level, and showing that this structure conforms to the grammatical rules, or the laws, of the theory.
The grammar thus gives a theory of these utterances in terms of such hypothetical constructs as the particular phonemes, morphemes, words, phrases, etc. of the language in question. As in the case of any scientific theory, only a certain subset of the observable events will have been observed at any given time. In the case of a grammar, we have, at any time, only a finite corpus of utterances, \(^{(1)}\) out of an infinite set of grammatical utterances.

With its law-like rules for the combination of elements, a grammar can thus be said to 'generate' a certain set of utterances on the basis of a given observed sample. As an analogue, consider a possible formulation of a part of chemical theory, in which, on the basis of such constructed notions as 'electron,' 'valence,' and so on, all possible chemical compounds might be described. A grammar presented in a different form, such that given an utterance, the grammar will mechanically provide an analysis of it in terms of each level, could be considered roughly analogous to a formalized system of qualitative analysis.\(^{(2)}\)

If we had such grammars for every language, we could attempt to abstract from them, and to construct a general theory of the elements of which languages are composed. Thus in a certain sense, the complete realization of the first goal, the construction of particular grammars, would lead to the construction of a general theory of linguistic structure.

2.2. On the other hand, we can scarcely describe a language at all, except in terms of some previously assumed theory of linguistic structure. For this reason, a large part of
the work in modern structural linguistics has been directed towards providing a methodology for the analysis and description of linguistic behavior, that is, a methodology for the construction of these particular scientific theories. The purpose of these methodological investigations has been, in part, to provide an essential mechanical method for constructing an appropriate grammar for each particular language; that is, a method in which the linguist’s intuition and other intangibles play no rôle. If the second goal, that of constructing a general theory, were achieved in a strong enough form, the construction of particular grammars would be a mechanical matter, requiring no ingenuity. Thus these two goals are in a sense interdependent. Given particular grammars, we could generalize to an abstract theory. Given a sufficiently powerful abstract theory, we could automatically derive grammars for particular languages.

2.3. Actually, of course, neither goal can be achieved independently. In constructing particular grammars, the linguist leans heavily on a preconception of linguistic structure, and any general characterization of linguistic structure must show itself adequate to the description of each particular language. The circularity is not vicious, however. It just points out the fact that linguistic theory has two interdependent aspects. At any given point in its development, we can present a non-circular account, giving the general theory as an abstract formal system, and showing each grammar is a particular example of it. Change can come in two ways — either by refining the formalism and finding new and deeper underpinnings for the general theory, or by finding out new facts about particular
languages, and simpler ways of describing them.

There are several possible ways of construing the relationship between particular grammars and the general theory. Certainly every grammar must be compatible with the theory in the sense that the elements set up in the grammar possess in a particular way the general properties required by the theory, i.e., that the system described by the grammar be what is called a 'true interpretation' of the theory. This is the weakest possible requirement. At the other extreme, the strongest requirement would be that the theory provide a practical means for literally constructing the grammar out of the raw data. That is, we might require that the grammar of each language be mechanically derivable from a sufficient corpus, once the theory is established. Let us call such a theory procedural. Thus given a sufficient corpus, a procedural theory will lead us directly to a grammatical description of the language, in some practical way, requiring, in principle, no ingenuity or intuition on the part of the linguist. A procedural theory gives what might be called a 'discovery procedure' for grammars. A weaker requirement than this would be that given a grammar, the theory must provide a practical mechanical way of validating it, i.e., of showing that it is in fact the best grammar of the language, in a sense specified by the theory. Extending the sense of 'procedural theory' to cover this case, we can say that a procedural theory provides a practical decision procedure for the notion 'grammar of a language.' This would seem to be the proper interpretation for the kind of theory that Harris is interested in building in his Methods in Structural Linguistics. A still weaker
requirement would be that the theory provide a method of evaluating any proposed grammar, so that, given two proposed grammars, there would be a practical and mechanical way for determining which is the better of the two. Notice that this last is still a strong requirement, much stronger than those imposed in natural sciences, where no one would seriously consider the possibility of a general, practical, mechanical method for deciding between two theories, each compatible with the available evidence. But in linguistics, given the nature of the data, it seems natural that our sights should be set at least that high. Each of these three kinds of relation has been qualified by the word 'practical.' The significance for linguistics of this vague and indefinable qualification will become clearer below. (3)

2.4. To recapitulate so far, we see that linguistic research has two aspects. It aims to provide for each language a theory of the structure of that language (i.e., a grammar), and at the same time to develop a general theory of linguistic structure of which each of these grammars will present a model. The particular grammars and the general theory must be closely enough related so that some practical technique be available for deciding between two proposed grammars as to which better exemplifies the theory.

3.1. Consider now the problem of justifying a grammar. It is clear in the first place that the form of the justification will vary with the purposes of the grammar. Thus a pedagogic grammar may be justified in terms of teaching success. But such considerations are obviously irrelevant for a linguistic grammar with no such special purposes,
a grammar which purports to show the 'structure' of the language.

A theory is justified by relating it to data. In the case of a linguistic grammar, we surely require that it meet certain external conditions of adequacy, that the generated sentences be acceptable to the native speaker, that the elements of the language as constructed in the grammar have certain observable correlates, etc. Let us assume that such criteria, however vague and incomplete, have been established for grammars. We then face the problem of choosing among the vast number of different grammars, each giving a different structure, and all meeting these vague and incomplete external criteria. This is the facet of the problem of justification which is most interesting at the present stage of linguistic research, and to which we will devote our primary attention in this study. But to this question, an answer has already been suggested in §2. The grammar can be justified by relating it to the general theory and showing that exactly the structure described arises from the data, given the general theory (or that the given grammar has the highest value in terms of the general theory). The general theory must meet the condition that all grammars to which it leads must satisfy whatever criteria of adequacy we can establish. Such a conception of the process of validation means that one indispensable aspect of the validation of a grammar of a given language is the construction of grammars for other languages. This conclusion follows from the conception of a valid grammar as one conforming to a linguistic theory which in turn must produce grammars meeting the external conditions of adequacy. Even weak conditions
of adequacy may impose severe restrictions on the choice of grammars for a given language $L_1$. While many grammars of $L_1$ may meet these conditions when $L_1$ is considered in isolation, it may be the case that very few of these grammars follow from some general theory that leads to grammars of the languages $L_2$, $L_3$, ..., all of which meet these conditions of adequacy. The fact that grammars of $L_2$, $L_3$, ... play a part in the evaluation of the grammar of $L_1$ is as it should be, since linguistics is not only interested in grammars of particular languages, but also in developing a general theory of linguistic structure. Whether or not we can determine grammars to uniqueness in this manner depends on how limiting are the conditions of adequacy and how stringent is the formulation of the general theory (which can in a sense be regarded as the definition of 'language'). (5)

It appears then that there are two factors involved in determining the validity of a grammar, the necessity to meet the external conditions of adequacy, and to conform to the general theory. The first factor cannot be eliminated, or there are no constraints whatsoever on grammar construction; the simplest grammar for $L$ will simply identify a grammatical sentence in $L$ as any phone sequence. Elimination of the second factor leaves us free to choose at will among a vast number of mutually conflicting grammars.

There has been some discussion recently as to whether the linguist 'plays mathematical games' or 'describes reality' in linguistic analysis of particular language, where the phrase 'playing mathematical games' refers apparently to the conscious development of a theory of linguistic structure for use in constructing and validating grammars. If by
'describing reality' is meant meeting the anterior conditions of adequacy, then in order to give content and significance to the requirement that the linguist must describe reality, it is necessary to give independent (i.e., outside the particular grammar) characterizations of these conditions, e.g., for sentencehood, by constructing informant response tests to determine the degree of acceptability or evocability of sequences. But within whatever bounds can be clearly set independently, the linguist's goal can only be to construct for each language a simple grammar which relates to the grammars constructed for other languages in such a way as to lead to a revealing general theory of which all are exemplifications. There seems no reason to consider that the constructs established in pursuit of these goals are in some sense invalid. If the methods developed with these goals in mind lead to unacceptable results, it is important to show this. But the alternative to ineffective methods is not no methods, but rather more effective methods.

3.2. What are the practical implications of this conception of validation for the program of building an objective science of linguistics? Suppose that a linguist constructs a grammar of a language in terms of such and such elements and rules of combination. To validate this grammar, it is not sufficient merely to give a formal characterization of the elements that have been constructed. Given any set of phonemes, words, etc., we can trivially find a precise and purely formal and operational way of deriving just this set from the data, namely, list such and such forms and call them words, list such and such words and call the class of these words a syntactic category, etc. Listing is as precise and formal a
procedure as we can find. To justify the contention that such and such are, e.g., the syntactic categories of the language, it is necessary to give a completely general characterization of the notion of syntactic category, and to show that the chosen categories meet this definition, whereas others do not. The important word here is 'general'. If the linguist wishes to justify a given assignment of words to syntactic categories in one language by appeal to a certain definition, formal or otherwise, he must be prepared to set up syntactic categories in every other language by exactly the same definition. (8) This condition rules out listing, but it also rules out many other customary devices for giving what is intended to be an objective justification for grammars. Thus a proposal that Adjective in English be defined as the class of words to which 'er' and 'est' can be added is ruled out, if intended as an objective way of setting up a syntactic class, because no general notion of syntactic category tells us that the suffixes should be taken to define a class. Similarly, the proposal that word or phrase be defined by certain (not all) suprasegmental features, that a certain syntactic category be defined by the fact of its exclusive occurrence in a given syntactic position, and many other suggestions become suspect. These suggestions may be perfectly legitimate when intended as a formal characterization of elements which have already been set up and justified on some basis, but not when they are intended as an objective means for setting up these elements in the first place, or as a significant and objective formal means of demonstrating that these and not some other elements should be constructed. Or they may be legitimate if understood as a shorthand way of
saying that these formal features are chosen because they lead to the simplest grammar. But now the real source of validation is simplicity, and it becomes necessary to analyze this notion, if the validation is to have any significance.

Such procedures give no support to the program of developing an objective and operational linguistics. There is only one way to determine whether a characterization of elements in some grammar does lead to serious support to this program. We must ask: in accordance with what general theory are the elements in question set up? Is this theory a rigorously constructed one, framed in terms of clear and applicable notions? Can this theory be applied to other languages, giving satisfactory results? We cannot set up an expression \( \mathbf{x} \) (or class of expressions, etc.) in a given language \( \mathbf{L} \) as a grammatical element of some sort simply on the basis of the fact that \( \mathbf{x} \) has the formal property \( \varphi \). Any element will have some formal property, and a purported 'justification' of this kind merely bypasses the only interesting question, namely, why \( \varphi \) was selected as a defining property in the case of this language. If we wish to use the formal property \( \varphi \) as the basis for setting up \( \mathbf{x} \) as a certain grammatical element in \( \mathbf{L} \), we must be prepared to state a proposition to the effect that in any language, an expression (or class, etc.) \( \mathbf{x} \) will be set up as a grammatical element of the kind in question if it has the property \( \varphi \) (or a property analogous in a stated respect).

Only formal properties for which we are prepared, at least tentatively, to make this claim, can carry the burden of serious validation.

3.3. There is a certain ambiguity in the word 'formal'
and the notion 'formal justification', which should perhaps be commented upon here. In the strict sense of the word, an argument, a characterization, a theory, etc. is 'formal' if it deals with form as opposed to meaning, that is, if it deals solely with the shape and arrangement of symbols. In this sense, any distributional theory or argument is formal. But the word 'formal' has misleading connotations, implying 'rigorous', 'clear', etc. Suppose that we use instead the word 'formalized', when we have this sense in mind. A formalized theory, then, is one that is formulated in accordance with certain clear canons of rigor and precision; definitions are given explicitly in such a way that defined terms are always eliminable, and the axioms and methods of proof are precisely stated. We can thus have a formalized theory purporting to be about form or about meaning. A formalized theory is, of course, not necessarily an acceptable, enlightening, or 'objective' theory. And the fact that a certain subject matter is stated to be the intended interpretation of a presented formalized theory does not confer any desirable properties upon this subject matter, or on the theory. It is possible to construct a formalized theory with no interesting interpretation, or with 'intuition', 'ghosts', etc., in its intended interpretation. What concerns us here is the possibility of a formalized theory of linguistic form, and the problems involved in constructing such a theory.

A possible instance of equivocation between the two senses of 'formal' is the case of formal justification, discussed above, for setting up such and such a category or element in a grammar, i.e., the notion that the establishment
of an element is justified when some distributional criterion has been given for it. Such constructions are formal in the sense of non-semantic, but the inadequacy of arguments based on formality in this sense, as we have already noted, is evident when we realize that listing is a formal operation. To reiterate our conclusion, the discovery of formal markers is not to the point insofar as we are interested in avoiding recourse to intuition in constructing or validating grammars. To do this significantly, it is necessary to demonstrate that the grammar follows from an general and abstract theory of linguistic structure.

This ambiguity may also lead to some confusion in considering the relevance of symbolic logic to linguistic theory. It is sometimes argued that syntactic treatments of systems of symbolic logic, or other so-called 'artificial languages,' with their formation and transformation rules, their syntactic categories, etc., may serve as useful models for the study of linguistic form. Although this could conceivably be so, it does not seem to me that this has been shown in any interesting sense, or that whatever similarities that can be found offer any interesting line of study. It may be that some of the conviction carried by the argument that linguistics should model itself on logical syntax is due to an equivocation in the word 'formal.' It is certainly correct that logic is indispensable for formalizing theories, of linguistics or anything else, but this fact gives us no insight into what sort of systems form the subject matter for linguistics, or how it should treat them. Neither from this fact, nor from the indisputable fact that work in logic has incidentally led to important insights into the use of
language, (9) can it be argued that the study of the formal (or semantic) properties of natural languages should model itself on the study of the formal (or semantic) properties of logic and artificial languages. Though logic can be applied with profit to the construction of a formalized linguistic theory, it does not follow that this theory is in any sense about logic or any other formalized system. (10)

4.1. We have seen that one requirement that justifications must meet if they are to be convincing is that of generality. Since the problem of justification and that of constructing a general theory of linguistic structure are, in part at least, essentially the same, another look at the structure of the general theory may clarify further requirements for justification of particular grammars.

The general theory will ultimately assume the form of a system of definitions, in which 'phoneme', 'word', 'sentence', etc., are defined, and their general properties and interrelations specified. If a system of definitions introducing a certain set of concepts is to have explanatory force, it must be based on some set of primitive, undefined terms. The interest and explanatory power of this theory will depend directly on the clarity and applicability of these primitive notions.

Among the primitive notions of this general theory of linguistic structure we might expect to find such terms as 'precedes' (so-and-so precedes such-and-such), 'is a fricative', etc. Suppose now that we know how to apply the primitive notions of this general theory to actual language material, i.e., that we know which form precedes which other form, which form is a fricative, etc. Then assuming that the
definitions in the theory meet certain formal conditions (conditions of constructivity), we have a mechanical procedure for discovering a grammar of the language. If 'phoneme', 'word', 'sentence', are each defined in terms of a set of notions $P$, $Q$, $R$, ..., and if we know in fact what are the $P$'s, the $Q$'s, the $R$'s, etc., of a given sample of the language, we can determine automatically the phonemes, words, and sentences. Thus, if given in a strong enough form, linguistic theory may actually provide a literal discovery procedure for grammars, though perhaps not a practical discovery procedure.

The form of theory that we have just described, where every notion appearing in the theory is completely analyzed in terms of a set of operational primitives, is a very strong one. A weaker conception of scientific theory can be given. But it seems to me that this is a correct way to state the goal of that aspect of linguistic theory that we are here considering.

Wells has pointed out recently that philosophers have, by and large, rejected, as a general criterion of significance, the strong kind of reductionism that we are suggesting as necessary for our particular purposes. He offers this in criticism of Bloomfield's program of avoiding mentalistic foundations for linguistic theory. It is true that many philosophers have given up a certain form of reductionism, of which Bloomfield's program (and our restatement of it) is an instance, as a general criterion for significance, and have held that such terms as 'soluble', 'has such-and-such a molecular structure', etc. (or the real number system), must be introduced into scientific
theory even if not amenable to the kind of analysis once sought. However, I do not believe that this is relevant to Bloomfield's anti-mentalism. The fact that a certain general criterion of significance has been abandoned does not mean that the bars are down, and that 'ideas' and 'meanings' become proper terms for linguistics, any more than it means that ghosts are proper concepts for physics. If this rejection of an old criterion is not followed by construction of a new one, then it simply has no bearing on the selection of legitimate terms for a scientific theory. Where it is followed by some new sense of 'significance', then if this new sense is at all adequate, it seems to me that it will rule out mentalism for what were essentially Bloomfield's reasons, i.e., its obscurity and general uselessness in linguistic theory. Thus Quine rejects reductionism, suggests an alternative, and rejects mentalism. (13)

Furthermore, I cannot agree that the kind of program we have outlined for linguistics is invalidated by the argument that it cannot work for physics (even if correct). There are only two kinds of terms in a theory, primitive terms and defined terms. The abandonment of the strong kind of reductionism in question means the decision to admit a great number of terms (e.g., disposition terms) as primitives, with the relations between them expressed by axioms (e.g., reduction sentences), rather than to avoid these terms, or to seek to introduce them by definition. Whatever the situation may be in other sciences, I think that there is hope of developing that aspect of linguistic theory being
studied here on the basis of a small number of operational primitives, and that introduction of dispositions (or mentalistic terms) is either irrelevant, or trivializes the theory. That this position is incorrect can not be shown by any general argument from the philosophy of science, but only by constructing an interesting theory beyond these (rather vague) bounds.

4.2. We have suggested that a grammar is justified by showing that it follows from application to the corpus of a properly formulated general theory. Clearly, for such a justification of a grammar to carry any conviction, it must be framed in terms of notions whose applicability can be determined in particular cases. Or, to put the same thing differently, we see that to use an explicit and formalized theory of linguistic structure for constructing grammars, it will be necessary that the data be presented to it in a certain form. What form this must be is determined by the primitive notions of this theory. The theory is significant just to the extent that we can demonstrate that the data does have the prescribed form, that is, just to the extent that we have unambiguous, cross-culturally valid tests for applying the undefined notions to data. Thus if one of the basic undefined terms of linguistic theory is 'intuition', and if we define phonemes in this theory as elements which our intuition perceives in a language, then the notion of phoneme is as clear and precise as is 'intuition'. We will be able to discover the phonemes of a language by applying this theory just in case our intuition applies without equivocation. And the justification of the grammatical statement at which
we arrive will be as convincing to others as are our intuitions.

It should be clear, then, why the linguist interested in constructing a general theory of linguistic structure, in justifying given grammars, or (to put the matter in its more usual form) in constructing procedures of analysis, should try to avoid such notions as 'intuition,' and others which fail in precisely the same way. With this in mind, we can turn to the question of what is an acceptable basis for linguistic theory.

5.1. The first question that arises in this connection is that of the role of meaning in linguistic analysis. This has been the subject of much debate in recent years. If this debate has not been conclusive, the reason may be in part that the question has been argued on the wrong basis. I think that this is in fact the case, and that a fresh approach can shed some new light on this problem.

The central objection to meaning as a criterion of analysis has always been the obscurity of semantic notions. I think it is indeed fair to say that we are currently in pretty much the same state of unclarity with regard to meaning as we are with regard to intuition. And this is a sufficient reason for refusing to admit meaning into linguistic theory. Suppose that we do manage to develop effective and unambiguous cross-culturally valid tests for synonymy, meaningfulness, and related concepts. Would this mean that the description of grammatical structure may be based on
semantic notions? I think that it would not -- that in fact
there is a deeper motivation for refusing to base
linguistic form on semantic notions than merely the obscurity
of such a foundation. What I would like to argue here is that
cannot be applied to the determination of
semantic notions are quite irrelevant to problems of formal
structure, that only their unclarity disguises their irrelevance,
and that when the claim is put forth that linguistic analysis
can not be carried out without the use of meaning, what is
really expressed is that it can not be carried out without
intuition.

5.2. There are two distinct sets of semantic notions.
Following Quine, we may distinguish between the theory
of reference and the theory of meaning. The theory of
reference is concerned with the relation between linguistic
expressions and the objects they name or denote, thus the
relation between 'book' and all books (or the class of all
books), between the word 'red' and all red objects (or the
class of these). Truth is a central notion in the theory
of reference.

The distinction between meaning and reference becomes
clear when we recognize that not all words have reference
(e.g., 'and', 'of'), even though they are presumably not
without meaning, and that even among referring expressions,
certain expressions with the same reference may not have the
same meaning. Thus 'centaur' and 'unicorn' have the same
(null) reference, but are not synonyms. And XXX 'elephant'
and 'largest animal found in captivity' are linguistic
expressions with the same reference, but obviously different
meanings. Similarly we note that if a horse dies (or, for that
matter, if all horses die), the meaning of 'horse' has not been
The central concepts of the theory of meaning are synonymy and analyticity and significance or meaningfulness. The word 'meaning' is often used in a broader sense including both meaning and reference. But in discussing the role of semantic notions in linguistic analysis, it is important to keep the distinction clear.

5.3. The linguist who claims that linguistic analysis can be done without meaning must demonstrate that the notions which he wishes to apply in describing languages can be given a completely general and abstract characterization in terms of a set of operational primitive notions, not including 'synonymy', etc. This is a formidable task. Suppose that he fails, conclusively. Does this mean that it has been shown that linguistic analysis must be based on meaning? Not at all. To show this it is necessary to carry out a parallel procedure, constructing a system in which the terms of grammatical description are defined and terms involving meaning do appear as primitives. The immediate availability of such a theory is often, quite mistakenly, taken for granted. The customary challenge "how can you construct a grammar with no appeal to meaning" begs, by implication, an important question in the unspoken assumption that naturally one can construct a grammar with appeal to meaning. But I think an investigation of this latter thesis will show that it is in fact not correct. It is impossible to give a literal demonstration that linguistic analysis cannot be based on semantic notions. But it is both possible and important to investigate in more detail some of the
particular cases where, it is urged, linguistics must (or can) rely on meaning.

It is almost a linguistic cliché, even among those linguists who consciously attempt to avoid meaning in their descriptive work, that in order to construct a phonemic system, while we do not need to know the meaning of expressions, we certainly must know whether or not expressions are different in meaning. (17) Put in this way, this assertion uses "meaning" in the broad sense including both meaning and reference. In the narrower use of "meaning" this claim reads: we do not need to know the reference of linguistic expressions, but only whether or not they are distinct in meaning. But if we know exactly which expressions of a given corpus differ in meaning, we also know exactly which expressions are the same in meaning. To know difference in meaning is also to know synonymy, and this is the central term of the theory of meaning. Hence this claim really reads: to construct a phonemic system, we do not need reference, only meaning. Put in this way, the word 'only' seems out of place. It would be much like saying: to solve this problem we do not need mathematics, only social science. For the theory of reference is certainly in much better shape than the theory of meaning. (18) If accepted then, this claim is an open admission that linguistics must be based on precisely the most dubious part of semantic theory.

Let us put aside the question of whether we can know synonymy relations in some clear and unambiguous way and assume that we have as much knowledge about synonymy and meaning as we please. It is by no means clear that we can
use this knowledge in constructing a phonemic system. If the problem at hand is stated clearly, and if the claim that difference of meaning can be relied on to resolve it is made explicit, then it becomes difficult to see how this claim could be correct.

5.4 We are concerned with one aspect of the fundamental problem of grammar, namely, the problem of determining the subject matter of grammatical description. There are several reasons why the subject matter of grammatical description cannot be identified directly with the linguist's corpus of observed utterances. For one thing, we cannot know, from merely observing the physical properties of observed and recorded utterances, which of these are repetitions of one another, which utterance tokens are instances of the same utterance type. This is what Quine has called the problem of determining the 'thick ness' of the corpus. (19)

Suppose, for instance, that an English corpus contains the three utterance tokens "I found the boy," "I found the boy," and "I found the toy," three distinct physical events. Our problem is to develop an analytic technique that will enable us to determine that the second of these utterances is a mere repetition of the first. If we carry out careful measurements, we find, of course, that the three are distinct from one another in their physical properties. The problem, then, is one of determining which phonetic differences are significant in the language in question, in that they determine non-repetition, or as we will call it, phonemic distinctness. To determine this, it is argued, we must
investigate the difference in meaning between utterance tokens of the corpus. The claim is that we can only find out which pairs of utterance tokens are phonemically distinct (are in contrast, form oppositions) by determining which pairs are different in meaning. Let us make this claim explicit. The strongest form that it could take is this: Given two utterance tokens $U_1$ and $U_2$,

(1) $U_1$ is phonemically distinct from $U_2$ if and only if $U_1$ differs in meaning from $U_2$

For clarity, we may think of the two utterances $U_1$ and $U_2$ as identified with the two pieces of tape $T_1$ and $T_2$ on which they are recorded. Since we are granting the accessibility of all semantic knowledge, let us assume that as $U_1$ and $U_2$ were produced and recorded on $T_1$ and $T_2$, respectively, their meanings and the reference of all their terms were somehow marked as well.

But it seems that because of homonyms on the one hand, and synonyms on the other, this proposed equivalence (1) is simply false in both directions. Let $U_1$ be the utterance "I saw him by the bank," meaning the bank of the Charles River, and let $U_2$ be "I saw him by the bank," meaning i.e., the First National Bank. Clearly the two utterances are different in meaning. Nevertheless they are phonemically identical. Hence (1) is false from right to left. It is not the case that if $U_1$ and $U_2$ differ in meaning, then they must be phonemically distinct. Notice that we cannot appeal here to the fact that these physically distinct utterances "I saw him
by the bank" and "I saw him by the bank" are two occurrences of the same sentence, two tokens of the same type, because the problem at issue is precisely to determine which utterances (i.e., which distinct pieces of tape) are repetitions of one another or tokens of the same type. To make this appeal is thus to beg the question at issue completely.

Now consider the equivalence from left to right. Let \( U_1 \) and \( U_2 \) be any two expressions with the same meaning, e.g., "he is a bachelor" and "he is an unmarried man". Or if one is inclined to deny the existence of absolute synonyms, consider such pairs as /ekonomiks/ and /iykanamiks/ ("economics"), "adult" and "adult", /reysan/ and /reyysan/ ("ration"), /rejiyeytar/ and /reydiyeytar/ ("radiator"), "advertisement" and "advertisement", etc., which often coexist in one person's speech and are clearly synonyms. Such pairs have the same meanings but are phonemically distinct. Hence (1) is false from left to right. It is not the case that if two utterances are phonemically distinct, then they must differ in meaning.

Thus there are clear counter-instances to (1) in both directions. There seems to be no special relation between phonemic distinctness and meaning. We cannot circumvent this argument by holding that this rule (1) holds for all cases except the rather special case of homonyms and synonyms. For one thing these are by no means peripheral cases. For another, homonymy and synonymy are simply the names we give to exceptions to this rule, and any rule works except for its exceptions. It appears than that even if we knew
everything about meaning, we would still know nothing about phonemic distinctness.

Or to put it differently, if we were to classify distinct utterance tokens together just in case they have the same meaning, we would find that the resulting classes do not correspond to utterance types, to classes of repetitions, in the intended sense.

5.5. We have just seen that the phenomena of homonymy and synonymy make it impossible to determine identity of phonemic constitution of utterance tokens in terms of the actual meanings of these tokens as they were produced. We might, however, try to establish a relation between synonymy and phonemic distinctness by reinterpreting the term "meaning" in (1). Let us define the "ambiguous meaning" of an utterance token not as its actual meaning as it was produced, but as the set of meanings that it might have had, produced under any possible circumstances. We might then claim that two homonymous tokens (e.g., an occurrence of "take the latter" and one of "take the ladder") have the same ambiguous meaning, since any meaning that one might have had (taken out of context) the other might have had too, though in fact the actual tokens as produced were quite different in meaning. This suggests an escape from the problem posed for a semantically-oriented approach by homonymity. We can consider rephrasing (1) with distinctness of ambiguous meaning as a sufficient condition for phonemic distinctness. That is, we say that $U_1$ is phonemically distinct from $U_2$ ($T_1$ from $T_2$) if any meaning that $U_1$ could have had, could have also been a meaning of $U_2$.

In this form this revision cannot be taken seriously. It requires that to determine phonemic distinctness, we must not only be able to discover the meanings of tokens, but also that we must be able to analyze the dispositional notion "could have such a meaning" (and that we must do this without reliance on the notion of "repetition" or "utterance type", if we are to avoid circularity). The prospects for a successful analysis of this notion seem so hopeless that we can safely disregard this proposal, as it stands.

We can, however, reformulate this approach in more satisfactory terms, making a weaker claim for the semantic orientation than (1). As we have seen, the attempt to define phonemic distinctness wholly in terms of difference in meaning seems doomed to
failure; but if the initial steps in the analysis of phonemic distinctness are taken on purely phonetic grounds, then the problem takes on a different aspect, and the prospects for a meaning-based analysis become somewhat brighter.

By a system of phonetic transcription we will mean a finite set of symbols, each of which is precisely defined in terms of fixed and distinct physical properties. Suppose that we have such a system. Furthermore, suppose that this system is guaranteed in advance to be more detailed than anything we will ever need in the phonemic analysis of any language. In other words, given this system, any utterance token in any language can be transcribed, in one and only one way; and we are informed in advance that if two utterances are phonemically distinct, then they will be differently transcribed (though not the converse, of course). Naturally, transcription of any one language will reveal an enormous amount of irrelevant detail.

Suppose that we transcribe each utterance token of a corpus in terms of this system, at the same time assigning to it its meaning and the reference of each of its components, which we assume once more to be known unambiguously. But now it will be the case that certain distinct utterance tokens are transcribed in the same way, since only certain distinctions can be recognized by this fixed system of phonetic transcription. In other words, part of the problem of phonemic distinctness is presumed to be solved on an a priori, purely physical basis.

But this slight incursion into the characterization of phonemic distinctness is quite important. It is presumably the case that if two sounds are assigned to different phonemes in a given language, then substitution of one for the other will lead to a difference in meaning in some context. Now that we are assuming part of the problem of phonemic distinctness to be solved on phonetic grounds by this a priori system of phonetic transcription, we can attempt to use this semantic property to complete the characterization of phonemic distinctness. We might now replace (1) by the condition that two differently transcribed tokens are assigned to different phonemes if there are two utterances which differ in meaning, but which are identically transcribed.
except for the transcriptions of the tokens in question. But the problem of homonymy again stands in the way of this formulation. Naturally, many phonemically identical tokens will be differently transcribed. Hence homonyms will often be differently transcribed, and this new condition breaks down just as did the old one. We might revi- vise the proposed condition, then, requiring for phonemic distinctness that every two utterances that differ just in the tokens in question be distinct in meaning. But this will not do, because of the existence of synonyms, for exactly the reasons we discussed above. We must require, then, that there be a sufficient number of pairs of utterances (in some sense which must be defined) which differ in meaning, and are phonetically differentiated just by the tokens in question.

Let us suppose that some more elaborate construction can succeed in avoiding these difficulties, thus providing us with a partially semantic approach to phonemic distinctness. A common attitude towards non-semantic procedures in linguistics is that even if they are possible, they are roundabout and tedious, and they involve an impossible expenditure of effort as contrasted with the presumably simple and direct semantic approaches. But this position can only be maintained when the semantic procedures are left unanalyzed. We will see below that in the case of phonemic distinct- ness, at least, the shoe is on the other foot. There is in fact a thoroughly non- semantic, operational device which does provide a direct and simple approach to the determination of phonemic distinctness. It may be possible, with a sufficiently detailed, logical construction, and with laborious investigation of the meanings of the tokens of an immense corpus transcribed in great phonetic detail, to arrive at the same result in partially semantic terms, though no satisfactory way to do this, even in principle, has yet been advanced.

This discussion of the possibility of a semantic approach to phonemic distinct- ness has been based on the assumption that all semantic information is available, and that it is possible to assign a meaning to each utterance token to be compared with other meanings. But clearly in making this assumption, without which the discussion could not even begin, we have granted far too much. We have not asked how we can
determine whether the meanings assigned to utterance tokens are the same or different. Surely this is a far more difficult problem than the problem of sameness or difference of linguistic forms that we hope to solve in terms of it. Furthermore, there is a conceptual difficulty that seems to undermine the whole approach in a more fundamental manner. We have not dealt with the question of whether the meanings assigned to distinct (but phonemically identical) tokens are identical, or merely very similar. I.e., does "I found the toy" have the same meaning, or only very similar meanings, every time it occurs? If the latter, then all the difficulties of phonemic distinctness are paralleled (and magnified, since now we are dealing with inherently more obscure subject matter) in the determination of sameness of meaning. We must provide a method for determining when two slightly different meanings are sufficiently similar. If, on the other hand, we try to maintain the position that the meanings are identical, that the meaning is a fixed and unchanging component of each occurrence, then a charge of circularity seems warranted. It seems that the only way to uphold such a position would be to conceive of the 'meaning' of an expression (a token) as the way in which tokens of this type are (or can be) properly used, the class of situations in which they could be used, the type of response they normally evoke, the type of change in the organism that they normally induce, or something of this sort. But it is difficult to make any sense at all out of such a conception without the prior establishment of utterance types (we disregard here the difficulty posed by the dispositional formulation). The degree of unclarity in this discussion makes the attempt to define phonemic distinctions in such terms appear rather pointless.

5.6. In the preceding sections we have pointed out that its vagueness and unclarity make the notion of meaning an ineffective criterion of analysis and an extremely dubious foundation for grammatical theory; and that it is difficult to see how it can be employed without a vicious circularity. Furthermore, we have observed that even disregarding these essential difficulties, if we take meanings seriously enough to assign meanings correctly to utterances, we seem to learn very little about phonemic distinctness. It is important to add that when we actually run into a real problem of
establishing phonemic distinctness, we in fact do not rely on meaning in any way.

A typical instance in which it is important to discover whether there is a phonemic difference between two forms is the case of the intervocalic post-stress allophone of /t/ and /d/ in certain English dialects, as in "latter" and "ladder", "rating" and "raiding", etc. If a linguist is interested in determining whether an occurrence of "take the latter" and an occurrence of "take the ladder" are phonemically distinct, there seems to be only one method that he can actually employ in practice, namely the pair test\(^{(24)}\) in one of its forms. To perform this test we take two utterances, let us say recorded on tape and labeled \(U_1\) and \(U_2\), play them repeatedly in random order to an informant, and see whether he can consistently and correctly identify them as \(U_1\) and \(U_2\). Of course we know that "take the latter" and "take the ladder" are different in meaning, but this information seems quite irrelevant to solving the problem at hand. Knowing it, we still must determine whether these forms are phonemically distinct. We are certainly interested in knowing whether the forms differ, but not, apparently, in whether they differ in meaning; rather in whether they differ in phonemic constitution. And even the linguist in the field must employ some such non-semantic device as the pair test to determine this. Only in this way can he find out how "take the latter" and "take the ladder", "go to the bank" (of the river) and "go to the bank" (First National) are related in a given dialect.

There are certain more complicated ways of attempting to establish a relation between meaning and phonemic distinctness that can be used to determine the latter, but I think it can be shown that these simply amount to special variations on the pair test, with a spurious explanation of the results of the test thrown in. The compulsion to give some such explanation seems to arise from a feeling that even if the pair test operationally makes the distinction we require between utterances, it still does not answer the question: in what respect are these utterances distinct? But to this question the answer can only be the tautology: in phonemic constitution. It is not clear what further answer could be required.\(^{(25)}\) There is no more need for a semantic explanation for the fact that "bill" is phonemically identical with "bill" than for the equally indisputable fact that
"bill" rhymes with "pill." If in fact difference in meaning did correspond exactly to distinctness as defined by the pair test, this would be an interesting correlation between independent notions. But since only the latter gives an operational account of the intuitive sense of distinctness of utterances that we are attempting to reconstruct in linguistic theory, only the latter is taken as the test of phonemic distinctness.

With this in mind, we can consider such comments on linguistic method as the following: "The kinds of meanings that the linguist needs can be defined with considerable objectivity in terms of behavioral discriminations. No appeals need be made to subjective intangibles in order to use meaning in a linguistic analysis."(26) It seems to be true that the kinds of reactions to language that the linguist needs can be defined objectively, for instance, by the pair test. But the responses to language marked by such operational devices as the pair test are not meanings. Calling them 'meanings' can only be explained as the result of an all too prevalent compulsion to introduce the word 'meaning' into the statement of linguistic method, no matter what violence is done to the ordinary sense of this term in the process.

The pair test is one of the operational cornerstones for linguistic theory, and as such it deserves much more study and elaboration. Its effectiveness can easily be determined by experiment, and many variations can be developed. Thus suppose that U₁ and U₂ are in free variation, but that the speaker in fact marks the distinction. This is an unlikely circumstance, since untrained speakers do not normally perceive subphonemic distinctions, but may occasionally arise. It is possible in this case to have the speaker make a series of repetitions of U₁ and U₂, and to run through the pair test with these repetitions. (27) If the speaker can consistently distinguish U₁ and U₂, with repetitions and other elaborations, then there is some linguistically relevant distinction between them. What this distinction is and where it lies, of course, the pair test cannot tell us. It is the task of the developed linguistic theory to determine this.

In administering the pair test, instead of asking the informant to identify the utterances as U₁ and U₂, we may ask him to identify them by meaning or reference ("the thing you climb on", "not the former", in the case of "take the ladder", "take the latter").
This has occasionally led to the idea that semantic notions are being 'sneaked in' surreptitiously. But this is a confusion. We might just as well have the informant label the utterances by signs of the zodiac, but this will not indicate that linguistics is based on astrology.

In determining phonemic distinctness by means of such devices as the pair test, we have gone outside of the corpus, and have made use of observations of certain very limited kinds of behavior. The question arises as to whether it is possible to determine phonemic distinctions solely in terms of the physical properties of the corpus. Such an approach would have to be framed in terms of some absolute physical properties of sounds, invariant from language to language, that can always be guaranteed to determine phonemic distinctness and to characterize it fully. It would thus be confronted with the difficulty that languages may differ greatly in the phonetic characteristics that fulfill a distinctive function, and that are consequently 'heard' as distinct by the speakers of these languages. It is well-known that a difference between sounds with a distinctive function in one language may not be perceived by speakers of another language, where this difference does not play a distinctive or phonemic role. We cannot prove that no such absolute and invariant distinctive properties exist—there always exist physical properties we have never thought of measuring. Hence we cannot rule out this approach on any a priori grounds, although it seems quite unlikely that it should actually be possible.

§7. We have seen that when we investigate the formula "not meaning, but only difference of meaning," we discover first that it actually amounts to a very serious reliance on meaning, in fact, on the most dubious part of semantic theory. Secondly, we see that when we try to give it an exact formulation, it seems to have nothing to do with the problem we actually face in linguistic analysis. One way to explain the wide acceptance of this formulation is to assume that there is a confusion here between 'meaning' and 'intuition about linguistic form.' Here, as often is the case elsewhere in the discussion of linguistic theory, the real content of the claim that the establishment of a linguistic element is based on meaning, seems to be that this element must be established on the
basis of intuition. But an intuition about linguistic form is not a meaning. Before we develop an objective and effective technique for constructing a phonemic system, we have only our intuition that "pit" and "pull" begin with the same phoneme, while "pit" and "bit" do not (though these forms have initial articulatory similarity). But there is nothing semantic about this intuition. This confusion of meaning with intuition indicates a failure to take meaning itself very seriously. The only thing that meaning and intuition have in common is their obscurity. Fortunately, in the case of phonemic distinctness the pair test enables us to avoid this reliance on intuition. And the major goal of methodological work in linguistics is to enable us to avoid intuition about linguistic form wherever we find it, replacing it by some explicit and systematic account. If the appeal to meaning proves to be similarly irrelevant at other points in the theory of linguistic form, then we will be entitled to say that this appeal functions as a dangerous bypass, in the sense that it simply indicates lack of interest in the problem of characterizing explicitly the linguistic notion in question—a dangerous bypass, because (as distinct from open recourse to intuition) it gives the illusion of being a real explanation, not merely avoidance of the problem.

Of course, the establishment of distinctive difference between utterances leaves us a long way from a phonemic analysis, and even if it is true that meaning is irrelevant to the determination of linguistically relevant distinction between utterances, this still does not prove that meaning can play no role anywhere in phonemic theory. It is impossible to give a proof for such a negative proposition. It is necessary to analyze each proposal case by case. But once we have recognized that a semantically oriented theory is by no means an immediately available alternative to a non-semantic theory, and that in fact a semantically-based theory must receive a painstaking and elaborate development and must meet the strict criteria of significance that are rightly posed for any other approach, then the semantic orientation loses much of its attractiveness. In place of the customary challenge "how can you carry out linguistic analysis without meaning," it is perfectly proper to ask "how can you carry out linguistic analysis with meaning?" It is not at all evident that there is any way to meet this challenge.
Until we are presented with some interesting and promising way to meet this challenge, we should certainly avoid such statements as "so-and-so is attempting to construct a grammatical theory on a grammar with no recourse to meaning." Even if this is literally true, it is quite misleading, since the implication that there is an alternative has not been demonstrated. Similarly, we can afford to be quite skeptical about the often-voiced claim that even if we can proceed without meaning, it is much easier to proceed with reliance on meaning, using this as a heuristic device to be eliminated in our careful reconstruction and validation of the results of linguistic analysis. It is certainly true that our intuitions about linguistic form may be useful in the actual process of gathering and organizing grammatical data, but this is not to say that our intuitions about meaning serve the same purpose. Whatever meaning is, it certainly is not intuition about form. This point is easily obscured in vague and general statements to the effect that to actually construct a grammar it is useful to know the language under analysis. If 'knowledge' (or understanding) of a language includes knowledge of the grammar, this statement is trivially true. If not, I see no reason to assume that it is true at all.

§ 8. It is difficult to evaluate many other suggestions about the role of meaning in grammar, largely because it is difficult to pin down the notion of meaning. However, I think that within the limits posed by the obscurity of these notions, it is reasonable to suggest 'intuition about linguistic form' as a more proper locution than 'meaning', wherever such suggestions are made.
Quine distinguishes two major notions in the theory of meaning, 'synonymy' and 'significance', and suggests that grammar relies on both for the determination of the subject matter of a linguistic description. The linguist must determine how many distinct forms constitute his corpus, and which forms not in his corpus must be described by the grammar. In the first case, he relies on synonymy (i.e., on difference in meaning), and in the second, on significance (i.e., the grammar must describe exactly the significant or meaningful sentences). The first point we have already discussed.

The grammar must generate a set of grammatical sentences on the basis of a limited corpus: Is it correct to identify 'grammaticalness' with 'significance'? I think that it is not. If we take 'meaningfulness' or 'significance' seriously, I think we must admit that

This is a round square
(2) I noticed a round square
or and
(3) colorless green ideas sleep furiously
are thoroughly meaningless and non-significant, but it seems to me that as a speaker of English, I would regard these as in some sense 'grammatical' sentences, and it can certainly be argued that the establishment of their non-significance falls outside the domain of grammar. On the other hand it hardly seems to the point to suggest that the reason why
(4) my father are here
is not to be generated by the grammar is because it is meaningless, or non-significant in some semantic sense. It
seems that (4) is more correctly described as an utterance that fails to meet certain formal requirements.

The situation we face is roughly this. We know that a speaker of the language, on the basis of a finite linguistic experience, can select, among sequences that he has never heard, certain grammatical sentences, and that he will do this in much the same way as other speakers. We might test this by a direct determination of some sort of 'bizarreness reaction', or in various indirect ways. Note for instance that a speaker of English, given (3), will normally read it with the standard intonation pattern of an English sentence. But given some permutation of the words of (3), e.g., the sequence formed by reading (3) from back to front, (5) furiously sleep ideas green colorless

he will read it with the intonation pattern characteristic of a sequence of unrelated words, each word with a falling intonation. Yet he has presumably never heard either (3) or (5), nor even any of the parts of these sequences, in connected discourse.

How can we describe this ability? The only thing we can say directly is that the speaker has an 'intuitive sense of grammaticalness.' But to say this is simply to state an abstract problem. Suppose that we can (i) construct a linguistic theory in which grammaticalness is defined, (ii) apply this linguistic theory in a rigorous way to a finite sample of linguistic behavior thus generating a set of 'grammatical' sentences, (e.g., 25) and (iii) demonstrate that the set of grammatical
sentences thus generated, in the case of language after language, corresponds to the 'intuitive sense of grammaticalness' of the native speaker. In this case, we will have succeeded in giving a rational account of this behavior, i.e., a theory of the speaker's linguistic intuition. This is the goal of linguistic theory. It is by no means obvious that it can be done, limiting ourselves to formal analysis. It is even more difficult to see how any semantic notion can be of any assistance in this program. In any event, there seems no reason to introduce the notion of 'significance' into this account.\(^{(26)}\) We begin by recognizing the existence of an intuition about linguistic form, which, if I am correct, does not correspond to the intuition about significance. We end, if successful, by giving an objective theory which, in a certain sense, explains this intuition. Before linguistic theory is constructed, the subject matter for linguistic description is determined not by significance and synonymy, but simply by reference to the speaker's intuitions about which forms are grammatical, and which pairs phonemically distinct.

After a successful theory has been constructed, the subject matter for linguistic description is determined by the theory itself. The success of the theory is determined in part by its efficacy in reconstructing intuition. Naturally, if it generates (5) but not (3) as grammatical, when we apply it to English, it will be judged unsuccessful. Insofar as any argument turning on words like 'meaning' and 'intuition' can be convincing, I think it is fair to say that in this case, too, any semantic reference is irrelevant to formulating
or resolving the problem.

If we keep clearly in mind the distinction between the task of constructing a general theory of linguistic structure and that of constructing particular grammars, we can restate Quine's contention that in order to set the grammarian's task, we must solve the "problem of defining the general notion of significant sequence." This statement is correct, where the grammarian's task is understood as the construction of a particular grammar. For this to be a significant project, we need a prior notion of 'grammatical'. But understanding the grammarian's task as the construction of a general theory, the development of this notion is not something needed to set his task, rather it is this task, or an integral part of it. This task is indeed prior to the construction of grammars. But to require that a definition of grammatical sentence precede the task of constructing this general theory would lead to an infinite regress, since, by the same reasoning, we could argue that this prior definition is not significant unless it reconstructs a predetermined class of grammatical sentences. The point is simply that the two-fold program of linguistic research has as distinct but related goals the construction of a general theory, in which terms like 'grammatical' are defined, and the construction of grammars validated by demonstrating that they follow from the theory. This dual program is significantly achieved only when it provides a systematic and integrated account of a good deal of linguistic behavior, an explanation of linguistic intuition, etc.

5.1

5-6. So far we have discussed the role of meaning in the
determination of the subject matter for linguistic analysis. Even if meaning is excluded here, there remains the possibility that specific linguistic concepts are definable in partially semantic terms. This cannot be ruled out in advance.

Nevertheless, I think it is important to note that the correlation between semantic units and the customary units of formal analysis is at best approximate. Thus although segments smaller than morphemes do not ordinarily have independent meaning, this is not always true, as we see, for instance, from the al- and gl- series in English, (39) where few would be inclined to give these elements morphemic status. And any attempt to define morphemes in terms of minimum semantic content with associated phonemic content runs into immediate difficulty when confronted with pairs like "hear" and "ear", "heat" and "eat", etc. (36) On the other hand, there are morphemes that would seem to have no semantic content at all, if 'semantic content' is taken seriously as an independent notion. It is difficult to see why "to" in "I want to go", or "en" in "they have taken it" should be considered to have any independent meaning at all, unless possession of independent meaning is understood as an automatic consequence of morphological status. These are simply forms which must appear in certain constructions, according to the grammatical rules. Similarly, no one would consider "of" to constitute a set of distinct morphemes, despite its many diversified 'meanings'. Recent work in defining the morpheme in formal terms (34) shows considerable promise. I do not think that the same can be said for any attempt at a meaning-based definition. For the present, the most reasonable supposition
seems to be that the morpheme is a purely formal unit, though we have been only partially successful in reconstructing explicitly the particular intuition about forms that enables linguists and native speakers to recognize morpheme boundaries quite consistently. As far as I can see, there are only two ways of explicating this intuition that are at all promising. The more satisfactory of these would be to define 'morpheme' in purely distributional terms, if this can be done. If it cannot, then we can seek to discover some behavioral test (like the pair test) for isolating morphemes. This would mean introducing 'morpheme' as an added primitive in linguistic theory. Since the point of a theory is to achieve the maximum expression of interrelation among the concepts entering into the theory, it is always preferable to keep the number of primitive terms to a minimum. A third approach would be to define morpheme in semantic terms, or partially in semantic terms. There has been little indication that a fruitful approach can be developed in this way, and when we are careful to distinguish 'meaning' from 'intuition about form', I think that much of the a priori likelihood it appears to have tends to vanish.

To mention just one more instance, it might be felt that such general relations between sentences as the active-passive relation have a semantic origin. Since such relations will be a major concern of this study, it may be well to mention that whether or not our formal characterization is judged successful, there is no obvious semantic definition for the active-passive relation. Even the weakest semantic relation,
that of factual equivalence. (38) fails to hold in general between actives and their corresponding passives. This is most clearly the case for sentences containing the "quantificational" words "all", "some", "several", numbers, etc. In order for (6) someone is liked by everybody
to be true, it is necessary (39) that there be some single person whom everybody likes, while

(7) everybody likes someone

may be true even if A and B have no common friends, as long as A likes C, and B likes D, etc. Similarly, "several of them know every European language" may be false, but "every European language is known by several of them" true, if the group in question contains several native speakers from each European speech-community, but no one who knows every European language. Non-quantificational occurrences can also be found where one of an active-passive pair would normally be considered true, and the other false. (39) While such instances do not prove the impossibility of a semantic definition, they do indicate that such a formulation will be no easy task. We will return to the active-passive relation below, where it will appear that it is a special case of a very general structural feature of language.

5.9. We have argued that the appeal to meaning in the determination of grammatical structure is actually a misnomer for the appeal to intuition, and hence is to be avoided. But it is important to distinguish sharply between the appeal to meaning and the study of meaning - must be sharply differentiated from the appeal to meaning. The study of meaning is an essential task.
of linguistics; it is certainly important to find some way of describing language in use. But this is not the study of grammatical structure. When these parallel studies are sufficiently advanced, it will be possible to explore the many indisputable connections between them. Exactly where the boundaries are between these studies, it is not easy to determine. We will see below that as the theory of linguistic form becomes more advanced, it can incorporate some of what might have been thought to belong to the theory of meaning. The important thing to remember in constructing a theory of linguistic form is that no matter what difficulties beset this endeavor, these difficulties in themselves in no way indicate that this theory should be based on meaning, or on any other given notion. It may be the case that a certain basis is too narrow. But to show that some particular notion should be added to this basis, it is necessary to demonstrate that when this notion is added, the difficulties fall away. In the case of meaning, this essential step is generally overlooked. We have seen that in several crucial cases, actually the introduction of semantic considerations seems to be quite beside the point, even when we overlook the difficulty of giving a clear account of meaning. For these reasons, it seems to me that the theory of linguistic form does not have semantic foundations.

At this stage of our knowledge and understanding of linguistic structure there seems to be little point in issuing manifestos for or against the dependence of the theory of linguistic structure on meaning. We can only try to construct
such a theory and investigate its adequacy, a part of this
investigation centering around its clarity and operational
interpretability. If we find that some notion can be
constructed in distributional terms, but can also be constructed,
even more significantly, perhaps in semantic terms, (39) (48)
this should not be taken as an indication that the theory
of form has a semantic basis. More significantly, we can
develop these theories independently, and note a correlation
between them in some larger theory that will integrate the
independently established theories of the form of language
and the use of language. If we find that a certain notion
cannot be given distributional grounds, but can be given clear
semantic grounds, we can either decide that this notion
does not belong to grammar, or that grammar has, in part,
a semantic basis. If we make the former decision in too
many cases, grammar will simply not be an interesting subject.
At present, it seems to me proper to say that whereas we
know of many grammatical notions that have no semantic basis,
we know of none for which a significant and general semantic
analysis is forthcoming. And for the present at least, this
justifies the tentative identification of grammar with
distributional analysis.

6.1. The actual selection of a basis for linguistic theory
depends on what kind of a description of language we intend
to give. We have already mentioned that one fundamental
requirement is that the set of utterance tokens be classified
into a set of utterance types. Thus one of our primitive
notions will be a relation of conformity among utterances,
operationally interpreted by the pair test (which, as we have
seen, can and should be elaborated). For this to be significant, the corpus of linguistic material must be broken up into a set of utterances which can be tested for conformity. Thus we have an initial segmentation of the corpus, which we will henceforth consider as a set of utterance tokens. This segmentation requires an operational interpretation. We will assume further that this initial segmentation occurs at sentence boundaries, though it is possible that the notion of sentence boundary can be derived. Thus we take the corpus to be a set of sentence tokens.

The sentence tokens are further segmented into a succession of discrete units which may be called 'phone tokens.' We might extend the primitive notion of 'conformity' to cover all sequences of one or more phone tokens, deriving the notion of phone type, and phone sequence type. Conformity of utterance tokens will then be a special case of conformity of phone sequence tokens (though we have suggested an operational interpretation only for this special case). (44)

The next step taken in linguistic theory is the development of a phonemic system. The major research in structural linguistics has been devoted to this problem, which lies outside the scope of our investigations. Exactly what foundation should be chosen for linguistic theory naturally depends on which phonemic theory is accepted. With the rise of acoustic phonetics, it seems likely that acoustically defined properties, the pair test, and an operational account of the initial segmentation might provide an objective and empirical foundation for linguistic theory.
6.2. However phonemic theory is constructed, the result of a phonemic analysis can be regarded as a certain system of representing the utterances of the language. We might go on to say that in §6.1 we have described three distinct but interrelated systems for representing the utterances of the corpus, where the corpus is taken as a set of sentence-tokens.

For clarity, we may understand these to be literally a set of tapes $T_1, T_2, \ldots$, though this step is not really necessary. The zero-th level representation of the corpus associates with each $T_i$ a sequence of symbols, each of which may be called a 'phone-token-name'. Each sentence-token $T_i$ has a representation completely distinct from that of every other sentence-token $T_j$. No phone-token-name occurs more than once in this set of representations. The zero-th level representation thus tells us nothing more than how many 'atomic segments' constitute each sentence-token $T_i$, and, perhaps, where the physical boundaries lie in $T_i$.

The first-level representation associates with each $T_i$ a sequence of symbols which we may call 'phone-type-names,' or simply, 'phones.' If a sentence-token $T_i$ is represented on the zero-th level by $n$ distinct symbols, then it is represented on the first level by a sequence of $n$ phones, but these are not necessarily distinct. In fact, two phone tokens are represented by the same phone in the first level representation just in case they conform.

The second level representation associates with each $T_i$ a sequence of symbols which we may call 'phonemes'. Exactly
what are the formal and acoustic (or articulatory) properties of this level, and how it is related to lower level representations, will be determined by the particular phonemic theory that we employ. Among the properties of the phonemic representation will be that two sentence-tokens have the same phonemic representation if, and only if, they conform. And the phonemic representation will be related to the zero and first level representations, for instance, in that there will be no more distinct phonemes than distinct phones for the corpus as a whole; the phone tokens corresponding to a given phoneme will be phonetically similar and in complementary distribution (in senses to be specified by the theory), etc. We may not require that a sentence token represented by a sequence of length \( n \) in terms of phones necessarily be represented by a sequence of length \( n \) in terms of phonemes, but the conditions under which a given phoneme may correspond to a sequence of phones and vice versa must be carefully specified.

Linguistic theory must be set up in such a way as to lead to uniqueness or near uniqueness of phonemic representation. If the structure and the interrelations of these levels of representation are given completely in terms of the primitive notions of the theory, then we have a definition of "phonemic representation". This may be much simpler than a direct definition of phonemes in terms of phone tokens, especially if complex relations between these notions (such as those discussed in the references of fn. 44 or in chap. III, below) are admitted.

6.3 The development of a system of phonemic representation does not conclude the process of linguistic analysis. We also
want to discover the morphemes, words, and phrases of the language, and to determine principles of sentence construction that could hardly be stated directly in terms of phonemes. Instead of giving a direct definition of these further notions, we can continue to construct levels of representation for sentence tokens. A sentence token can be represented as a sequence of phones and as a sequence of phonemes. But it can also be represented as a sequence of morphemes, a sequence of words, and, in various ways, as a sequence of phrases. Thus each sentence token will have associated with it a whole set of representations, each representation being its 'spelling' in terms of elements of one linguistic level. The zero level representation is in terms of phone-token-names. One intermediate representation will be in terms of morphemes. A very high level representation of a sentence token will be its representation by the single symbol 'Sentence'. And we will see below that a sentence token can have an even higher level representation in terms of a sequence of operations by which the sentence is derived from some more basic sentence.

On the lowest levels, up to the phonemic level, two sentence tokens are identically represented if and only if they conform. But on higher levels, conforming tokens may have different representations, and non-conforming tokens may have identical representations. For instance, on the morphemic level the phoneme sequence /ənəm/ may be represented either as "an aim" or "a name", and both of the phoneme sequences /wayf/ ("wife") and /wayv/ (as in "wives") are represented as "wife". But on the morphemic level, it is
still the case that no non-conforming sentence tokens have identical representations, unless cases of phonemic free variation, such as /ekænæmiks/, /ɪkænæmiks/, etc. (cf. above, 45.4), are represented by the same morpheme. On the level of phrase structure, of course, this is the rule. "John ate his lunch" and "Bill read the book" are both instances of the syntactic construction noun phrase-verb phrase, and will be so represented on this level. The case also provides many instances on this level. Thus "old men and women" can be represented as adjective - noun phrase (where the noun phrase is "men and women"), or as noun phrase - and - noun phrase (where the first noun phrase is "old men", and the second is "women"). Thus we can find higher level similarity of construction between phonetically dissimilar tokens, and higher level dissimilarities (alternative analyses) between conforming tokens. The latter case can be termed 'constructional homonymity'. We have already seen instances of this on the levels of morphemes and phrases. Ordinary homonymity is a case of constructional homonymity on the level of syntactic categories. Thus /riyd/ can be either the noun "reed" or the verb "read". We will find that constructional homonymity appears on overlying level we construct, even on the transformational level, with interesting consequences.

A grammar of a language must tell us exactly what are the grammatical sentence tokens, and exactly how these are represented on each level. Just how a grammar provides this information is an important problem, which will occupy us below. Linguistic theory, in this conception, becomes the
theory in which these systems of representation are constructed in an abstract manner, and the relations between them explicitly and abstractly characterized. In the next chapter, we will go into the theory of linear representational systems, or concatenation theory, in more detail. It can be regarded as a simple mathematical theory, borrowed for linguistics. Thus to the proposed basis for linguistic theory we must add terms (e.g., 'precedes') from which concatenation theory can be developed. By studying systems of representation and their interconnections, and attempting to formulate linguistic theory in these terms, we avoid the necessity for giving a very complex account of linguistic elements as classes, sequences of classes, classes of these sequences, etc. The definition of linguistic elements and the discussion of the relations holding between them would soon become unwieldy and inelegant if this direct course were followed, particularly in the light of the conceptions of chapter III.

We find, incidentally, that the suggested approach enables us to bypass certain of the problems of interpretation that arise when linguistic elements of higher levels (e.g., morphemes) are understood as literally constructed out of lower level elements (e.g., phonemes). Thus the verb "walked" is naturally analyzed into the morphemes "walk" and "ed", but there has been considerable discussion about the morphemic analysis of the structurally parallel verb "took." We might say that this form is composed of a discontinuous stem /t..k/ and an infix /u/ (which is an allomorph of the past tense morpheme), or that it consists of a stem /teyk/ and a morpheme "ay — u" which converts "take" into "took", etc. But this problem of determining the "content" of the individual morphemes involved does not arise when we understand the morphological level to be simply a higher level of representation of utterances, connected by certain "morphemic" relations to the phonemic level. On the phonemic level, "walked" is represented by the four-element sequence /w-.a-k-t/, and "took" by the three-element sequence /t-u-k/, where w, a, k, t, and u are indecomposable 'atomic' elements of the phonemic level. On the morphological level, "walked" is represented by the two-element sequence walk-past, and "took" by the two element sequence take-past, where walk, past, and take are indecomposable
atomic elements of the morphological level. The morphophonemic rules determine the phonemic "content" of both of these morphological representations. I.e., we must have rules to the effect that walk-past represents the same utterance as /w-2-k-t/, and that take-past represents the same utterance as /t-u-k/. The significant difference between these rules is that the first would appear as a special case of a more general rule, whereas the second would not. But this is the only interpretation of the facts which is available, in our terms. There is no need to identify part of /tuk/ as 'belonging to' the morpheme take, and part to the morpheme past. In precisely the same way, when we say that the phoneme sequence /t/ plus juncture is carried by the phonetic rules into the single phone [ T ], we need not attempt to specify which part of this phone 'belongs to' the /t/, and which part to the juncture. See also fn.60.

6.4. The motivation behind level construction can be briefly sketched in terms of the two-fold program of, on the one hand, constructing grammars, and on the other, developing a theory of linguistic structure.

In the case of a particular grammar, our first problem is to determine the subject matter of the description, that is, to determine which sequences are grammatical (whether or not they occur in the corpus), and which pairs are distinct. This completed, we want to present a description of this subject matter. Similarly in linguistic theory, we must define 'grammatical sentence' (cf. chapter IV), and characterize distinctness of utterances (by the pair test), and then present a methodology for giving this further description; that, we must construct a precise and general definition for each term entering into it.

A language is an extremely complex system. If we were to attempt to give a direct description of the set of grammatical sequences of phonemes, we would be faced with an immense and unmanageable task. Instead of attempting this directly, we analyze phoneme sequences into morphemes, morpheme sequences into words and phrases, etc., and we state the restrictions on occurrence of these 'larger' elements. We thus rebuild the complexity of this system piecemeal, extracting and separating out the contribution to this complexity of each linguistic level. In linguistic theory, we face the problem of constructing
this system of levels in an abstract manner, in such a way that a simple grammar will result when this complex of abstract structures is given an interpretation in actual linguistic material.

Though reduction of the complexity of grammar can be regarded as the major motivation behind construction of levels, we see that level construction has important secondary effects. Thus it leads to the recognition of higher order similarities between distinct utterances, and higher order dissimilarities (i.e., ambiguity) between conforming utterances. In general, establishment of higher levels permits us to account for these and other intuitions of the native speaker. The ultimate motivation behind the construction of a system of levels, then, is the motivation behind the construction of linguistic theory in the first place. Our aim is to construct simple grammars which explain and ground the linguistic intuitions of the native speaker, and which describe in a systematic way selected aspects of his linguistic behavior. These goals lead us directly to the conception of linguistic theory which we have sketched above, and will elaborate further in the following chapters.
construct simple grammars which explain and ground the linguistic intuitions of the native speaker, and describe in a systematic way selected aspects of his linguistic behavior. These goals lead us directly to the conception of linguistic theory which we have sketched above, and will elaborate further in the following chapters.

6.5. The question of interdependence of levels has been an important one in recent discussions of linguistic method. It has been correctly pointed out that if words or morphemes are defined in terms of certain properties of phoneme sequences, and if, at the same time, certain grammatical information is considered relevant to setting up a correct phonemic solution, then linguistic theory may be nullified by a real circularity. If a term is defined in a theory, then it must be expressible completely in terms of the primitive notions. If words are defined in terms of phonemes, and phonemes in terms of words, then (depending on exactly how the definitions are given) it may be that neither notion can be analyzed in primitive terms, hence that neither notion is properly defined. This must be avoided, but it is not a necessary consequence of interdependence of levels. The situation described above may be met without circularity by defining 'tentative phoneme set' and 'tentative morpheme set' in such a way that various sets meet these definitions; then defining a relation of compatibility between tentative phoneme sets and tentative morpheme sets; and finally, defining the pair of a phonemic analysis and a morphemic analysis as a compatible pair of a tentative phoneme set and a tentative morpheme set. We may attempt to provide a formal
measure of simplicity for grammars, so that part of the compatibility requirement will be the condition that the phonemic and morphemic systems jointly give the simplest grammar.

Putting this differently, according to the conception sketched roughly in the preceding sections, linguistic theory presents an abstract construction of a system of levels of representation related to each other in certain fixed ways, and formulated in terms that are operationally interpretable in a certain specified manner. The actual process of linguistic analysis, then, is the process of finding an interpretation for this system in a given corpus of linguistic material. We insist that the theory be formulated in such a way that each proposed interpretation can be evaluated, in some practical way, so that given two proposed interpretations we can determine which is superior, i.e., which definitely fails to be a correct interpretation by virtue of the fact that it is demonstrably not the best. Unless the abstract system of levels of representation is properly constructed, this program may collapse, in the sense that all grammars (or too many grammars) will be equally good interpretations. But the fact that levels are interrelated, i.e., that the systems of representation must be compatible in certain ways, does not necessarily lead to this result.

We cannot, therefore, rule out interdependence of levels in principle, on grounds of circularity. Instead, any proposed definition of linguistic levels must be tested
for significance, no matter to what extent interdependence of levels appears in it.

2.1. In §2.1 we pointed out that if linguistic research is to have content, there must be certain criteria of adequacy for each grammar, outside of those internal to linguistic theory. But we have said very little about what these criteria may be. Actually, there seems to be very little to say about this that is not uncomfortably vague. Hockett has pointed out \( 56 \) that the sequences generated by the grammar as grammatical sentences must be acceptable, in some sense, to the native speaker, and that the processes described in the grammar must conform somehow to the 'habits' of the native speaker. Above, in §5.2, we construed the linguist's task as one of reconstructing in some systematic and inspectable way the speaker's 'linguistic intuition'; i.e., as the task of giving definitions which, in the case of particular languages, isolate elements which correspond, by and large, to the native speaker's intuitions about grammatical form.

Thus if a theory of grammatical structure applied to English gave \( \text{"green"} \text{ or } \text{"there"} \) as grammatical, we would no doubt reject it as inadequate. Our reaction would be similar if the definition of 'word', applied to English, put only one word boundary in "John finished eating", perhaps between "finish" and "ed", or in some even less natural place; or if 'morpheme' were so defined that "er" in "mother", "father", "brother", "sister" were isolated as a morpheme (despite the common semantic content of these words, which could, with a meaning-bound concept of 'morpheme', be assigned to "er"). The speaker's intuition about form (sometimes misleadingly spoken of as 'semantic') poses, for each language,
conditions that must be met by linguistic theory. And, broadly speaking, we will regard a linguistic theory as successful if it manages to explicate and give formal justification and support for our strong intuitions about linguistic form within the framework of an integrated, systematic, and internally motivated theory.

But 'intuition,' of course, is an extremely weak support. The program of linguistic research would be a much clearer one if we could show experimentally that these intuitions have distinct behavioral correlates. Thus in §5.7, we noted a possible behavioral correlate for certain instances of the presystematic intuitive notion of 'grammatical sentence.' Whenever we have strong intuitions about language, it is reasonable to search from some testable and observable correlate to these intuitions. If we can develop a mass of data about the use and reaction to language, behavioral and otherwise, and independent of the theory of linguistic structure, then we can evaluate this theory not only in terms of its ability to reconstruct intuition about grammatical form, but in terms of its success in providing explanatory support and formal correlates for this independently derived material.

But we must be careful not to exaggerate the extent to which a behavioral reinterpretation of intuition about form will actually clarify the situation. Thus suppose we found some behavioral test corresponding to the analysis of "John finished eating" suggested above, or of "mother", "father," etc., as described above. Or, to choose a more interesting case, suppose that we manage to develop some
operational account (e.g., some effective eliciting technique) for synonymy and significance. As we have seen in 35, if these accounts are successful and to the point, then the notions characterized will not correspond to the notions of identity of phonemic constitution and grammaticalness, respectively, though there will be considerable overlap in each case. Thus it would be an error to suggest reconstruction of the now presumably operationally concept of synonymy as a criterion of adequacy for identity of phonemic constitution, or to generation of the class of significant sentences as a condition for the theoretical reconstruction of 'grammaticalness'. As a final example, consider the notion of order of statistical approximation to language, recently shown by Miller and Selfridge to have interesting behavioral correlates. We might be tempted to identify grammaticalness in English with high order of approximation to English, and non-grammaticalness with low order of approximation. But if we do this, though we will be characterizing something, it will not be grammaticalness, in the pre-systematic sense of this term. Perfectly grammatical English sentences can have a reasonable probability of occurrence only in zero order approximations to English, and as we move to higher orders of approximation, we simply exclude more and more grammatical sentences, while still including vast numbers of ungrammatical sentences. Hence these particular behavioral correlates are apparently not relevant to the characterization of grammaticalness.
It appears then that in a certain sense the ultimate criterion remains the speaker's intuition about linguistic form (and the possibility of giving an interesting and systematic account of it in clear terms), since only this can tell us which behavioral tests are to the point. We might hope that some more general account of the whole process of linguistic communication than we possess now may permit us to reconstruct the criteria of adequacy for linguistic theory in more convincing and acceptable terms. But for the present, it seems that we must rely, at least to some extent, on the speaker's intuitive conception of linguistic form.

The point of departure below will thus often be intuition. As motivation for the elaboration of linguistic theory, we will cite cases where previously constructed theory leads to counter-intuitive results, and will try to develop a natural reformulation which, among other gains, will lead to a significant correspondence with intuition, i.e., will supply formal grounds for intuition. But it is important to keep clearly in mind that this does not mean that linguistic theory itself is based on intuition, that 'intuition' and such notions appear in its basis of primitive terms. On the contrary, this basis is composed of the clearest and most objective notions we can find. Only in this way can linguistic theory serve the purpose of explicating our grammatical intuitions.

2.2. If a linguistic theory dictates solutions for a language
which are counter-intuitive, we have two possible courses of action. We may disregard the intuition, as fallacious (or as an intuition about something other than grammatical form), or we may reconstruct the theory. Between these two poles of reliance on the results of a given theory and reliance on intuition, there are many possible positions and attitudes. In this connection, we can return to the question of whether the linguist 'plays mathematical games' or 'describes reality' (cf., above, p. 53). To the extent that this discussion has any meaning at all, it seems to reduce to the question of where between these poles the proper approach lies. In the absence of clear criteria of adequacy and relevance, behavioral or otherwise, for theories, it is difficult to determine a correct position. The danger in the 'God's truth' approach (as it has been formulated, it sometimes verges on mysticism, ) and tends to blur the fact that the rational way out of this difficulty lies in the program of, on the one hand, formulating behavioral criteria to replace intuitive judgments, and on the other, constructing a rigorous account of linguistic structure and determining its implications for particular grammars.
(1) But note that all utterances of the corpus need not be considered grammatical, just as not every recorded observation need be accepted as a datum. We return to this question below, in chap. IV.

(2) This is essentially the 'operational' form of grammar discussed by Ber-Hillel, A Quasi-arithmetic Notation for Syntactic Description, Language, 1953. Most published grammars are of essentially the first type. Several methods of presenting grammars of this type are discussed by Harris, Methods, 20-3. Cf. Jakobson, Russian Conjugation, Word, 1948, for examples of grammars of this general form.

(3) Cf. chap. IV, also chap. III. It may be worth mentioning that the importance of the vague distinction between a discovery procedure and a practical discovery procedure is not specific to linguistics. One obvious example is in the theory of design of switching circuits, where it is important to find the shortest equivalent of a given truth functional expression. There is always a mechanical method for finding this, but a good deal of effort has been expended in discovering a practical method.

(4) We return to this assumption below, in 4.2.

(5) This definition must be restrictive at least to some extent -- no one confuses language with, say, basket making -- but exactly how much behavior should be called linguistic is a question to be determined in part by the possibilities for constructing an integrated linguistic theory.

(6) If, for some particular linguistic element or relation, it is possible to set up an unambiguous, cross-culturally valid and effective behavioral test, then this element or relation can be characterized in the general theory by this test (i.e., by an appropriate primitive). But even where
elements and relations can be characterized in this way, it is still necessary to carry out theoretical studies in terms of patterning, simplicity, etc. so as to discover how apparently independent notions are interrelated, and to give a rational account of intuitions about reality. It is immaterial whether these tests, where available, are given as criteria of adequacy for defined terms, or as operational accounts of primitive terms of the theory, in which case the relations between these terms will be expressed by theorems. What is important is that the relationships be expressed in the theory.


(8)(p. 14) Of course such a definition may have many cases. It may apply to one language in one way, and another language in a different way. But it must be formulated abstractly if it is to be significant. We must be able to determine somehow from other characteristics of the language, which case of the definition applies. The difficulty of giving an abstract formulation for some criterion suggested for linguistic analysis can be easily underestimated.

(8')(p. 15) It should be noted that there can be no basic quarrel between the linguist who attempts to justify his grammar in this way, and the linguist who is satisfied to present the grammar without any such justification, and to utilize it or explore its implications for other purposes. They are simply interested in different problems.

(9)(p.18)E.g., the distinctions between meaning and reference, use and mention, ordinary and counterfactual conditionals, etc.


(11) See for instance Quine, Two Dogmas of Empiricism, Phil. Review (1951) (reprinted with minor revisions as chap. 2 of From a Logical Point of View) or Hempel, A Logical Appraisal of Operationism, Scientific Monthly, 1954).

(13)(p.20) Quine, From a Logical Point of View, p. 48 (The chapter entitled Meaning and Linguistics). Cf. Goodman, Fact, Fiction, and Forecast for a discussion of the problems touched on in these remarks. As Goodman puts it (p. 58) "Lack of a general theory of goodness does not turn vice into virtue; and lack of a general theory of significance does not turn empty verbiage into illuminating discourse."

(p.23)


(15) An analytic sentence is one which is taken to be true by virtue of the meanings of its component expressions, with no reference to the actual facts of the outside world being necessary, e.g., "every tall man is a man", or "all bachelors are unmarried." A synthetic sentence requires outside support for the determination of its truth or falsity, e.g., "there are more than two billion people in the world." I do not want to imply that this distinction is a clear one. In fact its status is a subject of much discussion in contemporary philosophy of language. Cf. Quine, op. cit., articles by White, Goodman (Likeness of Meaning), Mates in Linsky, ed., Semantics and the Philosophy of Language, and many other articles.

(16)(p.24) This is by no means a clear term—cf. Hempel, op. cit. But we may take it for granted for our purposes.

(17)(p.25) Almost every descriptive linguist concerned with phonemics has on some occasion maintained this position, and this view has been reiterated by representatives of neighboring fields.

(18)(p.25) Though it can be argued that the theory of reference as it exists today offers little help to the linguist given his particular problems.


(20)(p. 39) Cf. Harris, Methods, p. 33, op. cit. (could be p. 1-3)

(21)(p. 36) Other than a physiological or neurological investigation into what occurs when one reacts to a distinctive difference,
Footnotes

(20) This assumption can be rephrased in terms of a fixed set of physical scales, and a notion of clustering of values on these scales, it being guaranteed in advance that if two utterances are phonemically distinct, then they will, at some place, fall into different clusters on at least one of these scales. Cf. chapter V, §39-2, for further elaboration.

(21) Literally, that differ just in tokens with the same transcriptions as the tokens in question.

(22) Namely, the pair test. Cf. Harris, Methods, p.32f. Of course it is an empirical assumption, not a logical necessity, that the pair test and some semantic approach will provide the same analysis of utterance tokens. But it is an empirical assumption which we cannot now verify or disprove, since only one of its components (the pair test) can be evaluated by any operational means that we now possess or envisage.

(23) Note incidentally that in our context this is no stronger than the assumption that we can determine difference in meaning. Given the latter, we could meet the requirements of revision of (1), for example, by ordering the utterance tokens as $X_1, \ldots, X_n$, assigning the number 1 to the first token $X_1$ (as its 'meaning', for our purposes), and assigning the number $q$ to any token $X_i$ in the sequence just in case either (i) there is an earlier token $X_j$ ($j < i$) which is the same in meaning as $X_i$, and to which $q$ has been assigned, or (ii) there is no such $X_j$, and the highest number that has not yet been assigned is $q-1$. 
fn. 24 (cont'd). It might be argued that a semantic alternative to the pair test is to ask the informant whether the utterances in question are the same or different in meaning. It is difficult to comment on this proposal, because it clearly fails the minimal standards for any procedure designed to determine the application of a notion of linguistic theory; that is, this procedure is based not on observation of the speaker's behavior, but on his reports about his behavior. Not only is such information extremely unreliable, but it also leaves us completely in the dark as to the status of the linguistic notion in question. Suppose that this procedure is proposed as the method of applying the notion "phonemically distinct". Has the speaker actually utilized any semantic information in giving his answer? This is a question which we obviously could not hope to answer. Hence this procedure sheds no light on the question of what is the relation between meaning and phonemic distinctness. If we try to introspect, it seems most reasonable to assume that the informant who is asked whether "bank" and "bank" differ in meaning, reinterprets the question as: are "bank" and "bank" the same word? (i.e., are they phonemically distinct?). This then is no procedure at all. It amounts to asking the informant to do the linguist's work, and to do it in a way which is not open to observation. Consider furthermore how the speaker would react if asked whether such phonemically distinct forms as "adult" and "adult", etc., differ in meaning.
or fails to react to a non-distinctive difference. Whether or not this is possible or interesting, it is obviously not within the scope of linguistics.


... not acquainted with any proposal for the use of meaning in phonemics beyond the claim (which, I have argued, is fallacious) that synonymy can be used somehow as a criterion of phonemic distinctness.

(27)(p. 35) We are using the speaker's ability to repeat what he hears in an operational test to determine whether two fixed utterance tokens are repetitions of one another. Note that there is no circularity in this.

See Harris and Voegelin, Eliciting in Linguistics, Southwestern Journal of Anthropology, 1953, for a discussion of the distinction between repetition and imitation, based on observations by Goldstein, Language and Language Disturbances.

(28)(p. 37) op. cit.

(29)(p. 40) I.e., we determine the extension of the predicate 'grammatical' in this interpretation of the theory.

(30)(p. 39) Further evidence that no element of significance is involved here comes from noticing a similar phenomenon on the phonemic level. Here too, we can distinguish 'grammatical nonsense (e.g., "glip") from ungrammatical nonsense (e.g., "lisp"). It seems reasonable to suppose that the processes involved are similar. Schematically put, they have much the same character (cf. , chap. IV, below). But surely no notion of significance is involved in the phonological process. A non-semantic characterization of both processes makes it possible to seek an interesting generalization here.

(31)(p. 34) op. cit., p. 53.

(32)(p. 40) In terms of the logic, not the history of investigation.

(33)(p. 40) Cf. Harris, Methods, p. 177, Bloomfield, Language, p. 156. Cf. Also Jespersen, Language, chap. XX for many further similar instances.
(35) (p. 43) Cf. Bolinger, Complementation should Complement.
(36) (p. 41) Cf. Bolinger, Complementation should Complement.

Studies in Linguistics, 1950, for many such instances.


(32) (p. 42) Cf. in this connection fn. (6), above.

(33) (p. 43) Two sentences are factually equivalent if both are true or both are false. Hence 'factual equivalence' is a notion of the theory of reference. 'Logical equivalence' and stronger notions can be constructed, corresponding to the case where two sentences are 'necessarily' either both true or both false, by virtue of the meanings of their component expressions. Cf. Carnap, Meaning and Necessity. Cf. also L. Scheffler, On Synonymy and Indirect Discourse, Philosophy of Science, for a discussion which, by implication, touches on the linguistic relevance of these further constructions.

(34) (p. 43) More properly, this is the case for one interpretation of (6). There is enough ambiguity in the meaning of all these sentences to make the construction of a clear cut example, positive or negative, quite difficult. This ambiguity is itself a good reason to steer clear of a semantic orientation. We will see below many instances where ambiguity of interpretation is related to ambiguity of formal structure (i.e., constructional homonymy, cf. p. 50), but this is not one of them. Semantic ambiguity with no discernable structural correlate is another of the many problems of the theory of meaning.

(35) (p. 43) E.g., A has unwittingly and unintentionally done something which B took as an insult, it might be correct to say "B was insulted by A", but not "A insulted B".

(36) (p. 43) Just as in the case of grammaticalness (cf. fn. 26), we will see here too that when the problem is framed in non-semantic terms, considerable generalization is possible. The active-passive relation will turn out to be a special instance of a transformational relation, where synonymy obviously could not be offered as an explanation for other instances.
In this connection, it is interesting to note a proposal to incorporate parts of the theory of meaning into the theory of reference. See Goodman, Likeness of Meaning (reprinted in Lingky), More on Likeness of Meaning, Analysis 1953(?) Part of the difficulty with the theory of meaning is that 'meaning' tends to be used as a catch-all term to include every aspect of language that we know very little about. Insofar as this is correct, we can expect various aspects of this theory to be claimed by other approaches to language. (45)

(45) (p. 45) Personal names might be an instance. Suppose we find that we can establish this set on distributional grounds and independently, on semantic grounds (but in the theory of reference, not meaning). The semantic characterization might be a more revealing, in some sense, than the characterization of this class in distributional terms.

(46) (p. 46) For a brief characterization of the extended sense in which I am using this rather vague term, see the last paragraph of chap. III.

(46) (p. 46) An operational interpretation might be suggested for the more general sense of conformity on the basis of the pair test for utterances. Assuming the segmentation into phone-tokens, and taking our utterances as tapes on which utterances are recorded, we can literally substitute a segment of the tape A bearing a phone token sequence a for a certain segment of B bearing a phone token sequence b, yielding a new tape C. a and b will be said to conform if the pair test shows C to be a repetition of a tape which the pair test has already show to be one of the repetitions of B. Conformity of utterances is thus a special case of this operation. If some such approach is used to characterize conformity of phone token sequences, we must be prepared to accept the consequence that allophones of distinct phonemes may conform (cf. Schatz, The Role of Context in the Perception of Stops,
Language, 1954). Naturally, various allophones of one phoneme will not in general conform in this sense. A phonemic theory based on distinctive features will not require this extension of conformity, since 'phonetic similarity' will be determined in terms of a fixed set of phonemic features, acoustically defined in the general theory, and available, presumably, for all languages, hence constituting a part of the definition of 'language'.


(p. 47) A phone-token-name, taken in this way, is a symbol in the metalanguage to the language being described. It denotes (or represents) a certain small segment of L, namely, a segment that we called a phone token in §6.1. At this point, there seems to be no point in thus shifting all notions up one language level, but we see directly that this leads to a simpler and more unified linguistic theory.

(p. 48) Cf. Harris, Methods, §5.2, §9.21 for discussion of such problems.

(p. 49) That these are sometimes pronounced differently is irrelevant. The example is valid if in some instances they conform.

(p. 50) Insofar as it has syntactic correlates, i.e., the distinction between "read" and "reed", but perhaps not between "sun" and "son".

(p. 51) And in principle, we must provide operational interpretations for these, though this task can safely be
disregarded, as trivial.

Cf. Hockett, "Language," (p. 51) and Hoehn, "Language," (p. 53) for a similar approach to linguistic levels.

Cf. Bar-Hillel, On Recursive Definitions in Science, XI Congress of Philosophy, 1953, and Logical Syntax and Semantics, Language, 1954, for a discussion of the possibility of using non-circular recursive definitions in linguistics. My own feeling is that Bar-Hillel is much too optimistic about the possibility of using recursive definitions to reconstruct the kind of interplay between levels that we find in linguistic analysis. And if we limit our goals to the establishment of a practical evaluation procedure, rather than a practical discovery procedure, then the method described here will avoid circularity with only direct definitions.

Review of Recherches Structurales, IJAL, 1952, (p.53) For instance, Mandlebrot's work (cf. Structure formelle des textes et communication: deux études, Word, 1954, and references cited there—also Miller, Communication) is an important case of independent material which can serve as criterion of adequacy for linguistic theory. From general and intuitively appealing considerations, Mandlebrot derives certain statistical conditions which elements of a well constructed communication system might be expected to meet, and shows that words in many languages meet these conditions. If we can give a definition of 'word' in linguistic theory in such a way that in particular languages, the elements defined to be words are distributed in texts with the relative frequencies required by Mandlebrot's 'canonical law', then this can certainly be taken as a significant indication of the adequacy of the construction.

(59) We have mentioned above (42.1) that even a slight reliance on the clear cases may impose stringent limitations on the construction of linguistic theory, since this theory must be adequate to provide grammars meeting these relatively clear conditions for many languages. For further discussion of this problem, often referred to as the problem of the psychological reality of linguistic elements, cf. *Empirical Linguistics* by Sapir, *La réalité psychologique des phonèmes*, translated in *The Selected Writings of Edward Sapir*, Mandelbaum, ed. Also Trubetzkoy,..., Harris and Voegelin, *Eliciting*, Harris, Distributional Structure. Aside from correspondence to intuition, we have other (equally vague) criteria that grammars and linguistic theory, like any scientific theory, must meet, e.g., simplicity, naturalness, etc. Cf. Chap. III, especially 41.2.

(60)(p.59) As the alternative to 'playing mathematical games' has been styled by Householder, *Review of Harris, Methods, IJAL*, 1952. *End of next page*
fn. 60 Cont'd. There are other consequences of the formulation of the "God's truth" - "hocus-pocus" dichotomy that appear to me equally unfortunate. There is apparently a strong temptation to label as hocus-pocus views to which one is opposed on some grounds, usually unstated. E.g. e.g., Hockett in Manual refers to Harris' treatment of juncture as "most misleading and unfortunate hocus-pocus", because Harris states that each juncture is itself phonetically zero, functioning solely as an environment for other phonemes which undergo various changes in the environment thus constructed. It appears to me to be a perfectly clear and well-defined interpretation of juncture, and it is a perfectly natural interpretation, given the conception of levels sketch above, and developed by Hockett himself in Manual, p. (Furthermore, I do not see how it differs significantly from the conception that Hockett advances). Junctures are regarded as elements of the phonemic representation. Each juncture is itself correlated with the zero element of the lower level phonetic representation, and the sequences X-juncture (for various phonemes X) are correlated with various pre-junctural allophones of the phoneme X in question. One may or may not approve of this characterization, but it certainly cannot be accused of obscurity or "hocus-pocus" in any sense. A characterization in the general theory can be criticized on the basis of the results to which it leads, or the nature of its formulation. Since this characterization presumably leads to the same results as Hockett's, and since it is formulated in a perfectly clear and logical fashion, I fail to see any basis for the allegation that hocus-pocus is involved, and that it fails does not meet the standards of the so-called "God's truth" approach (nor does Hockett show how this judgment arises). There are many similar cases in the literature. I mention this not to defend one or another interpretation of juncture, but to cite what seems to me an unfortunate tendency in the discussion of linguistic theory.

(61)(p. 59) Thus the linguist is enjoined not to be misled by theories, but just to describe 'the facts,' the 'real structure' of language, etc.
Chapter II. Linguistic Levels

§ 1. In considering the nature of linguistic theory, we have been led to regard the theory of linguistic structure as being, essentially, the abstract study of 'levels of representation.' Before going on to develop certain of the specific structures of linguistic theory, we will investigate the general nature of such systems of representation, and we will develop some simple ideas that will be useful later on.

A linguistic level is a system \( \mathbb{L} \) in which we construct unidimensional representations of utterances. Thus a level has a certain fixed and finite 'alphabet' of indecomposable atomic elements, which we will call its 'primes'.

Given two primes of \( \mathbb{L} \), we can form a new element of \( \mathbb{L} \) by an operation called 'concatenation', symbolized by the arch ' \( \cdot \) '. Thus if \( \alpha \) and \( \beta \) are primes of \( \mathbb{L} \), we can form \( \alpha \cdot \beta \) and \( \beta \cdot \alpha \) as new elements of \( \mathbb{L} \). Concatenation is essentially the process of 'spelling', where primes are taken as 'letters'. Given the element \( \alpha \cdot \beta \) and the prime \( \gamma \), we can form a new element by concatenating \( \alpha \cdot \beta \) with \( \gamma \). This element is denoted \( \gamma (\alpha \cdot \beta) \). It is exactly the element formed by concatenating the prime \( \gamma \) and the element \( \alpha \cdot \beta \). That is, \( \gamma (\alpha \cdot \beta) = (\alpha \cdot \beta) \gamma \). "\( \alpha (\beta \cdot \gamma) \)" and "\( (\alpha \cdot \beta) \gamma \)" are names for the same element. Because of this property of associativity of concatenation, we can drop parentheses, and write this element as \( \alpha \cdot \beta \cdot \gamma \). E.g., on the phonemic level in English we have the primes \( p, i, \) and \( n \), and we can form the compound elements \( p \cdot i, i \cdot p, n \cdot i \), etc. Concatenating \( p \cdot i \) with \( n \) we form \( (p \cdot i) \cdot n \), which is identical with the element formed by concatenating \( p \) with \( i \cdot n \). This can thus be written unambiguously as \( p \cdot i \cdot n \), the representation on this level of the word "pin".

In general, given two elements \( \alpha \) and \( \beta \) of \( \mathbb{L} \), whether primes or not, we can form by concatenation two new elements \( \alpha \cdot \beta \) and \( \beta \cdot \alpha \), and concatenation is associative for such compound elements. The elements of \( \mathbb{L} \) will be called strings (2)

It is convenient to assume that the system \( \mathbb{L} \) contains an identity element.
which, when concatenated with any string \( x \), yields again the string \( x \). We will call this element, which is unique, the unit \( \mathbf{u} \) of \( L \). Thus for any string \( x \),

\[ x \mathbf{u} = \mathbf{u} x = x \]

We will see that the postulation of a unit element in each level greatly simplifies the construction of levels and the development of the formal relations between them.

The unit element \( \mathbf{u} \) of \( L \) must, however, not be confused with any so-called 'zero elements' which may be set up as primes of \( L \), and for which we use the symbols \( \mathbf{q}_1 \), \( \mathbf{q}_2 \), ... (\( \mathbf{u} \) will not be considered a prime, but of course it is also not a compound string). Zero morphemes, etc., are quite familiar in linguistic work. We can characterize a zero element of the level \( L \) as a prime of \( L \) which happens to correspond to the unit element of some lower level. For example, consider the two levels \( P_m \) and \( M \), the phonemic and the morphemic levels, respectively. We can 'spell' a certain utterance in terms of phonemes or in terms of morphemes; i.e., we can associate with this utterance a certain string in \( P_m \) and a certain string in \( M \), as in the final paragraph of \( \text{§} 2.2 \). But these strings are related. The 'morphophonemic rules' tell us which strings of phonemes corresponds to a given string of morphemes. We can thus define a 'mapping' on \( M \) which carries any string in \( M \) into a string in \( P_m \). This morphophonemic mapping will tell us how to specify a morpheme in a given context as a string of phonemes.

If a certain morpheme is mapped into the unit \( \mathbf{u} \) of the phonemic level, i.e., if it disappears on this lower level, then we call this morpheme a 'zero morpheme', and we write it with the symbol \( \mathbf{q}_1 \), for some \( i \). We might say, then, that the element \( \mathbf{q}_1 \) has real morphemic content, but no phonemic content. Thus a string of a zero morphemes has a perfectly clear meaning. Zero morphemes cannot be added or dropped at will in representations in the level \( M \), and they are established on the same grounds as other morphemes.

\[ \text{§2.} \quad \text{To recapitulate more formally, a linguistic level } L \text{ has} \]
the basic form of a concatenation algebra, where a concatenation algebra $\mathcal{G}$ is a

\[ \mathcal{G} = \langle \mathcal{O}, \preceq, = \rangle, \quad \mathcal{O} \]

being a class of elements, \preceq a binary operation, and

$= a binary relation satisfying the following system of axioms.

**Ax.1.** $= is an equivalence relation.

**Ax.2.** If $a, b, c, d \in \mathcal{O}$, and $a = b$ and $c = d$, then $a \circ c = b \circ d$.

**Ax.3.** If $a, b \in \mathcal{O}$, then $a \circ b$ is a uniquely determined element of $\mathcal{O}$.

**Ax.4.** If $a, b, c \in \mathcal{O}$, then $a \circ (b \circ c) = (a \circ b) \circ c$.

**Ax.5.** $= = \preceq$, and $a \circ b = a \circ c$ or $b \circ a = c \circ a$, then $b = c$.

**Ax.6.** If $= = \preceq$, and $a \circ b = c \circ d$, then there is an $x$ such that either

$a \circ x = c$ or $c \circ x = a$.

**Ax.7.** There is an $x$ in $\mathcal{O}$ such that for all $y$ in $\mathcal{O}$, $a \circ x = y \circ x$.

[It follows that this element is unique. We will designate it '$U$', the
ut element of $\mathcal{O}$.]

**Ax.8.** There is an $x$ in $\mathcal{O}$ such that if $x \circ y = p$, then $x \circ y$ and $x \circ y$.

[An element $p$ with this property will be called a prime. Thus $U$ is not

a prime. The class of primes of $\mathcal{G}$ we denote '$\mathcal{P}$'.]

**Ax.9.** If $a$ is in $\mathcal{O}$, $a \not\approx U$, then there is a $p, q, \equiv x, y$ in $\mathcal{O}$ such that $p$ and $q$ are

primes and $a \circ p \equiv x \equiv y \circ q$.

**Def.1.** '$a \preceq b'$ is if there is an $x = y$ in $\mathcal{O}$ such that $x \preceq a \preceq x \preceq b$, and $x \not\approx U$ or $x \not\approx U$.

[i.e., $a$ is a proper substring of $b$]

**Ax.10.** There is no infinite sequence $\langle a_n \rangle$ of elements of $\mathcal{O}$ such that for all $n$, $a_{n+1} \preceq a_n$.

**Ax.10** is called by Rosenbloom the descending chain.
condition: It follows that each element of $\mathcal{C}$ has a unique factorization into primes; i.e., a unique spelling in terms of the 'alphabet' of primes. We will usually characterize a concatenation algebra $\mathcal{G}$ as a triple $[\mathcal{G}, \cdot, \cdot]$, where $\mathcal{G}$ is the set of primes.

10. A linguistic level is not determined completely by the statement that it is a concatenation algebra. We must also specify its relations to other levels (i.e., the conditions of compatibility between levels, e.g., $\mathbb{I} \cdot \mathbb{I}$) and there may be further algebraic properties within the given level.

We have already noted that a grammar of a language must state the structure of each grammatical utterance of the language on each linguistic level. In carrying out linguistic analysis, then, we must construct on each level $\mathbb{L}$ a set of elements (which we will call "$\mathbb{L}$-markers"), one of which is assigned to each grammatical utterance. The $\mathbb{L}$-marker of a given utterance $T$ must contain within it all information as to the structure of $T$ on the level $\mathbb{L}$. The construction of $\mathbb{L}$-markers, for each level $\mathbb{L}$, is thus the fundamental task in linguistic analysis, and in the abstract construction of linguistic theory we must determine what sort of elements appear as markers on each level. In the case of most levels, markers will be simply strings. Thus on the level $\mathbb{P}$ of phonemes, each grammatical utterance will be represented as a string of phonemes; the $\mathbb{P}$-marker of "pin" in English will be just the string $p^i\cdot n$. However, markers are not always simply strings. We will see below in chapter VI that it is necessary on the level $\mathbb{P}$ of phrase structure to take as $\mathbb{P}$-markers certain sets of strings in $\mathbb{E}$, if we are to provide complete information about the structure of each utterance on this level.
In addition to $\bar{L}$-markers, we may want to define other elements in $L$ as well as to state various relations among the elements of this level, thus extending the algebra of $\bar{L}$. The relations between $\bar{L}$ and other levels can be presented as a set of mappings defined on $\bar{L}$. One mapping of particular importance carries the $\bar{L}$-markers into grammatical utterances. This mapping we will denote "$\Phi$". $\Phi$ thus assigns $\bar{L}$-markers to utterances of the language (though $\Phi$ may apply to elements other than $\bar{L}$-markers as well, i.e., $\bar{L}$-markers may be only a proper part of its domain). By associating an $\bar{L}$-marker with an utterance $T$, the mapping $\Phi$ specifies the structure of $T$ on the level $\bar{L}$, since $\bar{L}$-markers will be designed in such a way as to contain within them all structural information about the utterances to which they are associated by $\Phi$.

In terms of the mapping $\Phi$ and the notion of $\bar{L}$-marker, we can give a clear and general sense to the important notion of *constructional homonymity* (cf. above, \(H_0.2\)). We have a case of constructional homonymity on the level $\bar{L}$ when the mapping $\Phi$ assigns two or more $\bar{L}$-markers to a single utterance (more explicitly, to a set of conforming utterances). This utterance then falls in the overlap of two distinct patterns, and if our theory is adequate, such utterances should be, intuitively, cases of structural ambiguity.

We need not require that the values of $\Phi$...
exhaust the set of grammatical utterances. Thus, some grammatical utterances may simply not be represented on a certain level. Below, we will see the usefulness of constructing the level of phrase structure in such a way that certain utterances have no markers on this level. (5)

In very general terms, then, a level \( L \) is a system

\[
L = \left[ L, ^*, R_1, \ldots, R_m, ^*, \Phi, \phi_1, \ldots, \phi_n \right]
\]

where

(i) \( L \) is a concatenation algebra with \( L \) as its set of primes.

(ii) \( R_1, \ldots, R_m \) are properties and relations defined within \( L \). \( R_1 \) is the identity relation =.

(iii) \( ^* \) is the set of \( L \)-markers --- objects of some sort constructed in \( L \).

(iv) \( \Phi \) is a mapping which, in particular, maps \( ^* \) into the set of grammatical utterances. We can regard \( \Phi \) as a single-valued mapping into the set of equivalence classes of conforming utterance tokens. (6)

(v) \( \phi_1, \ldots, \phi_n \) are relations between \( L \) and other levels.

We will use subscript and superscript to indicate to which level \( R_1, ^*, \Phi, \phi_1, \phi_n \) belong, when several levels have been established.

In linguistic theory, then, we construct a set of levels of
the general form (2). The notions developed on each level must be
fully characterized in terms of the primitive basis of linguistic
theory. The resulting system of levels can be understood as
offering a definition of 'language',(7) and the operational tests
for the primitives delimit the area of behavior that, in terms of
the theory, can properly be called linguistic.

11.2. It is interesting to inquire into the question of how
closely we can specify the primes of the various levels, within
general linguistic theory. The program of developing a general
linguistic theory is reminiscent, in certain respects, of much
erlier attempts to develop a universal grammar. These attempts
have generally concerned themselves with such questions as whether the
categories of Noun, Verb, etc. are universally applicable. We can
restate this as the question of whether it is possible, within
general linguistic theory, to construct a fixed set of elements
which must (or may) appear as primes on some level in the interpretation
of the theory for every language.

For example, in the abstract development of the level of
syntactic categories, we might attempt to actually define "Noun",
"Verb", etc., as primes of this level, fixed elements that may
occur in the description of many languages. Or we may simply define
"syntactic category" in such a way that the nouns of English, for
instance, turn out to be constitute a single syntactic category,
though there is no way of associating this syntactic category with
some category in some other language which we might also like to
consider to be nouns. The former solution, with the actual
construction of a fixed prime which can appear in the grammatical
description of various languages, is, of course, a much more powerful
and difficult one, if it is at all possible. In our discussion of
syntactic categories and phrase structure we will not be able to
approach such a construction. We will merely suggest certain formal conditions that the primes of these levels must meet, and we will make no attempt to construct primes with, in some sense, a fixed 'content' for all languages. It is not clear what sort of basis of primitive notions would be required for this much more ambitious undertaking. Certainly such constructions would require a much richer primitive basis than anything we will consider. In the discussion of transformational structure, however, we will be able to give a much more concrete interpretation of certain of the primes of this level, and it will make sense to ask, in some cases, whether distinct languages have the same transformations.

12. Suppose that we have, as primes of the word level \( \mathcal{W} \) for English, the elements "New", "York", "City", "is", "in", "State". Then we can form various strings by concatenation of these \( \mathcal{W} \) elements, e.g., (8)

\[
(3) \enspace \text{New}^\wedge \text{City}^\wedge \text{is}^\wedge \text{in}^\wedge \text{New}^\wedge \text{York}^\wedge \text{State}.
\]

(3) is a string in \( \mathcal{W} \), in fact a \( \mathcal{W} \)-marker of the utterance "New York City is in New York State," since on the level \( \mathcal{W} \), markers will be simply strings in \( \mathcal{W} \).

For many purposes we need a notion of 'occurrence', which will enable us to refer unambiguously to the second occurrence of "New" in (3), etc. Using a device of Quine's(9) we can identify an occurrence of a prime \( \mathcal{X} \) in a string \( \mathcal{Y} \) as that initial substring of \( \mathcal{Y} \) that ends in \( \mathcal{X} \). Thus the second occurrence of "York" in (3) would be, literally, the string

\[
(4) \enspace \text{New}^\wedge \text{York}^\wedge \text{City}^\wedge \text{is}^\wedge \text{in}^\wedge \text{New}^\wedge \text{York}
\]

and the first occurrence of "York" in (3) would be the string

\[
(5) \enspace \text{New}^\wedge \text{York}.
\]
occurrence of "York" in (3) would be literally, the string

New York City is in New York,

We will also have to speak of the occurrences of non-primes, and it is convenient to construe as being, simultaneously, an occurrence of York, New York, in New York, ..., and of itself.

Def. 2. \( Z \) is an occurrence of \( X \) in \( Y \) if there is a \( W_1, W_2 \) s.t.

\[ Y = W_1 X W_2 = Z W_2 \]

Any of these elements may be unit elements. If \( W_1 = U \), then \( Z = X \), that is, \( X \) is an initial substring of \( Y \), and is thus an occurrence of itself. If \( X = U \), then \( W_1 = Z \). Thus any initial substring of \( Y \) is an occurrence of \( U \) in \( Y \).

Though this method of introducing the notion of occurrence is somewhat artificial, it appears to be adequate for our purposes. A different approach would be to develop an inscriptionsal concatenation theory, in which any word would be considered as a string of the eight distinct primes, each a unique occurrence of itself. One of the relations on each level would then be the relation of conformity, holding, for instance, between the first and the sixth primes of (3). While this approach is more natural, it requires somewhat more elaborate constructions, and for this reason, we have not followed it in this study.

13.1. Substitutability is the major distributional relation that will concern us below. Given a string \( X \), and an occurrence \( Z \) of \( W \) in \( Y \), we will often require a notation for the string that differs from \( X \) only in that \( Y \) is substituted
for \( W \). We will denote this new string by \( \bar{X} \) the symbol \( \bar{A} = \bar{(Y/W,Z)} \.)

**Def. 3.** \( X(Y/W,Z) \) is the string \( \bar{X} \) such that either: for some \( \bar{X}_1, \bar{X}_2 \):

1. \( \bar{X} = \bar{X}_1 \uparrow Y \updownarrow \bar{X}_2 = \bar{X}_2 \updownarrow \bar{X}_2 \)
2. \( \bar{X} = \bar{X}_1 \uparrow Y \updownarrow \bar{X}_2 \)

or: there is no \( \bar{X}_1, \bar{X}_2 \) such that (i).

Thus if \( W \) actually is part of \( X \) (occurs in \( X \)), then \( Z \) is an occurrence of \( W \) in \( X \) and \( Z = \bar{Z}_1 \uparrow W \) and \( \bar{Z}_1 \updownarrow Y \) is an occurrence of \( Y \) in \( \bar{X} = X(Y/W,Z) \). If \( W \) does not occur in \( X \), then \( X(Y/W,Z) = X \). If, for example, \( X = (3) \), \( W = \text{New York} \), \( Z = (4) \), and \( Y = \text{an Eastern} \), then \( X(Y/W,Z) \) will be

(6) \text{New York City is in an Eastern State.}
for \( W \), we will denote this new string by \( X(Y/W,Z) \).

\[ X(Y/W,Z) \] is the string \( X \) such that, for some \( Z_1, Z_2 \),

1. \( X = Z_1 \wedge W \land Z_2 = Z_2 \)
2. \( X = Z_1 \wedge Y \land Z_2 \) or \( \exists 1 \leq i \leq n \) s.t. \( Z_i = X \)

Thus if \( X = (3) \), \( W = \text{New York} \), \( Z = (8) \), and \( Y = \text{an Eastern} \),
then \( X(Y/W,Z) \) will be

\[ \text{New York City is in an Eastern State} \]

This notation can be extended to cover a more general case of substitution. Given a string \( X \), and an occurrence \( Z_1 \) of \( W_1 \), \( Z_2 \) of \( W_2, \ldots, Z_n \) of \( W_n \) in \( X \), we will denote by

\[ X(Y_1/W_1, Z_1; Y_2/W_2, Z_2; \ldots; Y_n/W_n, Z_n) \]

the string formed by replacing \( W_i \) by \( Y_i \), where \( W_i \) precedes \( W_{i+1} \).

\[ X(Y_1/W_1, Z_1; \ldots; Y_n/W_n, Z_n) \] is the string \( X \) s.t.

1. \( \exists 1 \leq i \leq n - 1 \) s.t. \( Z_i \wedge V_i \wedge W_{i+1} = Z_{i+1} \), for \( 0 \leq i \leq n - 1 \),
   where \( Z_0 = Y_0 \) and \( Z_i \) is an occurrence of \( W_i \) in \( X \)
2. \( X_1, \ldots, X_n \) s.t.
   a. \( X_1 = X(Y_n/W_n, Z_n) \)
   b. for \( i > 1 \), \( X_i = X_{i-1}(Y_{n-i+1}/W_{n-i+1}, Z_{n-i+1}) \)
   c. \( X = X_n \)

\[ X = \cdots \wedge W_1 \wedge \cdots \wedge W_{n-2} \wedge Y_{n-1} \wedge \cdots \wedge Y_n \cdots, \] etc., until finally,
\bar{X} = x_1 \cdots x_2 \cdots x_{n-1} \cdots x_n \quad \text{The special form of this definition is dictated by the manner in which we have defined 'occurrence'.}

Suppose that \(X = (3), X_1 = Y_2 = Z_1 = \text{New York}, Z_2 = (4), x_1 = \text{the largest.} \) Then

\(X(x_1/Y_1, Z_1; x_2/Y_2, Z_2)\) is

(8) the largest city is in an Eastern State

and the intermediate step \(x_1\) is (6).

14. In the discussion above, then, we have given no actual content to the elements or relations of any linguistic level, but have left the matter open for specific constructions. The requirements that a level must meet are quite broad, and we are free to develop the formal structure of the various levels in many different ways.

In particular, though the \textit{unidimensional} representations that we construct we have not required \textit{unidimensional} on any linguistic level are unidimensional, there is no reason that left to right order of representation correspond directly to temporal order in the represented element. Discontinuous elements provide a common \textit{mix} instance (though not the only one) of non-correspondence between order of
\( \bar{X} = X_n \cdots X_1 \cdots X_2 \cdots \cdots X_{n-1} \cdots X_1 \cdots \)

The special form of this definition is dictated by the manner in which we have defined 'occurrence'.

Suppose that \( \bar{X} = (3), \ W_1 = W_2 = Z_1 = \text{New York}, \ Z_2 = (4), \ Y_1 = \text{the largest}. \) Then \( \bar{X} (Y_1, M_1; Z_1; Y_2, M_2; Z_2) \) is \( (3) \) the 'largest city is in an Eastern State and the intermediate step \( \bar{X}_1 \) is \( (4) \).

The systems of representation that we construct are unidimensional, but there is no reason for left to right order of representation to correspond directly to temporal order in the represented element. Discontinuous elements provide a common instance (though not the only one) of non-correspondence between order of representation and temporal order. Thus Semitic stems are composed of discontinuous roots and vowel patterns (e.g., Hebrew "yeled" is broken down into two morphemes, the root "y..l..d" and the vowel pattern "...e..e.." \( \Rightarrow \) Appendix to chap. 5, below), but they can be represented quite adequately as having the order root-vowel pattern (or vice versa). Any other cases of temporal discontinuity of elements can be handled easily. On the other hand, the general case of discontinuity presents problems. By accepting a linear system of representation, we rule out the possibility of certain kinds of discontinuity. If more general kinds of discontinuity than we can handle occur in language, a more complicated theory of representations will be necessary.

The question of left-to-right and temporal correspondence can be rephrased in this way. Suppose that on the level \( L \),
L-markers are simply certain strings in \( L \), and one of these markers is the string \( X = X_1 \wedge X_2 \wedge \ldots \wedge X_n \) (\( X_i \) a prime). This marker can be taken as denoting a certain set of grammatical utterances (perhaps only one), where \( \Phi \) (cf. §10, above) is understood as the relation of denotation. Let \( T_1 \) be one of the denoted utterances. \( \Phi \) is defined in such a way that each prime of \( X \) denotes a certain segment of \( T_1 \). We might expect \( X_1 \) to denote an initial segment, \( X_2 \) the following segment, \( \ldots \), \( X_n \) the final segment, where these segments do not overlap. But there is nothing in our definition of levels that necessitates such an interpretation. That is, we have not required that \( \Phi \) be an order-preserving mapping. As long as no such requirement is laid down, we are free to regard as a linguistic level any structure that has the formal properties of \( (\text{I}) \). We will make use of this freedom below to suggest transformational analysis as an additional level of linguistic structure. Here, the order of representation bears no relation whatever to temporal order, though the system can be interpreted as a concatenation algebra, and it appears to me that there is good motivation for doing so. For instance, we will see that this leads to a natural and effective extension of the notion of constructional homonymy. It would be possible to require as a condition on levels, that \( \Phi \) be order-preserving, or partially so, but I see no motivation for this, and in view of the existence of discontinuous elements (and the desirability of taking markers as elements other than strings on some levels) it might be difficult to formulate such a requirement.
Footnotes—Chap. II

(1) (p. 60) The particular form of these constructions was suggested by an unpublished paper of Henry Hiż, entitled "Positional Algebras and Structural Linguistics". In the details of notation and axiomatization, we generally follow P. Rosenbloom, Elements of Mathematical Logic, Appendix 2, The Algebraic Approach to Language.

(2) A string is regarded as a single 'object', as distinct from a sequence. Thus the string \( X_1^1 X_2^2 \ldots X_n^n \) must be sharply distinguished from the sequence \( X_1, \ldots, X_n \), or the sequence \( X_1^1 X_2^2, X_3^3 X_4^4, \ldots, X_{n-1}^{n-1} X_n^n \), etc. We might develop concatenation theory differently without making this distinction. This would give a more economical theory, but would require certain artificial devices (cf. Quine, Mathematical Logic, §5).

(3) (p. 61) The distinction between these representations is sometimes confused by the fact that morphemes are generally given in phonemic spelling.

(4) So far we have provided only one way of constructing elements in \( L \), namely, concatenation. But we will assume with no further question (or mention) that each level includes a full set theory, so that we can also form sets of strings, sequences of strings, etc. This assumption may be dispensable, but it simplifies our constructional task (or at least the exposition of it). Quine, Mathematical Logic, ch. 7, for general background on various kinds of concatenation theory. Chomsky, Systems of Syntactic Analysis, for discussion of the possibility of constructing linguistic theory with very meager formal apparatus, using many devices developed by Goodman (Cf. The Structure of Appearance) and Goodman and Quine (Cf. Steps towards a Constructive Nominalism, Journal of Symbolic Logic, 1947).

(5) (p. 64) Thus, it will appear (cf. below, ch. 8) that these utterances acquire 'partial phrase structure' in other ways.

(6) (p. 64) But conformity, as discussed in ch. 8, I was a matching relation, which is not transitive from any logical
necessity. We must therefore either make the empirical assumption that it is actually an equivalence relation, or we must construct an equivalence relation from it. The assumption that conformity is actually an equivalence relation is one form of what is called the fundamental assumption of phonology.

(9) Certain levels may be missing in the analysis of particular languages. The exact conditions under which this may be the case can not be stated until the levels are actually constructed.

(10) Henceforth we will denote a linguistic expression either by that expression within quotes, or by the expression italicized, whichever is more convenient.


(12) Cf. Chomsky, op. cit., for some elaboration of this inscriptive approach to developing linguistic theory.

(13) If \( W_i \) and \( W_{i+1} \) overlap, this substitution becomes meaningless. (4), def. 4, is formulated so as to rule out this possibility for non-units.

(14) Harris, Method, for discussion of discontinuous morphemes. Cf. also Wells, Immediate Constituents, Language 1947, W. Hockett, Problems of Morphemic Analysis, ibid., \#15, Method, \#21, for other relevant situations.


(16) In this study, suprasegmental features (pitch, stress, juncture) have not been considered. Ultimately, of course, these phenomena must be incorporated into any full syntactic theory, and it may be that this extension will require a more elaborate system of representation.

See Hockett, Two Models, for some discussion of this possibility. See Accent Inference for discussion of a phonemic transcription for English that includes stress, juncture, but preserves linearity of representation in a natural way.
Chapter III - Simplicity and the Form of Grammar

15. Suppose that we have completed the abstract development of a system of levels of the type described in Chapter II. We construct a grammar of a language L by showing how, in particular, this system of levels applies in the case of L. For each grammatical sentence S of L, the grammar will state the constituency of S in terms of elements of each level. That is, the grammar must provide the means for constructing a set of representations for each grammatical sentence S, with each representation being a 'spelling' of S in terms of one type of element.

A grammar must also state the set of markers of each level, where these may be distinct from the representing strings (cf. \[\alpha\]). Let us assume that on every level \(\mathbb{L}_i\), the \(\mathbb{L}_i\)-marker of a grammatical sentence S can always be derived in a mechanical manner from the set of representations of S on the level \(\mathbb{L}_i\), once this set is given. This will always be the case in our constructions, and this assumption will enable us to limit our discussion of the form of grammars to the problem of generating the set of representations of grammatical utterances on each level. We are thus laying down as a condition on levels (or to put it differently, as a definition of 'structural information') that all structural information about grammatical utterances is derivable from the set of strings that represent utterances in each level. And in determining the form of grammars we must make clear exactly how a grammar provides this structural information, i.e., how it determines representations of utterances and \(\mathbb{L}_i\)-markers.

We want linguistic theory to enable us to choose among proposed grammars. Every consideration that is relevant to this choice should be built into linguistic theory, into the actual definition of linguistic elements. So far, the only condition on grammar has been the requirement that the actual system of levels determined (in a manner yet to be given) by the grammar be of the form required
by linguistic theory, i.e., that each level have the proper structure, that levels be properly interrelated, and that they be related in the proper way to the corpus of utterances of the language under analysis. But this formulation leaves out one of the most important and characteristic features of grammar construction. In careful descriptive and methodological work, we almost always find that one of the considerations involved in choosing between analyses is the 'simplicity' of the grammar in which these analyses are presented. If we can set up elements in such a way that very few rules need be given about their distribution, or that these rules are very similar to the rules given for other elements, this fact certainly seems to be a valid source of support for this analysis. But such considerations often are best stated, or necessarily stated, in terms of features of the grammar, rather than directly as features of the elements set up for the language. It seems reasonable, then, to inquire into the possibility of defining linguistic notions, in the general theory partially in terms of properties of grammars. If this course is to be followed, then we must carefully investigate the sense in which a grammar can be said to describe the structure of a language, and we must lay down precise conditions on the form that a grammar may assume.

It is important, incidentally, to recognize that considerations of simplicity are in general not trivial or 'merely esthetic'. It has been recognized of philosophical systems, and it is, I think, no less true of grammatical systems,
that the motives behind the demand for economy are in many
ways the same as those behind the demand that there be a system
at all. (1)

16.1. We have seen that the grammar must provide a set
of representations for each sentence S. Let us call such
a set the sentence-grammar of S. We might then, as a start,
take the grammar to be a list of all sentence-grammars. One
difficulty with this is that we want the grammar to be of
finite length, while there are infinitely many sentence-
grammars. But even aside from this, we would certainly
require a more elegant and revealing description.

Suppose that the highest level representation of any
grammatical sentence is the representation Sentence. Then
a sentence-grammar will be a sequence of representations
\( R_n, \ldots, R_1 \), where \( R_n = \text{Sentence} \), and \( R_1 \) is, let us say,
a phonetic spelling, and where the \( R_i \)'s \((1 \leq i \leq n)\) are intermediate
representations in terms of phrases, words, morphemes,
etc. (2) Given \( R_i \), we know a good deal about \( R_{i+1} \) and \( R_{i-1} \),
since there are severe restrictions on the interconnections
between levels. Thus we might replace the list of sentence
grammars with a set of rules that state what the representation
of level \( i-1 \) may be, given an \( i^{th} \) level representation.
A finite set of rules can serve to generate an infinite
set of sentence grammars, if we define 'generation' recursively
in some way. These rules have the general form

\[ \alpha \rightarrow \beta \]

where \( \alpha \) and \( \beta \) are representations, and (1) is interpreted as
asserting that wherever we have an \( \alpha \) level representation, we may have an \( \beta \) level representation. Applying this set of rules, we construct the set of sentence-grammars. If many \( \alpha \) level representations are carried into \( \beta \) level representations in essentially the same way, then we will achieve a good deal of economy by giving a single rule to this effect, applying to all sentence-grammars. We may call each rule of the form (1) a \textit{conversion}. A sequence of such rules we will call a sentence-grammar thus bears a certain formal analogy to a formal proof. A proof of a theorem in a formalized theory is a finite sequence of lines, each of which is either an axiom, or follows from preceding lines by the rules of inference, and the last member of which is the theorem in question. In the case of a sentence-grammar, the representation \textit{Sentence} can be regarded as the single 'axiom' (i.e., all sentence-grammars have as their first line the representation \textit{Sentence}), and the 'rules of inference' are the rules of the grammar of the form (1). The 'theorems' are the \( \beta \) level representations (here then the analogy breaks down).

16.2. It is not necessary that \( \alpha \) and \( \beta \) in (1) be full sentence representations. Thus instead of giving a set of rules

(a) \textit{Proper Noun''Verb''} \rightarrow \textit{John''Verb''}...

(2) \textit{...''Verb''Proper Noun} \rightarrow \textit{...''Verb''John}

e tc.

we can give a single rule

(3) \textit{Proper Noun} \rightarrow \textit{John}

which will apply in both cases.

Or, more interestingly, we may introduce a level of
representation solely because it enables us to replace a
great many rules by a single rule about a single element of
this new level. An example of this would be a morphophonemic
statement. Suppose that the $i$th level representations of
English are in terms of morphemes, and the $i-1st$ in
terms of phonemes. Instead of associating with each
morpheme a set of phoneme strings (its allomorphs, perhaps
only one), along with the conditions dictating the occurrence
of each, it is often possible to rewrite analyze each morpheme into as
a string of morphophonemes, these being elements such that
a relatively small number of statements about the phonemic
forms which they assume in various contexts will suffice to
determine the whole set of conversions of morphemes into
phonemic representations. We will then say that a language
'has' such and such morphophonemes if this treatment is
simpler (in terms of a criterion of simplicity yet to be
established) than the former one. In English, for instance,
we have a set of rules

\begin{align*}
\text{(a)} \quad \text{wife}^\text{pl} & \rightarrow /\text{wav}/^\text{pl} \tag{ultimately, "wives"} \\
\text{(b)} \quad \text{wife}^\text{x} & \rightarrow /\text{waf}/^\text{x} \tag{\text{x}^\text{pl}}
\end{align*}

\begin{align*}
\text{(a)} \quad \text{knife}^\text{pl} & \rightarrow /\text{naf}/^\text{pl} \tag{"knives"} \\
\text{(b)} \quad \text{knife}^\text{x} & \rightarrow /\text{naff}/^\text{x} \tag{\text{x}^\text{pl}}
\end{align*}

\begin{align*}
\text{(a)} \quad \text{leaf}^\text{pl} & \rightarrow /\text{lyv}/^\text{pl} \tag{"leaves"} \\
\text{(b)} \quad \text{leaf}^\text{x} & \rightarrow /\text{lyf}/^\text{x} \tag{\text{x}^\text{pl}}
\end{align*}

\text{etc.}

where \text{pl} is the plural morpheme, and "/.../" encloses phonemic
representations. A much simpler analysis than this would be to
'spell' these morphemes in terms of morphophonemes, thus

(7) \textit{wife} \rightarrow \textit{way}^{F}

(8) \textit{knife} \rightarrow \textit{nay}^{F}

(9) \textit{leaf} \rightarrow \textit{liy}^{F}

e tc.

We can then give a set of simple rules for conversion of morphophonemes into phonemes, e.g.

(10) \begin{align*}
(a) & \quad \text{F}^{\prime} \text{pl} \rightarrow \text{F}^{\prime} \text{pl} \\
(b) & \quad \text{F}^{\prime} \text{a} \rightarrow \text{F}^{\prime} \text{a} \quad (\text{pl})
\end{align*}

Words that are not subject to this change (e.g., "fife", "fifes") are spelt with the element "fl" on the morphophonemic level.

If these morphophonemes have a wide distribution, and if there are many such cases, complexly interconnected, this procedure of analysis can lead to significant economy.(3)

The same type of argument can be used to motivate and justify the introduction of the level of phrase structure, once a criterion of simplicity is established. Suppose that the \textit{1-th} level representation of every sentence is \textit{Sentence}, and the \textit{n-th} level is in terms of, let us suppose, word classes. Then we will require a vast number of \textit{rules} of the form \textit{Sentence} \rightarrow \ldots, where \ldots is some permissible string of word classes. However, we find that we can classify strings of word classes into phrases in such a way that relatively few strings of phrases occur. In this way we may hope to reduce the overwhelming complexity of a direct analysis of word class strings to manageable proportions. To state the phrase structure of sentences and the word-class structure of phrases
is jointly much simpler than to state the word class structure of sentences directly (though we will see below, chap. VII, that this assumption is only partially successful).

17.1. We see then how considerations of simplicity can function in determining linguistic structure. Given a measure of simplicity defined in the general theory, we can define the set of linguistic elements of a given level \( \mathbb{L} \) as being a set of elements that satisfy the axioms of \( \mathbb{L} \) (and are properly related to other levels), and that furthermore appear in the simplest grammar of the language under analysis. Linguistic theory is thus constructed in a meta-meta-language to any natural language, and a metalanguage to the language in which grammars are constructed. In applying this theory to actual linguistic material, we must construct a grammar of the proper form, and demonstrate that the set of sentence-grammars derived from this grammar satisfy the conditions of level structure laid down in the general theory. Among all grammars meeting this condition, we select the simplest. If we are to meet the fundamental requirement of providing an evaluation procedure for grammars, the measure of simplicity must then be defined in such a way that we will be able to evaluate directly the simplicity of any proposed grammar.

17.2. Suppose that we have constructed linguistic theory in such a way that given a corpus of utterances for which we know in advance that there is some grammar, it is the case that:

(i) All systems having the prescribed form for grammars, given this corpus, can be enumerated in order of simplicity, starting with the simplest;

(ii) given any such system, it is possible to
determine in a mechanical way whether the generated set of sentence-grammars has the prescribed level structure. In this case, the general theory will provide a method for literally constructing the grammar, though perhaps not a practical method. The theory we discuss below is too sketchy and incomplete for us to be able to discuss it seriously in these terms, but it seems likely that such a goal can be reached by linguistic theory. \( (\text{I}) \) will be met, in particular, if the set of sentence-grammars is finite. We will see below that there are good reasons for determining the adequacy of a grammar, in terms of a certain finite set of sentence-grammars, even though there are infinitely many grammatical sentences \((V, \text{chap. IX})\). Condition (1) can be met if 'simplicity' is defined in such a way that, given a proposed grammar, we can automatically evaluate its simplicity measure \((5)\) and if, furthermore, there is a finite upper bound, determined as a function of the length of the corpus (e.g., in phone tokens), to the number of grammars that must be investigated. If a simplicity measure such as the one we discuss below proves adequate, we can weaken this to the requirement that the number of distinct atomic symbols from which the grammar is constructed is bounded, given the corpus.

These remarks have no practical importance. We are far from being able to construct a linguistic theory of even a much weaker kind. But it is important to keep in mind that such a program is possible.

18.1. In §16 we developed a rough conception of the form of
grammars. How can the simplicity of such grammars be measured? This is essentially the question of determining what features of grammars are to be taken into account in choosing among them.

In constructing a grammar, we try to set up elements having regular, similarly patterned, and easily stateable distributions, and subject to similar variations under similar conditions; in other words, elements about which a good deal of generalization is possible and few special restrictions need be stated. It is interesting to note that any simplification along these lines is immediately reflected in the length of the grammar. We have a generalization when we replace a set of statements, each about a single element, by a single statement about the whole set of elements. We can generalize the notion of generalization to cover the situation where we have a set of statements (each about a single element) which are identical in part. Then the common substatement can be extracted out and stated only once, applying to all elements, with a consequent shortening of the grammar. The demand for 'patterning' would appear, in many instances, to reduce to a demand for generalization in this broader sense.

It is tempting, then, to consider the possibility of devising a notational system which converts considerations of simplicity into considerations of length. This would amount to constructing a set of 'notational transformations', to be applied to grammars which are sets of statements of
the form (1), in such a way that the simplicity of such grammars is a function of the number of symbols in the grammar. (More generally, simplicity might be determined as a weighted function of the number of symbols, the weighting devised so as to favor reductions in certain parts of the grammar.) Full generalizations and elimination of special restrictions automatically reduce the length of grammars, and the notational system can be so constructed as to facilitate 'extracting out' of common components of distinct statements, by the use of brackets and parentheses in fairly familiar ways.

18.2. It is important to recognize that we are not interested in reduction of the length of grammars for its own sake. Our aim is rather to permit just those reductions in length which reflect real simplicity, that is, which will turn simpler grammars (in some partially understood, pre-systematic sense of this notion) into shorter grammars. Within linguistic theory, we must define once and for all the 'notational transformations' that are to be available in evaluating the complexity of grammars. It can easily be seen that to permit notations to be devised anew for each particular grammar would quickly lead to absurdity.

Suppose, for instance, that a language has \( n \) phones \( a_1, \ldots, a_n \). We might express the grammar in as complicated a way as we like, say as \( Q(a_1, \ldots, a_n) \), where \( Q \) is some immensely complicated schema. Then defining 'f(\( x_1, \ldots, x_n \))' as '\( Q(x_1, \ldots, x_n) \)', we could construct the extremely short grammar, literally, 'f(a_1, \ldots, a_n)'. It is clear that restrictions must be placed on the notations available for grammar construction so as
to exclude such notations as \( f(a_1, \ldots, a_n) \), allowing only notations which in general serve to consolidate similar statements, etc. Thus the choice among grammars for a given language is not only influenced by the grammars of other languages (as we have seen above, \( \text{II.} \cdot \text{II.}, \text{chap.I} \)), but also by the choice of notations available in the metalanguage. When length, under notational transformations, is used as a measure of simplicity, we can adjust our notations so as to reflect different senses of the notion of simplicity, thus affecting the choice among grammars, and consequently, altering our conception of what are the elements of given languages.

The problem of choosing the correct notations is much like that of evaluating a physical constant. Given criteria of adequacy for grammars of certain languages, we can arrive empirically at proper notations by determining a set of notations with the property that the grammars meeting the criteria of adequacy are in fact the shortest, given these notations. In other words, we define simplicity so that, in certain clear cases, simplest grammars are in fact the correct ones. As long as we do not take simplicity to be an absolute ideal, thoroughly understood and specified in advance of theory construction, this procedure is no stranger than attempting to define 'morpheme' in such a way that what we know to be morphemes in some language turn out to be morphemes when we apply the theory to a corpus of utterances in this language. That this program can be realized is immaterial. It can be realized in a non-trivial fashion if we can give
a general and abstract definition of 'simplicity' (just as of 'morpheme') which in the case of particular languages leads to adequate grammars, and if the general theory of grammatical structure, in which this definition appears, meets certain considerations of significance that apply to any scientific theory. For instance, the theory must have internal systematic motivation for its constructions (it cannot be simply a list of all known results, as would be the case, essentially, if it permitted such notations as 'f', or permitted the validation of elements by the discovery of formal markers, cf. §3.2), it must have predictive power (i.e., in the case of new languages, it must lead to correct grammars), etc. These considerations of effectiveness and simplicity are not specific to linguistic theory. Note that when we speak of the simplicity of linguistic theory, we are using 'simplicity' in the still vague sense in which simplicity is an ideal for any science, whereas when we speak of the simplicity of grammars, we are using it in a sense which we hope to make as precise as the definition of 'phoneme' or 'morpheme'. The simplicity of linguistic theory is a notion to be analyzed in the general study of philosophy of science; the simplicity of grammars is a notion defined within linguistic theory.

An analogy can be drawn between philosophy of science and linguistic theory. Philosophy of science deals with all sciences, physics, chemistry, linguistic theory, etc., and seeks to determine, among other things, the criteria that lead to the construction of theories and the choice among
conflicting theories in the case of each of these sciences. Linguistic theory deals with a set of rather special scientific theories called 'grammars' (cf. \( g.1 \)), and seeks to establish rigorous principles that lead to the construction and choice of grammars for particular languages. In terms of this analogy, a conception of the nature of science such as that advanced recently by Quine provides motivation for the introduction of a rigorous notion of simplicity as a criterion for choosing among grammars. In linguistic theory, where the material under investigation is relatively clear and limited, we may hope to carry out in an effective way the task of literally defining simplicity for the theories in question, namely grammars, and of setting up an effective evaluation procedure for these theories in terms of criteria of simplicity.

19.1 We are given grammars which are a set of statements of the form (1), and are faced with the problem of evaluating in terms of their simplicity. To do this we construct notations which permit the consolidation of similar statements, and we then measure the 'degree of generalization' (which we have tentatively taken as the measure of simplicity) as length, when these consolidations have been carried out. The determination of correct notations will involve detailed study of the effects of various choices on actual grammars. Almost no rigorous work has been done in this important direction. Hence we can only make the most tentative constructions. In this section we define certain notations for subsequent use in pursuing this type of inquiry with actual
language material. I've notations which we introduce are not unfamiliar, but we will attempt to use them in a thoroughgoing and systematic manner.

The notations which are to be used for evaluating grammars are introduced definitionally in the general theory by what are known as 'contextual definitions'. Each such definition of a given notational element provides a procedure for converting any expression using this notation into a list of expressions in which this notational element does not appear at all. Thus given a consolidated grammar, i.e., a list of expressions containing the notations developed in linguistic theory, we can expand it to a list of expressions of the form (1), where none of the consolidating notations appear.

We will see below that the order of the statements of a grammar becomes quite important, although up to this point we have been considering grammars as unordered sets of conversion statements of the form (1). To prepare for this later development, we will require that the definitions of our notations provide a procedure for expanding any consolidated expression into a unique sequence (not an unordered set) of expressions not containing this notation. If order plays no role, this requirement is of no harm -- it is simply a useless refinement. If order is significant, the requirement is essential. An ordered set of statements of the form (1) we will call a linear grammar.

19.2. Suppose that we have a statement \( \alpha \rightarrow \beta \), where \( \alpha = a^*b^c \), and \( \beta = a^*b^c \). Then we can say that \( b \rightarrow b' \) in the environment \( a \rightarrow c \).

**Def. 1.** The statement: (A1) \( b \rightarrow b' \) in env. \( a \rightarrow c \) stands for: (B1) \( a^*b^c \rightarrow a^*b'^c \)
(Bl) is of the form (1), and Def. 1 tells us how to restate anything of form (Al) as a statement of form (Bl). This is the fundamental notational abbreviation, since it separates the element undergoing the change from its context, which may be quite complex. When we have a statement of form (Al), we will call \( a \rightarrow c \) the conditioning context for the conversion \( b \rightarrow b' \), where \( a \) is the initial conditioning context, and \( c \) the final conditioning context.

We will use brackets in the following manner. The statement

\[
(11) \ldots \begin{cases} \begin{array}{c} a_1 \\ \vdots \\ a_n \\ \end{array} \end{cases} \ldots
\]

will be an abbreviation for the sequence of statements

\[
(1) \ldots a_1 \ldots \\
(11) \ldots a_2 \ldots \\
(12) \ldots \\
(13) \ldots a_n \ldots
\]

in this order. If two sets of brackets with the same number of rows occur, then in the expansion, the \( i^{\text{th}} \) row of the first corresponds to the \( i^{\text{th}} \) row of the second. Thus

\[
(13) \ldots \begin{cases} \begin{array}{c} a_1 \\ \vdots \\ a_n \\ \end{array} \end{cases} \ldots \begin{cases} \begin{array}{c} b_1 \\ \vdots \\ b_n \\ \end{array} \end{cases} \ldots
\]

will be an abbreviation for
Similarly, in the case of more than two sets of brackets with the same number of rows. We must give the general definition for this case by a recursive definition. Before doing this, we note one further generalization. It will be convenient to introduce various types of brackets. We may indicate the type by a subscript to each left and right bracket. Assuming indefinitely many types, we can define (11), (13), etc., by

**Def. 2.** The statement: \((A_2)\) \[
\begin{align*}
&\begin{cases}
a_{11} \\
a_{12} \\
a_{1n}
\end{cases} \quad \begin{cases}
a_{21} \\
a_{22} \\
a_{2n}
\end{cases} \quad \cdots \quad \begin{cases}
a_{m1} \\
a_{m2} \\
a_{mn}
\end{cases} \\
\end{align*}
\]
stands for the sequence of statements:

\[
\begin{align*}
(i) & \quad \cdots a_{11} \cdots a_{21} \cdots a_{1m} \cdots \\
(ii) & \quad \cdots a_{12} \cdots a_{22} \cdots a_{1n} \cdots \\
(n) & \quad \cdots a_{1n} \cdots a_{2n} \cdots a_{mn} \cdots
\end{align*}
\]

for \(i=1,2,\ldots\) where \(j\) indicates the type of bracket.

If brackets of different types appear in a consolidated statement, we expand according to the following principle.

If \(j<k\), then the statement

\[
\begin{align*}
(15) & \quad \begin{cases}
a_1 \\
a_2 \\
a_n
\end{cases} \quad \begin{cases}
b_1 \\
b_2 \\
b_m
\end{cases} \\
\end{align*}
\]
will be an abbreviation for the sequence of statements

\[
\begin{align*}
\text{(i)} & \quad \ldots a_1 \ldots \begin{cases} b_1 \\ b_2 \end{cases} \ldots \\
& \quad \begin{cases} b_m \\ b_i \end{cases} \\
\text{(ii)} & \quad \ldots a_2 \ldots \begin{cases} b_1 \\ b_2 \end{cases} \ldots \\
& \quad \begin{cases} b_m \\ b_i \end{cases} \\
& \quad \vdots \\
& \quad \begin{cases} b_1 \\ b_2 \end{cases} \\
\text{(n)} & \quad \ldots a_n \ldots \begin{cases} b_1 \\ b_2 \end{cases} \ldots \\
& \quad \begin{cases} b_m \\ b_i \end{cases}
\end{align*}
\]

which in turn, by def. 2, is an abbreviation for the sequence

\[
\begin{align*}
&\begin{cases} \text{(i,i)} & \quad \ldots a_1 \ldots b_1 \ldots \\
& \quad \text{(i,ii)} \quad \ldots a_1 \ldots b_2 \ldots \\
& \quad \vdots \\
& \quad \text{(i,m)} \quad \ldots a_1 \ldots b_m \ldots \\
& \quad \text{(ii,i)} \quad \ldots a_2 \ldots b_1 \ldots \\
& \quad \text{(17)} \\
& \quad \text{(ii,m)} \quad \ldots a_2 \ldots b_m \ldots \\
& \quad \vdots \\
& \quad \text{(n,i)} \quad \ldots a_n \ldots b_1 \ldots \\
& \quad \text{(n,m)} \quad \ldots a_n \ldots b_m \ldots 
\end{cases}
\end{align*}
\]

This is to say that higher order brackets are to be expanded before lower order brackets. This rule, general in the manner of def. 2, can be given as a definition; replacing def. 2.
Def. 3. The statement (A2) (Def. 2) stands for the sequence of statements (B2), where the highest order brackets appearing in any statement of (B2) are of order \( j-1 \), or, if no brackets occur in (B2), then \( j=1 \).

Def. 2 is now dropped, and Def. 3 is the general definition for all orders of brackets. Note that if two sets of brackets of the same order occur, they have the same number of rows, and the process of eliminating them is analogous to matrix multiplication. If brackets of different orders occur, they may have different numbers of rows, and elimination is analogous to the formation of the Cartesian product. It follows from the definition that expressions whose brackets of a given order never appear within lower order brackets of a given order appear within higher order brackets. Below we will revise this (cf. 12.2).

The effect of this definition is that we have the following rules for expanding consolidated statements of the grammar, where the expansion proceeds by stages.

Rule 1: Higher order brackets are expanded before lower order brackets.

Rule 2: The expansion proceeds as in Def. 3, i.e., as the expansion of (11) to (12) if there is only one pair of highest order brackets, as (A2) to (B2), if there is more than one pair.

Rule 3: A pair of brackets and the enclosed expression are treated as a single element when inside a containing pair of brackets. E.g., the main bracket in
(18) \[
\left\{ \begin{array}{c}
\left\{ \alpha_i \right\} \\
\alpha_2 \\
1 \\
\left\{ \gamma_i \right\} \\
\gamma_1 \\
1 \\
\left\{ \beta_i \right\} \\
\beta \\
\end{array} \right\}
\]

has two rows.

To indicate how many rows a pair of brackets contain, we might use the unit element \( U \) if no element \( a_i \) occurs. Instead of this, we will often use '---'. Thus

(19) \[
\left\{ \begin{array}{c}
\alpha \\
--- \\
\beta \\
\end{array} \right\}
\]

has three rows.

Note that only main brackets are considered at any stage in the expansion of a consolidated statement of the grammar.

19.3. We will use angles \( \langle \rangle \) to indicate that an element may or may not be present. Thus

(20) \[ a \langle b \rangle c \]

stands for the sequence of statements

(21) \( a^\circ b^\circ c \)

(i) \[ a^\circ b^\circ c \]

(ii) \[ a^\circ c \]

where \( a, b, \) and \( c \) may be any expressions of the grammar, however complex. We have a notion of paired angles and main angles, exactly like paired and main brackets (cf. fn (8)), and it is necessary to generalize the definition to the case where there are several main angle pairs.
Def. 4. The statement:

\[(A4) \quad a_1^{b_1} a_2^{b_2} a_3^{b_3} \cdots a_n^{b_n} a_{n+1}\]

stands for the sequence of statements:

\[(i) \quad a_1^{b_1} a_2^{b_2} a_3^{b_3} \cdots a_n^{b_n} a_{n+1}\]

\[(B4) \quad a_1^{a_1} a_2^{a_2} a_3^{a_3} \cdots a_n^{a_n} a_{n+1}\]

where the \(b_i\)'s meet the condition on the \(a_{i+1}\) of Def. 2
(cf. fn (8))

We have left unspecified the order of expansion except to require that the statement with all 'angled' elements precede every other statement, and that the statement with none follow every other statement. Without going to the trouble of specifying the order further, we simply assume that some such specification has been given. This happens never to be relevant to our later constructions.

The rules of expansion 1–3 followed from Def. 2. But now we must add an independent rule to apply to the case where a statement contains both main brackets and main angles.

**Rule 4:** The expansion of main brackets of any order precedes the expansion of angles.

If we regard angles as zero order brackets, this is a special case of rule 1, but a case independent of definitions 3, 4.

**Rule 5:** The expansion of angles is in accordance with Def. 4.

We now have an explicit step by step procedure for
converting any statement with brackets or angles into an ordered sequence of statements of the form \( \alpha \rightarrow \beta \), where \( \alpha \) and \( \beta \) are representations, i.e., strings in \( L \), for some \( L \). This is a properly formed statement with brackets and angles can be converted into a linear grammar in the sense of \( 4.1 \).

Devices permit certain selected features of similarity among statements of the grammar to effect a decrease in length, so that grammars whose rules have these features become more highly valued. Thus these constructions can be understood as offering an analysis for certain aspects of simplicity. Whether they afford a correct account of simplicity can only be determined by investigating the effects of these simple constructions in actual grammatical work. In the syntactic studies of chaps. \( V \) and \( VII \), below, we use only these notational devices. In the morphophonemic study in the appendix to chap. \( VI \), several notations will be added.

12.4. As an example of the functioning of these rules of development, consider the following case:

\[
(22) \quad \begin{cases} \\
\{ \langle \alpha_1, \alpha_2 \rangle \} \\
\{ \alpha_3 \} \\
\{ \beta_1, \langle \beta_2 \rangle \} \\
\end{cases} \quad \longrightarrow \quad \begin{cases} \\
\{ \gamma_1 \} \\
\{ \gamma_2 \} \\
\end{cases}
\]

The expansion is given step by step as follows, with the rule governing each step.

\[
(23) \quad \begin{cases} \\
\alpha: \{ \langle \alpha_1, \alpha_2 \rangle \} \rightarrow \{ \gamma_1 \} \\
\beta: \{ \beta_1, \langle \beta_2 \rangle \} \rightarrow \{ \gamma_2 \} \\
\end{cases} \quad \text{by \ Rules 1, 2}
\]
\( (ii) \)  
\[ a_1 : \langle \alpha_1 \rangle \alpha_2 \rightarrow \gamma_i \]  
\[ a_2 : \alpha_3 \rightarrow \gamma_2 \]  
\( \text{by Rule 2} \)

\[ b_1 : \beta_1 \langle \beta_2 \rangle \rightarrow \gamma_i \]  
\[ b_2 : \beta_1 \langle \beta_2 \rangle \rightarrow \gamma_2 \]  
\( \text{by Rules 2, 4} \)

\( (iii) \)
\[ a/\alpha : \alpha_1 \bar{\alpha}_2 \rightarrow \gamma_i \]  
\( \text{by Rule 5} \)

\[ a/b : \bar{\alpha}_2 \rightarrow \gamma_i \]  
\[ a/\alpha : \alpha_3 \rightarrow \gamma_2 \]  

\[ b/a : \bar{\beta}_1 \bar{\beta}_2 \rightarrow \gamma_i \]  
\( \text{by Rule 5} \)

\[ b/b : \bar{\beta}_1 \rightarrow \gamma_i \]  

\[ b/2a : \bar{\beta}_1 \bar{\beta}_2 \rightarrow \gamma_2 \]  

\[ b/2b : \bar{\beta}_1 \rightarrow \gamma_2 \]  

The expanded sequence of statements of the grammar is (23iii), in that order. The use of five pairs of brackets and angles permits the elimination of ten occurrences of the elements \( \alpha_1, \alpha_2, \) etc., which may themselves be long and complex expressions.

20.1. The preceding discussion has been an informal elaboration of the syntactic structure of the system in which grammars are written, i.e., of the metalanguage which is to be available for linguistic description. This description could be given much more precisely, and ultimately, the analysis of the language of grammar will have to be undertaken as a serious study in its own right. This necessity is a consequence of the decision to bring features of the grammar (i.e., simplicity) into consideration as a determining factor in the validation of grammars. The
purpose of the foregoing discussion was simply to explain the symbolism which will be used in our introductory investigations of simplicity below, and to suggest an approach to the formal analysis of simplicity.

The precise reconstruction of a system such as that discussed above can follow familiar lines, and offers no essential difficulties. We first note that the construction can be only schematic. The basic elements of the grammar are the representations of utterances, and, within the limits set by the formal conditions on the set of levels, these representations will vary from language to language.

Suppose that the set of representations is given. That is, we assume a set of properly constructed and related levels \( L_1, \ldots, L_m \). We use Greek letters \( \alpha, \beta, \ldots \) as variables over strings in any \( L_i \), and Latin letters \( a, b, \ldots \) as variables over expressions of the grammar quite generally, whether containing strings in some \( L_i \) or not. We can now construct the language of grammars \( G \) as a formal system.

The 'vocabulary' of \( G \) contains all strings in \( L \) (\( 1 \leq i \leq m \)), and the symbols:

\[
\begin{align*}
(i) & \quad \{i\} \\
(ii) & \quad \langle i, j \rangle \\
(iii) & \quad \rightarrow
\end{align*}
\]

Instead of writing elements in brackets horizontally, we write them vertically so as to facilitate the formal construction of \( G \), which we now outline.
We can recursively define "well-formed element (WFE) of order $k$" as follows:

**Def. 2.** (i) Any string of $a_1$ is a WFE of order 0.

(ii) If $a_1$, $a_2$, $a_3$ are WFE's of order $\leq k$, and one of $a_1$, $a_2$, $a_3$ is of order $k$, then

$$a_1 \langle a_2 \rangle a_3$$

is a WFE of order $k$.

(iii) If $a_{11}, \ldots, a_{1n}, a_{21}, \ldots, a_{2n}, \ldots, a_{m1}, \ldots, a_{mn}, b_1, \ldots, b_{m+1}$ are WFE's of order $< k$, then

$$b_1 \langle a_{11} ; \ldots ; a_{1n} \rangle b_2 \langle a_{21} ; \ldots ; a_{2n} \rangle \ldots b_m \langle a_{m1} ; \ldots ; a_{mn} \rangle b_{m+1}$$

is a WFE of order $k$.

(iv) These are the only WFE's of order $k$, $k > 0$

**Def. 6.** A well-formed statement (WFS) in $G$ is any statement of the form:

$$a \rightarrow b \text{ in } env. \quad c \rightarrow d,$$

where $a$, $b$, $c$, and $d$ are WFE's of orders $k_1$, $k_2$, $k_3$, $k_4$, respectively.

**WFS**

Given a formula of $G$ containing well-formed elements of order $\leq k$, we can, by reapplying the rules of the preceding section, derive from it a unique sequence of formulas of order not higher than $k$. 
Def. 7. Suppose that $x$ is the WFS

$$a_1 \rightarrow a_2 \text{ in env. } a_3 \cdots a_4$$

Then we define the derived formula sequence (DFS) of $x$ as follows:

(i) if each $a_1$ is a string in $L$, for some $i$, then $x$ itself is the DFS of $x$.

(ii) suppose that $x$ contains exactly $r$ pairs of main brackets of order $k$, and no main brackets of order greater than $k$. Thus $x$ is

$$a_1 \left\{ d_{11}; \ldots; d_{1n} \right\} a_2 \cdots a_r \left\{ d_{r1}; \ldots; d_{rn} \right\} a_{r+1},$$

these being all of the highest order main brackets in $x$. (9) Then the DFS of $x$ is the sequence formed by rule 2, i.e., it is the sequence

$$(1) a_1 d_{11} c_2 d_{21} \cdots c_r d_{r1} a_{r+1}$$

$$(2) a_1 d_{1n} c_2 d_{2n} \cdots c_r d_{rn} a_{r+1}$$

(iii) suppose that $x$ contains no main brackets, but contains exactly $r$ pairs of main angles. (9) I.e., $x$ is

$$a_1 \langle d_1 \rangle a_2 \cdots a_r \langle d_r \rangle a_{r+1},$$

where these are all of the main angles in $x$. Then the DFS of $x$ is the sequence formed by Rule 5, i.e., it is (B4) of Def. 4 with $a_1$ in place of $a_1$, and $d_1$ in place of $b_1$ (r in place of n).

Thus given a WFS in $\mathfrak{g}$, we can form its DFS, which will be a sequence of WFS's. Given a sequence of WFS's, we can form a unique DFS of the sequence, by establishing an order among the derived statements, formula sequences.
Def. 8. If $x_1, \ldots, x_n$ is a sequence of WFS's, and for each $i$,

$$x_1 \xrightarrow{a_i} \ldots, x_{i+1}$$

is the DFS of $x_i$, then the DFS of

$x_1, \ldots, x_n$ is the sequence:

$$x_1, \ldots, x_{i+1}, x_{2i}, \ldots, x_{2i+1}, \ldots, x_n$$

Each term is again a WFS. We can thus continue to form a DFS of this, a DFS of the DFS, etc., until we have a sequence of WFS's containing only strings in $L_1$, and symbols of types (iii), (iv), (v). We can thus give a recursive definition of elementary derived sequence (EDS).

Def. 9. If $x_1, \ldots, x_n$ is a sequence of WFS's, the EDS associated with it is the sequence of WFS's containing $m$ brackets or angles formed by repeatedly taking the DFS (i.e., it is the DFS which is identical with its own DFS).

This EDS is a unique sequence of WFS's of the form

(25) $\alpha \rightarrow \beta$ in env. $\gamma \rightarrow \delta$

Applying Def. 1, 919.2, we can convert this sequence into a unique sequence of statements of the form

(26) $\alpha \rightarrow \beta$

i.e., of the form (1).

Def. 10. A grammar is a sequence of WFS's which is not a DFS of some shorter sequence of WFS's.

Given any grammar, we can expand it uniquely to a sequence of statements of type (1). However, it is not the case that for any sequence of statements of the form (2), there is a unique grammar, consolidated grammar.
The main point of this digression into the structure of the 'syntax language', the language of grammars, is to present an example of the kind of investigation that must be carried out if we wish to take the simplicity of grammars seriously as a means of validating grammars. We must develop, in an abstract manner, a general schematic account of the form of grammars, and we must give a general definition of simplicity for grammars of the proper form.

20.2. We can simplify our notations somewhat (although by so doing, we complicate the description of $G$, the language of grammatical description) by dropping subscripts for brackets, and using brackets of different shapes in place of brackets of a single shape with subscripts. Whenever we can get along with only two kinds of brackets, we will use \[ \{ \} \] and \[ \left[ \right] \] as brackets of order 0 and 1, respectively. The order is important, since it determines the order of development. Thus if we have a statement containing main brackets of both types, \{\} will always be developed first.

In stating the structure of $G$, we required that brackets of order $k$ never appear within brackets of order $\leq k$ in well-formed elements. However it is clear that we can often break this rule with no ambiguity, as long as main brackets are always expanded before brackets contained within them. Doing this will permit a reduction in the number of bracket forms that we need to employ. Before, it was a consequence of our definitions that main brackets were expanded prior to contained brackets, but now we must add this as a special rule of development.
Rule 6: At each stage in the expansion of a statement, only main brackets are developed, with \{\} being prior to [ ].

The rules 1-6 will be adopted below in our grammatical examples as characterizing these notations.

21.1. Just as the decision to validate grammars, in terms of simplicity compels us to lay down the exact form of grammars with great care, it also necessitates a clear and precise formulation of the notion of 'generation'. There must be a mechanical way to derive all structural information\(^{(11)}\) from any grammar proposed for a language. No step, no matter how obvious, can be left to the intelligent reader, for we cannot know to what extent formal incorporation of this step would have detracted from the simplicity of the grammar.

Given a grammar, as in Def.10, we must define the sense in which it generates a set of sentence-grammars. Since we can convert a grammar into a sequence of statements of the form \((26)-(1)\), it is sufficient to define generation for sequences of such statements. This is the question discussed in §16. We can reconstruct this account as follows:

Suppose that the grammar is expanded into the elementary-derived sequence of conversions

(i) \(\alpha_1 \rightarrow \beta_1\)

(ii) \(\alpha_2 \rightarrow \beta_2\)

(27)

(iii) \(\alpha_n \rightarrow \beta_n\)

i.e., \(\alpha_1, \alpha_2, \ldots, \beta_1, \beta_2, \ldots\) contain no notational elements other than strings in some herd.
Then by a derivation of a representation $X$ (which must be a lowest level representation) we will mean a sequence of strings $\gamma_1, \gamma_2, \ldots, \gamma_m$, where $\gamma_1$ is $\alpha_1$ (i.e., it is the representation Sentence) and $\gamma_m$ is $X$, and each string $\gamma_i$ of the sequence follows from the preceding string $\gamma_{i-1}$ by means of the application of one of the rules of conversion of (27).

More precisely, we define:
Def. 11. \( \beta \) follows from \( \alpha \) by (27) if, for some \( \gamma, \delta, \alpha_1, \beta_1, \rho_1(\gamma \rho \delta) \),
\[
\alpha = \gamma \alpha' \delta \quad \text{and} \quad \beta = \gamma \beta' \delta
\]

Def. 12. A sequence \( \gamma_1, \ldots, \gamma_m \) is a derivation of \( \gamma_m \) if
(i) \( \gamma_1 = \alpha_1 \)
(ii) for \( i > 1 \), \( \gamma_{i+1} \) follows from \( \gamma_i \) by (27)
(iii) \( \gamma_m \) is a lowest level representation.

A derivation is not yet a sentence-grammar. A sentence-grammar is a sequence of representations of an utterance (a sentence), exactly one from each level. When a sentence is generated step by step by a derivation, it will have many representations partially in terms of one level, partially in terms of another. There will be a great many rules of (27) that convert morphophonemes into phonemes. Suppose that (27*) and (27+1) are two such rules, both of which are needed to convert \( \alpha \), a string of morphemes, into \( \beta \), string of phonemes. Then after the application of (27*) to \( \alpha \), and before the application of (27+1), we have a partially converted string, containing both phonemes and morphemes. In general, only very few strings of a derivation are unmixed, and thus belong to the sentence-grammar.

Def. 13. Given a derivation \( \gamma_1, \ldots, \gamma_m \), a sentence-grammar is
a subsequence \( \gamma_{a_1}, \ldots, \gamma_{a_k} \), where \( \gamma_{a_1} = \gamma_1, \ldots, \gamma_{a_k} = \gamma_m \), and each \( \gamma_{a_j} \) is an 'unmixed' representation.

Clearly, then, for this definition to have any significance,
levels must be defined in the general theory in such a way that we can determine from the grammar which strings belong to which level. Or, and this seems a more reasonable alternative, we must mark each statement of (27) to indicate to which levels it refers. This suggests grouping together those rules of (27) that convert phrase sequences into morphemes, those that convert morphemes into phonemes, etc. It turns out that there are much more compelling reasons for this grouping.

21.2. The analogy drawn above (§16.1) between the rules in formalized systems (27) and rules of inference is quite misleading in certain respects. For one thing, there are restrictions on the order in which the grammatical rules are applied in constructing derivations. The rules that convert phrases into morpheme strings will naturally have to be applied before the rules that convert morphemes into phoneme strings. It follows that there is a natural grouping of rules in terms of the order in which they apply. We can utilize the fact that (27) is a sequence of rules to establish this order. That is, we can place higher level rules before lower level rules, and we can indicate the level to which a rule belongs very simply by marking the place in the sequence where the rules belonging to a new level begin. In terms of the construction of derivations, then, we do have a hierarchy of levels.

The rules belonging to a higher level will automatically apply before the rules belonging to a lower level, but we have placed no restriction on the order of application of statements within a given level. Actually, such restrictions
can lead to considerable simplification of the grammar. Suppose we have on a certain level, a set of rules of the form (25). In general, there are heavy and complex restrictions on the conversion of $\alpha$ into $\beta$. That is, the statement of the conditioning context $\gamma \rightarrow \delta$ will be quite intricate. A good deal of this complexity can be avoided by restricting the order in which the rules apply.

Suppose, in particular, that we have a rule $\text{R1}$ converting $\alpha$ into $\beta$ in the context $\gamma \rightarrow \delta$, and a rule $\text{R2}$ converting $\alpha$ into $\beta'$ in every other context. Thus

$$\text{R1: } \alpha \rightarrow \beta \text{ in env. } \gamma \rightarrow \delta$$

$$\text{R2: } \alpha \rightarrow \beta' \text{ in } \gamma' \rightarrow \delta'$$

where $\gamma' \rightarrow \delta'$ is a specification of all other contexts. We see immediately that if we specify that $\text{R1}$ precedes $\text{R2}$ in application, then the conditioning context $\gamma' \rightarrow \delta'$ need not be specified in $\text{R2}$, since the only instances of $\alpha$ which 'get past' $\text{R1}$ are those in $\gamma' \rightarrow \delta'$. Thus we have simply

$$\text{R1: } \alpha \rightarrow \beta \text{ in env. } \gamma \rightarrow \delta$$

$$\text{R2: } \alpha \rightarrow \beta'$$

Suppose that $\gamma, \delta$ are quite simple, and that $\text{R3}, \ldots, \text{Rm}$ are various rules that convert $\gamma, \delta$ into a variety of complex forms. Then $\text{R1}$ must be extended to cover every form that the conditioning context $\gamma \rightarrow \delta$ may assume, or else, if $\text{R1}$ is applied before $\text{R1}$, $\text{R1}$ will be inapplicable to the result, and $\text{R2}$ will (incorrectly) apply. But the necessity to so extend $\text{R1}$ can be avoided by imposing an order of application on $\text{R1}$ in such a way that $\text{R1}$ must apply before $\text{R3}, \ldots, \text{Rm}$. Then only $\gamma \rightarrow \delta$, and not all of
the forms that these elements may assume in further
development, need be stated as the conditioning context
for the conversion $\alpha \rightarrow \beta$. This is a fairly standard type
of situation, and it is thus obvious that we can effect a
considerable simplification by ordering the rules in a
proper way.

The simplest and most elegant way to effect this ordering
will be to use the given ordering of the rules in the linear grammar (27),
elementary derived sequences, in the sense of 2a-1, 2c.

We can thus make a single statement in the general theory that establishes the order
of application of statements once and for all, for all grammars.

**Condition 1:** If $\gamma_1, \ldots, \gamma_m$ is a derivation of $\gamma_m$, given (27),
$\gamma_1$ follows from $\gamma_{i-1}$ by (27e), and $\gamma_{i+1}$ follows
from $\gamma_i$ by (27g), then $\gamma_i$.

If this condition is met, then, a rule can be applied
several times in forming a derivation (e.g., if the element $\alpha_i$
to which this rule applies appears several times in the
string being considered), but no rule can apply after a
rule which follows it in the $\gamma$.

We return to this question
in more detail in chap. VI.

We would like to have the rule $R_1$ (of (28)) apply before
the rules that convert its conditioning context $\gamma_i$ beyond
the form which is relevant to the determination of the
conversion of $\alpha$ into $\beta$. On the other hand, $R_1$ cannot
apply before the rules $R_2, \ldots, R_m$ that produce $\gamma_i$ from
some earlier elements, if the form of $\gamma, \delta$ is relevant to the conversion of $\alpha$ to $\beta$. In other words we want to place R1 in the sequence of rules in such a way that the conditioning elements $\gamma, \delta$ will have been developed to exactly the extent relevant to the determination of the conversion of $\alpha$ into $\beta$. If it is possible to place each rule in the sequence in such a way that its conditioning contexts are developed just to the relevant degree of detail, we can say that the grammar meets a certain condition of optimality. Putting it roughly, a grammar will meet this optimality condition if, when the rules are given in the maximally condensed form, it is possible to arrange the resulting statements in a sequence in such a way that

(i) we can form all derivations by running through the sequence of rules from beginning to end

(ii) no conversion $\alpha \rightarrow \beta$ will appear twice in the sequence (i.e., no rule need be repeated in several forms at various places in the grammar

(iii) each conditioning context is developed to exactly the extent relevant for the application of the rule in which it appears.

This statement is too vague to be literally applicable to the evaluation of grammars, but such an optimality condition can be given in a more refined and explicit manner, and can be weakened and further analyzed. This seems to me to be a worthwhile endeavor, but one which is perhaps premature now, before we have actually settled with any finality on an effective notation for grammar (i.e., definition of 'simplicity'). And this decision must be based on considerable experimentation
with the description of actual language material.

Nevertheless, we can use this crudely stated optimality condition as a guide to the correctness of analyses. It is not at all evident that it is possible to set up grammars meeting such a stringent condition, with such a rigid hierarchy of statements even within a level. (13) If it can be met, we not only have very simple grammars, but an extremely neat general theory as well, with the very simple condition 1 incorporated into def. 12 as the sole determinant of order of application of grammatical rules. When we find it possible to construct a linguistic level, for a given language, in such a way as to meet or approach this condition, we have good reason to suspect that we are on the right track.

When we consider a grammar to be a sequence of rules to be applied in order in constructing derivations, we must extend our characterization of the form of grammars in several ways. For one thing, the line of demarcation between the set of statements concerning one level, and the set dealing with another, must be sharply marked, as we mentioned above, so that we can reconstruct the system of levels from the grammar. Since we will discuss each level separately in the following pages, this will not concern us. Secondly, we must distinguish between those statements that are obligatory, and those that are optional. For instance, a rule which essentially lists the membership of the class $A$ as $a_1, a_2, \ldots, a_n$ will appear in the grammar in the form
(30) \[ A \rightarrow \{ a_1, a_2, \ldots, a_n \} \]

But only the bottom line of the expansion of (30) can be obligatory. In applying these notions below, we will consider various ways of meeting such situations.

Note that by introducing the notion of ordering, we have added a new feature to the analysis of simplicity. Now a given description of a language is more highly valued not only if it permits generalizations, but also if it leads to a simple, but quite rigid, hierarchical organization. The introduction of the dimension of order of statements was motivated by the recognition that, in many instances, the grammar can be simplified in quite a natural and effective way if certain restrictions on order of application of statements are imposed. We then, in condition 1, fixed upon a particularly simple general ordering condition for grammars. If this condition is built into the general theory, then grammars which have the formal properties fixed upon in this condition (i.e., grammars which best approach some optimality condition, as posed above), are more highly valued.

22. We have seen that as a consequence of the decision to consider the simplicity of grammar to be a factor in the choice among grammars, hence a factor in the determination of linguistic structure, we find it necessary to make a detailed study of the form of grammars and the method by which we derive structural information from a grammar. We have outlined a conception of a grammar as an ordered set of
rules, of the form $\alpha \rightarrow \beta$, and have discussed the question with respect to how such a grammar might be evaluated in terms of its simplicity. Sentence-grammars are generated by running conversions through the sequence of statements from beginning to end. This conception is incomplete in many respects. For one thing, we have not really stated an evaluation procedure, but only indicated how one might be stated. To complete the statement, we would have to indicate exactly how the symbols of the grammar should be weighted, e.g., should brackets and angles count in calculating length, should a simplification on the phonemic level be counted as more important than a simplification on the syntactic level, etc. Relevant here is the question of how much complexity should be tolerated on each level. The addition of a statement of fixed length to the description of a very simple structure will no doubt be judged far more serious than the addition of a statement of the same length to the description of an already complicated structure. Some of the most interesting questions in grammar construction, as we will see below, concern just this problem of weighing complication of one level against complication of another.

It would be quite easy to fill in these gaps one way or another, but it would also be quite pointless, at this stage of our knowledge. There is no reason to prefer any one of a number of different ways of completing the construction. Our approach from this point on will be experimental. We will test the workability of these notions by attempting to apply them very precisely to linguistic
material, and determining whether the results of this application are acceptable, illuminating and suggestive. When a great deal of such material has been collected, it will be possible to make some more rational decision as to how to proceed further with the constructional task. In particular, this problem of weighting the contribution to the total complexity of the description of each level is an important problem for investigation.

While the problems briefly cited above are unresolved, it seems that they can be accommodated within the present framework conception. But other defects of our analysis of simplicity are more serious. For instance, suppose that we find a rule that applies to all but one of a certain class of morphemes, this one being an exception requiring a separate rule. Suppose there is a pair of alternative rules, with the same total complexity, and describing the same situation, one of which applies to all but one member of the class, the other to this 'exception', the two rules being of equal complexity. In our terms the two solutions have the same value, but there is a real sense, that we have not captured, in which the first is superior. There are unquestionably many other facets of simplicity not covered by the simple conception sketched above, and it is quite important to investigate these. But it will appear that even on the basis of this limited conception, we can go quite far in studying language structure.

With this, we complete our preliminary investigation of the program and tools of linguistic research. The notions
that enter into linguistic theory(14) are those concerned with the physical properties of utterances, the formal arrangement of parts of utterances, conformity of utterance tokens (as determined by the pair test), and finally, formal properties of systems of representation and of grammars. Considerations of other kinds can be admitted if shown to be clear and relevant. We will refer to linguistic analysis carried out in these terms as 'distributional analysis.' This usage seems to me to correspond to the practice of what has been called distributional analysis.

Before entering into the direct analysis of the level structure of language, we must study the crucial question of determining 'possible' or 'grammatical' sentences. This will occupy us in the next chapter.
Footnotes - chapter III
(p. 73)
(1) This statement is paraphrased from Goodman, On the Simplicity of Ideas, Journal of Symbolic Logic, 1943, where the reference is to a special sense of simplicity, namely, economy in the basis of primitives. But the same can be said about other senses of simplicity that will concern us directly. See Quine, From a Logical Point of View, for discussion of the role of simplicity in setting up scientific theories.

(2) (p. 73) Actually, we will see below that there is a higher level representation than sentence, but this need not concern us now. We have not given any grounds for distinguishing higher from lower levels — these are simply a set of interdependent structures. For the purpose of this discussion, we may simply assume that the levels are arbitrarily, in the obvious way. Cf. § 21. 2. Furthermore, we will see below in the detailed development that sentence is best considered a representation in the level of phrase structure, but this need not concern us.

(3) (p. 76) In the appendix to chap. V we discuss an actual instance of a complex morphophonemic structure.

(4) (p. 78) But we will see below, chap. VII, that this reduction is only partially successful.

(5) (p. 78) This is a condition that must be met by the definition, or we will not even have an evaluation procedure (cf. § 2. 2, chap. I).

(6) (p. 81) I.e., we begin by investigation of languages where we are so insistent on certain results that giving these can be taken as a criterion of adequacy for the general theory. Cf. § 3. 1, § 7.


(8) (p. 88) This definition actually requires further qualification, because of the occurrence of brackets within brackets. Notice that we are interpreting the a here not only as strings of some level, but as any expression of the grammar, with or without grammatical constituents. We then must define paired brackets, in the obvious way, and define main brackets as paired brackets not occurring within paired brackets. We
then require that no $a_{ij}$ is enclosed within paired brackets, and that each $a_{ij}$ contains only paired brackets (if any at all).

(9) (p. 95) Main brackets (angles) being paired brackets (angles) not enclosed in brackets or angles.

(10) (p. 97) This is not a mere change of notation. It actually permits certain consolidations that would not otherwise be possible. Thus the systems of §20.1 and §20.2 provide analyses of simplicity that are not quite the same, and in the search for an adequate account of simplicity, these (and many other alternatives) should be separately studied with respect to their empirical consequences.

(p. 98)

(11) /i.e., all sentence-grammars. cf. §15.

(12) (p. 99) In this oversimplified introductory account we overlook the fact that concatenation of elements of different levels (different concatenation algebras) has literally no meaning. We discuss this, below, in the case of particular levels. We can understand each level as being embedded in the next higher level.

(p. 104)

(13) /Unless, of course, we relax many other requirements for grammars. Note that as we have given this condition, only a finite number of sentence-grammars can be generated from a grammar by running through the sequence of rules once. We may disregard this problem for the moment. Below (chap. VI) we will suggest a way of extending generation, but finally we will come to the conclusion that the infinite extension of the set of derivations comes about in a different way.

(14) (p. 108) We are referring, of course, to that central area of linguistic theory concerned with grammar. We have noted that there are other legitimate areas of linguistic research with other purposes and other tools. It would perhaps be preferable to replace 'linguistic' by 'grammatical' throughout this discussion.
Chapter IV: Grammaticalness

23. The first problem that the linguist must face in constructing the grammar of a language is that of determining the subject matter of his description. Given a corpus of sentences, this problem breaks down into two parts, as we have already seen in §5.7. First, he must determine which of these utterances are phonemically distinct. Second he must determine which utterances, whether in the corpus or not, are grammatical, hence to be described in the grammar, to be generated in the manner discussed in chap. III. The first problem is met by the pair test, the second will concern us now.

It is clear that the set of grammatical sentences cannot be identified with the linguist's corpus of observed sentences. Not only are there many (in fact, infinitely many) non-observed grammatical sentences, but, in addition, certain sentences of the corpus may be ruled out as ungrammatical, e.g., as slips of the tongue. Thus we must project the class of observed sentences to a larger, in fact, infinite class of grammatical sentences, and a linguistic theory we must define 'grammatical sentence' in terms of 'actual, observed sentence.'

24.1. Investigating the conditions that we want this definition to meet, we find that a partition of utterances into just two classes, grammatical and non-grammatical, will not be sufficient to permit the construction of adequate grammars in terms of what we have broadly described as distributional analysis. If we wish to distinguish between
two elements $X$ and $Y$ in distributional terms, we must be able to discover some significant class of contexts such that $X$ occurs in the contexts of $K$, forming grammatical sentences, by $Y$ does not. If $X$ also occurs in the contexts of $K$ in grammatical sentences, then we will have lost the distributional grounds for distinguishing $X$ from $Y$. A difficulty arises when it is the case that occurrence of $Y$ in $K$ is somehow 'near grammatical', that is, closer to grammatical than certain completely non-grammatical sequences. If $Y$ is excluded from $K$, we lose the grounds for distinguishing the sequences formed by placing $Y$ in $K$ from complete ungrammatical nonsense. To demonstrate by example that this difficulty does actually arise requires a decision as to what are significant contexts, i.e., it requires a grammatical theory. Since there are a variety of suggestions about grammaticalness, but few attempts at a precise statement, we could give a convincing demonstration only by presenting instances to counter any such suggestion. Instead of attempting to pursue this rather aimless course, we limit ourselves to a few examples that may make the general argument plausible.

Certainly any adequate grammar of English will have to distinguish such proper nouns as "Jones" from abstract nouns like "sincerity", or from nouns like "golf". At the same time, it will have to indicate that such sentences as

\[(a) \{ \text{Sincerity} \} \text{ admires Bill} \]

\[(1) \{ \text{Sincerity} \} \text{ had lunch with Bill yesterday} \]

are in some sense, near-grammatical sentences, much more so
than, for instance,

(2) (a) the admires Bill
     (b) sincerity the of

However, if we hope to set up proper nouns such as "Jones" as a distributional class, excluding "sincerity" and "golf", we will need some discriminatory class \( K \) of contexts. But such contexts as

(3) (a) -- admires Bill
     (b) -- had lunch with Bill yesterday

seem to be just the contexts relevant(3) to making this distinction, since certainly

(4) Jones \( \begin{cases} \text{admires Bill} \\ \text{had lunch with Bill yesterday} \end{cases} \)

are grammatical sentences. If we have only a two part classification, into grammatical and ungrammatical, then either such sentences as (1) are also regarded as grammatical, in which case we apparently lose the distributional grounds for making the distinction we require among nouns, or they are rejected as ungrammatical (as in some sense, they surely are) in which case there is no way to distinguish (1) from (2).

The same kind of argument can be somewhat differently applied to another sort of instance. An adequate grammar would have to indicate that the relation between "bring" and "brought" is the same as that between "like" and "liked", i.e., that "brought" is morphologically "bring" plus the past tense morpheme. We might try to show this distributionally by pointing out that just as we have
(5) They like it today.
(b) They liked it yesterday.

we also have
(a) They bring it today.
(b) They brought it yesterday.

But alongside of these we also find

(a) They break it today
(b) They broke it yesterday

Now, then, do we know that the pairs paralleling 
like-liked are bring-brought and break-broke, rather than 
bring-broke, and break-brought. The most reasonable 
distributional grounds for making the correct pairing would 
seem to lie in the fact that we have a class K of contexts as 

(a) Wars -- disease and famine
(b) They -- into a store

Now consider the class K_1 of sentences formed by putting 
"break", "broke" in the blank of (3 a), etc., and "bring", 
"brought" in (3 b), etc. If these sentences are ruled as 
ungrammatical, as in some sense they surely are, then we can 
make the pairing correctly on distributional grounds, since 
clearly "bring" and "brought" can occur in (3 a), and 
"break", "broke" in (3 b). However, this solution would 
fail to indicate that the sentences K_1 are somehow more 
grammatical than the class of sentences K_2 formed by placing, 
e.g., "Jones", in the blanks of (8) On the other hand, if 
K_1 are ruled to be fully grammatical, then we lose the basis
for making a correct distributional analysis of the verbs in question (and, at the same time, we admit some rather odd sentences into the language).

This leads us to observe that an adequate linguistic theory will have to recognize degrees of grammaticality, so that (4), (5)-(7), (8a) with "bring", "brought", and (8b) with "break", "broke", are all fully grammatical, (1) and $K_1$ are partially grammatical, and (2) and $K_2$ are completely ungrammatical.

24.2. We can approach this matter in somewhat different terms. Clearly one of the fundamental facts about linguistic behavior is that a speaker, on the basis of a finite and somewhat accidental experience with language, can produce utterances which are new both to him and to other speakers, but which are immediately recognizable as utterances belonging to the language. We might restate this ability, somewhat figuratively, (5) by saying that in learning a language, the native speaker has done much more than merely absorb a large set of sentences which he can now reproduce. He has also abstracted from this set of sentences, somehow, and learned a certain structural pattern to which these sentences conform. And he can add new elements to his linguistic stock by constructing new sentences conforming to this structural pattern.

Is it possible to reconstruct this ability within linguistic theory? That is, can we develop a method of analysis which will enable us to observe a corpus of sentences,
to abstract a certain structural pattern from this corpus, and to construct, from the old materials, new sentences conforming to this pattern? This is a question of fundamental importance. Our working hypothesis is that we can give an account of this process of generation or projection within the limits of distributional analysis. That is, we aim to construct in linguistic theory a formal model of this behavior in such a way that by applying the methods of linguistic analysis to a corpus of sentences, the linguist can reproduce this process of generation in his determination of grammatical sentences. It is by no means obvious that an even partially adequate reconstruction of this behavior can be given in distributional terms, i.e., in terms of the structural characteristics of observed utterances. It might be the case that many other factors in the particular history and development of the individuals concerned may be responsible for this ability. Hence if this hypothesis can be validated and demonstrated to any considerable extent, this will be a significant result. The endeavor program of developing methods of linguistic analysis, or, in our terms, a theory of linguistic structure, might be interpreted as being basically an attempt to reconstruct this ability to speak and recognize new grammatical utterances. We can expect such a reconstruction to be complex, with contributions formulated in terms of many different linguistic levels. Our direct concern, in this chapter, is to reconstruct the first and most elementary step in this process of projection. This will occupy us in §25-27. In §35, we suggest how the process can be continued in
terms of higher levels, and these suggestions are elaborated in succeeding chapters.

But we must be quite careful in determining just what we are to reconstruct. In the only pre-systematic terms we have (cf. §5.7, §7), we can say that we are trying to reconstruct and explicate that intuition about linguistic form that enables a speaker to distinguish grammatical sentences like (2), (3), [chap. 1] and (4), [this chapter] from such non-grammatical sequences like (5), [chap. 1] and (2), above. We have already noted in §5.7 that grammaticalness cannot be identified with meaningfulness or significance. (6)

Grammatical sentences may or may not be significant, and we must distinguish between grammatical nonsense, like (2), (3), [chap. 1] and non-grammatical nonsense, like (4), as in the (5), [chap. 1] and (2) [this chapter] cases cited above.

More generally, there is little doubt that speakers can fairly consistently order new utterances, never previously heard, with respect to their degree of 'belongingness' to the language. Thus the following sentences might all be new in English:

(9) look at the cross-eyed elephant
(10) " " " kindness
(11) " " " from

but I think it is clear that any native would arrange them in this order with respect to 'belongingness' to English. In other words, we might say that a speaker projects his finite and somewhat accidental linguistic experience to
a set of more and more comprehensive extensions. Corresponding
to this fact, linguistic theory must develop a notion of
'degree of conformity to the structural pattern', so that,
given a reasonable sample of English not containing (9),
(10), or (11), we might predict that (9) (like (4)) is
perfectly grammatical, (10) (like (1)) is partially grammatical,
and (11) (like (2)) is not grammatical at all.

Thus from several points of view, we are led to
consider as the goal of our reconstruction, the notion
of 'degree of grammaticalness'.

25.1. The most reasonable model for explaining and
reconstructing this projectibility seems to be too based on
the notion of syntactic category. Let us assume that we have
a finite set of sentences, the corpus, with word division
marked. The corpus might contain, for instance,

(a) John came
(12) (b) Bill ate
(c) John saw Bill
etc.

We assign these words to classes. Let us call this
assignment a syntactic analysis of the words of the language,
leaving open for the moment the question of whether these
categories (i.e., have no members in common).
categories are disjoint. We can now associate with each
sequence of words a sequence of classes, replacing each
category word by the class to which it belongs. Thus if we assign
'John', 'Bill' to the class N, and 'came', 'ate', 'saw'
category to the class V, we will have NV, NV, and NVN as sequences
categories of classes corresponding, respectively, to (12a), (12b), and
(12c). There will in general be many fewer sequences of
categories than sequences of words. Each sequence of categories may be called a sentence form, and we can construe the generated language of grammatical, but perhaps non-occurring sentences, as those which conform to one of these sentence forms. In this example, for instance, 'John ate,' 'Bill saw John,' etc., would be sentences, perhaps not in the corpus, but conforming to the sentence forms constructed from the syntactic analysis.

The notion of degree of conformity to the structural pattern is easily derived. Instead of considering only one syntactic analysis of words into categories, we consider several analyses of various orders. That is, we have broad syntactic categories like Noun and Verb, subcategories of these, subsubcategories, etc. Thus we might have a first order syntactic analysis into a hundred categories, a second order analysis into seventy categories, down to, e.g., a tenth order analysis into one category. For each order of subcategories we have a set of sentence forms in terms of the categories of that order. For an order with many categories, hence small categories, we have sentence forms which are more selective and which generate few sequences. For an order with few categories, hence large categories, we have sentence forms which generate a great many sequences. A non-occurring sequence then has a higher degree of grammaticalness if it conforms to the more selective sentence forms stated in terms of a many-category (hence low order) analysis.

Referring to the previous example, assume that (13) look at the cross-eyed man

(13) look at the cross-eyed man

does occur in the corpus. We see that (9) has a high degree of grammaticalness, since 'man' and 'elephant' are
presumably co-members of the small subclass of Animate Common noun, and thus (9) conforms to the selective sentence form stated in terms of this small class. (10) is less grammatical, since 'man' and 'madness' are co-members of no class smaller than the larger class Noun, and (11) is still less grammatical, since the only class containing both 'man' and 'from' is presumably the class of all words.

*25.2. For the sake of clarity and uniformity of terminology, let us describe more precisely the kind of system we have in mind.

We have a system $\mathcal{C}$ of classes of words,

$$\mathcal{C} = \left\{ \mathcal{C}_i^n \right\}, \text{ where (i) } 1 \leq n \leq N$$

(ii) $1 \leq i \leq a_n$

(iii) $a_1 > a_2 > \cdots > a_N$

(iv) $w \in \mathcal{C}_i^n \Rightarrow w \text{ is a word}$

(v) $\mathcal{C}_1^n \neq \emptyset$, $\mathcal{C}_1^n \cap \mathcal{C}_1^n \Rightarrow i = i$

There are many other conditions that we can put on $\mathcal{C}$, but this will suffice for our immediate discussion.

For each $n$, the set $\mathcal{C}_n = \left\{ \mathcal{C}_1^n, \ldots, \mathcal{C}_{a_n}^n \right\}$ will be called the syntactic analysis of order $n$, and the $\mathcal{C}_i^n$'s are called categories of order $n$.

We are given a corpus $K$ which we take to be a set of sequences of words. A sequence $\mathcal{C}_1^{n_1}, \ldots, \mathcal{C}_m^{n_m}$ of categories of order $n$ is said to generate the word sequence $w_1, \ldots, w_m$ if $w_1 \in \mathcal{C}_1^{n_1}, \ldots, w_m \in \mathcal{C}_m^{n_m}$. Thus the set of word sequences
generated by a category sequence is, the Cartesian product of the categories, or, as Wells has called it, (8) a **sequence-class**.

A sequence of categories of order \( n \) is said to be a **grammatical sentence form** of order \( n \) if one of the word sequences generated by it is in \( K \).

The grammatical sequences of order \( n \) are those generated by the grammatical sentence forms of order \( n \). Thus the highest degree grammatical sentences are those of order 1, the order with the largest number of categories. If \( a_N = 1 \), then all sequences are grammatical of order \( N \).

26.1. Assuming that such a model can give an adequate account of projectibility, the question is: how can we construct a formal and abstract notion of syntactic category that will lead to an appropriate system of categories, when we are presented with an adequate sample of linguistic material. Or, to put it differently, we must develop a procedure such that given a corpus of sentences with word division marked (i.e., a set \( K \), as in §25.2), we can, by means of this procedure, construct a system of classes \( \mathcal{C} \) which will give the correct account of grammaticalness.

The most obvious approach to this problem seems to be some sort of substitution technique. The very nature of our goal dictates that these categories \( C^p_1 \) be classes of elements that are, in some sense, mutually substitutable. But attempts to describe this sense by developing a substitution technique run up against serious difficulties. We will now
briefly consider some of these difficulties and the possibilities of elaborating a direct substitution technique in such a way as to avoid them. Then, in §27, we will discuss in greater detail a somewhat different approach to the problem.

26.2. Consider first the problem of constructing the smallest categories, the categories $C^1, \ldots, C^1_{g_1}$ of order 1. This is the problem of defining highest degree grammaticalness. The use of a substitution technique for this construction faces two immediate difficulties. In any sample of linguistic material, no two words can be expected to have exactly the same set of contexts. On the other hand, many words which should be in different categories will have some context in common. There are certain contexts like

\[(15) \quad \ldots l's \ldots\]

where elements of many different categories can appear (e.g., "John", "red", "mine", "here", \ldots). Thus substitution is either too narrow, if we require complete mutual substitutability for co-membership in a syntactic category $C^1$, or too broad, if we require only that some context be shared. Thus neither of the simplest approaches to substitution is adequate, without elaboration.

Though we can scarcely hope to find two words mutually substitutable everywhere in a linguistic corpus, we might find some behavioral test for mutual substitutability, so that we could run through our (finite) corpus, testing every pair of words for this property in each context. Then the $C^1$ could be defined as classes of words among whose members complete mutual substitutability holds.
The unacceptability of this approach lies in the fact that it simply avoids the very question with which we are concerned. An operational test for grammaticalness would simply record the speaker's ability to project, but a systematic approach which defines grammatical sentences in terms of the rules of combination set up for actual observed utterances could actually be said to provide an explanation for this ability in formal terms, and as such, it would represent a fundamental contribution to our understanding of linguistic behavior. Replacing a systematic account of grammaticalness by a behavioral test would mean abandoning this goal, which is both important, and, it seems, quite possible to attain.

To put it differently, this approach amounts to taking first order grammaticalness as a primitive notion in linguistic theory. The purpose of a theory is to investigate to the fullest extent the relations between the notions that enter into the theory. Complete success in unearthing and expressing these interrelations is marked by the elimination of primitive terms from the primitive basis. The discovery of an effective behavioral test for first order grammaticalness, then, should be taken as posing (rather than solving) the problem of accounting for this fact (now presumably behaviorally marked) of first order projection.

This discussion recalls two earlier ones. The problem of the preceding paragraphs came up in the brief discussion of morphemes in \( \text{5.7} \). Also, in discussing the question of phonemic distinctness in \( \text{5.5-6} \), we observed that it might be
possible to determine the phonemic distinctions in a given language by systematic investigation of the formal and physical properties of the utterances of the language, and nothing else. But we also expressed doubt that any such approach, based on some absolute notion of relevance, can be successful, and we developed a simple operational test, the pair test, to resolve the problem of determining phonemic distinctions. If these remarks on phonemic distinctness and projection are supported by further study, then one difference between the status of these twin problems in linguistic theory is that in the case of the former we must be satisfied with a behavioral test which merely records the behavior with which we are concerned, while in the second case we may actually hope for a systematic explanation of the behavior, in formal and structural terms.

26.3.1 If we could eliminate such contexts as (15) as illegitimate on some grounds, we could define the terms of the relation of substitutability in some context. We might, for instance, take the $C^1_i$'s as the equivalence classes determined by the ancestral of the relation "substitutable in some legitimate context"), or as classes of elements every pair of which share some legitimate context, etc.

Let us make the assumption that contexts like (15) can be excluded on some grounds, and that the $C^1_i$'s are defined. We return to this assumption below. Meanwhile, we will see that still further problems must be faced by a direct substitution technique.
26.3.2. It is evident that substitutability is directly relevant only to the establishment of the simplest categories. For words which belong to the same higher order (i.e., larger and less selective) category, e.g., the nouns 'horse' and 'justice', it would be difficult to find any natural sentence in which they can replace one another.

Putting it differently, if we are to use substitutability in the manner sketched above to construct the first order categories, then we cannot use it to establish higher orders. If the \( C_1 \) are correctly constructed, then 'horse' and 'justice' will be in different categories; i.e., all contexts shared by abstract and common nouns will have been excluded along with (15).

For this reason, it is necessary to develop the methods of substitution somewhat further. One way to do this is as follows. If we find two words in the same context, we assign them to the same category. We now rewrite the corpus, replacing words assigned to the same category by a single symbol everywhere they occur. In other words, we disregard the distinction between words assigned to the same category. This reduces the number of distinct contexts, so that new pairs of words become mutually substitutable, and we can build larger categories. We can thus proceed to analyze the corpus in a constructive manner, using the information obtained at each step in determining substitutability relations for the succeeding step. Such a procedure will lead to a system of categories from which an interpretation of the form \( \mathcal{C} \) must still be selected. Although the problem
of defining grammaticalness is thus not completely solved, given such a technique, it is tremendously reduced in scope. (9)

It seems that a substitution technique must be at least as powerful as this if it is to be effective. This outline of a substitution technique can be developed into a formal construction in several different ways. It is interesting, among other things, to investigate the effect of applying various formulations of a substitution technique to different kinds of language. Given a classification of languages on some structural basis, we can determine which formulations of a substitution technique are equivalent in the sense that they lead to the same syntactic categories for a given kind of language. Any results obtained in this study can in turn be applied to determining the structural type of a given language, by investigating the results of applying various forms of category analysis to the language in question.

Such techniques can be extended immediately to include substitutability of phrases (i.e., sequences of words) as well as of single words. This leads to a generalization of the notion of syntactic category to cover at least a partial analysis of phrase structure, and to an extension of the notion of grammaticalness. One important result of this extension is that now the projection of the corpus can include sentences longer than those of the original corpus. We may be able to develop the infinite generative power required for a full account of grammaticalness by continually repeating the analysis for the successive projections of the corpus. Naturally, it may result that even the word categories are
extended when we generalize substitutability in this way, since a much more extensive analysis of sentence structure is now possible. (10)

The extension to include substitution of word sequences raises a further problem of segmentation. We cannot freely allow substitution of word sequences for one another. Thus we cannot set up a category containing "from New York was here" and "left", even though both occur in the context "my friend -- "(11) Thus it may be necessary not only to exclude certain contexts as illegitimate, but also to rule out certain sequences as improper substituends.

26.3.2. A final difficulty for a substitution technique is the problem of homonyms. Such words as "will", /tuw/ ("to", "too", "two"), /riyd/ ("read", "reed"), etc., are best understood as belonging simultaneously to several categories. If this approach to homonymity is followed, in general the categories of a given order will not be disjoint. Hence the technique of substitution must be designed in such a way as to lead to a system C with the categories of a given order overlapping in the syntactic homonyms.

The problem of homonymity is an important one. In preparation for later discussions of homonymity, we note now that only certain cases of homonymity are relevant to grammar, namely those that can be interpreted syntactically by assignment of the words in question to the overlap of syntactic categories of some order. Thus "will", /tuw/, and /riyd/ are instances of syntactic homonyms in this sense, while /san/ ("son", "sun"), or "will" in its various nominal uses, are perhaps not. To a certain extent, this
distinction is relative to the subtlety of our grammatical analysis. Thus a more refined account of high order grammaticalness may convert certain cases of apparent purely semantic homonymity into cases with syntactic correlates.

Among the syntactic homonyms, there is one further distinction of importance. In the overlap of Noun and Verb, we find both "walk" and /riyâ/, but clearly these are very different kinds of homonyms. In the first case, we are inclined to say that we have to do with a single word which is in two categories, while in the second, we are more inclined to describe this as a case of two words, one a noun, one a verb, with the same phonemic shape. This distinction might be felt to be a wholly semantic one. We cannot deal with it at this stage in our investigations, but below, we will suggest syntactic grounds for it (4, chap. IX).

One way to approach the problem of homonymity within the framework of a substitution procedure as outlined above would be to investigate elements of narrower distribution before investigating elements of broader distribution. Since homonyms are elements which have the distributional features of all categories to which they belong, it follows that non-homonyms will be investigated first. We can proceed systematically in this way, building up the basic classes first, then investigating elements of wider distribution in terms of the categories already constructed. This proposal is of course too inexact to stand, but it can be made more
precise in various ways, and it suggests that substitution techniques can perhaps be elaborated to deal with homonymity.

26.4. A procedure such as that of §26.3 requires for its success that we make an intelligent choice of the elements to which we apply it. If we do so, we can pass over such contexts as (15), and we can begin our investigation of distribution with non-homonyms. Linguistic theory must replace this element of 'intelligent choice' by an exact specification of the conditions under which each step is taken in analyzing a language. In the preceding paragraph, we mentioned one possible way to make the proper choice for the case of homonyms. But even if we find an effective method for choosing, at each stage of the analysis, the proper substituends, we still must face the problem that a blind choice of contexts may admit such contexts as (15), and thus lead us to establish improper distributional relations even between properly chosen elements.

One way in which a procedure which fails in this manner might be converted into an adequate procedure is by repeating it many times with randomly chosen sequences of contexts. Suppose we apply the procedure of §26.3, choosing a context at random and assigning the elements that occur in it to the same category, choosing a second context at random, etc. We stipulate that if, at any point in this process, we come upon a context that would throw together large classes already established, then this context is rejected. (13) Suppose that we apply this procedure many times, each time with a randomly selected
sequence of contexts. If, in one of these applications, a context like (15) occurs early in the sequence, then this application will fail to give the correct analysis. But if contexts like (15) are relatively infrequent, then there will be many applications in which they do not occur early in the sequence, so that we will have grounds, by the stipulation suggested above, for dropping them from the class of contexts in this application. Such applications will then lead to the correct analysis. In other words, if contexts like (15) are relatively infrequent, then we will find that in the sequence of applications of this procedure, a certain syntactic analysis will keep recurring over and over quite frequently. And this frequently recurring analysis can be selected as the correct analysis. (14) Considerations of this kind may help to avoid many of the obstacles confronting a direct substitution procedure such as that of §26.3.

26.5. Another way to approach this complex of problems is through the analysis of clustering. In §26.2, we ruled out as uninteresting the possibility of defining syntactic categories of the first order as classes of words which are mutually substitutable in all environments, and in §26.3-4 we have been discussing various ways of elaborating a conception of syntactic categories according to which mutual substitutability in some environment is the basis for assignment to the same category. Another line of thought is suggested by a decision to assign words to the same category if they share a certain proportion of...
contexts. There are many ways of elaborating such an approach in detail. We will briefly touch upon a few.

For each word there is a set of contexts in the corpus in which this word occurs. This set we define as its distribution. We can define the distributional distance between two words as the number of contexts they share divided by the total number of contexts in which either can occur, i.e., by the logical product of their distributions divided by the logical sum, a number between 0 and 1. More generally, given $n$ words $a_1, \ldots, a_n$ with distributions $A_1, \ldots, A_n$, respectively, we can define the cohesion of this class as the cardinal number of the logical product of $A_1, \ldots, A_n$ divided by the logical sum, i.e., as $A_1 A_2 \cdots A_n / A_1 \lor A_2 \lor \cdots \lor A_n$, so that distance is a special case of cohesion. Clearly we cannot determine the cohesion of a class from knowledge of pairwise distances between its members.

We are interested in choosing categories in such a way as to maximize the amount of mutual substitutability among the words belonging to the given category. Let us call the quantity that we would like to maximize the cluster-value of the category. We can determine the cluster value of a category as a function of the cohesions of its subsets. There are various ways in which this can be done, each making precise a certain sense of the vague notion of mutual substitutability. Suppose we fix on one such function, thus defining the cluster value of a category. We might then define maximum the joint-cluster-value of a set of categories as a function of the cluster
values of the categories in this set, e.g., their average. Or we might define this directly in terms of the cohesions of subsets of categories, in various ways. Given such a definition we would choose the set of categories as the set with the highest joint-cluster-value.

This only gives us the best analysis into \( n \) categories for each \( n \). Thus we still have the problem of selecting a system \( C \) from this system of classes by choosing the \( n \)'s for which the \( n \)-class analysis is to be selected as a syntactic order. This might be done, for instance, by considering, for each \( n \), what the best joint cluster value of an \( n \) class analysis would be if words were distributed randomly in sentences, and choosing those \( n \)'s for which the deviation of the joint cluster value from this value is above a certain amount, or is at a relative maximum, etc.

If we can develop an effective notion of clustering, this can be used either to establish a system \( C \) directly, or to provide a basis for application of a system of the kind outlined in §26.3. That is, we might use cluster analysis to exclude contexts like (15) (these being characterized by the fact that elements belonging to several clusters appear in them) and to determine homonyms (as elements belonging to several clusters). These two methods of applying cluster analysis would not necessarily lead to the same system of categories.

This outline can be filled in with formal detail in several different ways, leading to alternative definitions of clustering. We would be led to favor one of these formulations if it were shown to have particularly interesting
formal properties, or (and this is the ultimate test) the correct empirical consequences. A certain amount of investigation on the formal side of the question has not led to any conclusive reason for choosing one of several formulations, and it would clearly require a tremendous amount of data to present even the most fragmentary empirical validation. For these reasons, I will not go on to present in detail any one of the various ways of realizing the program just outlined. Nevertheless, the study of this complex of notions seems to be of considerable importance and promise for distributional analysis, and despite its difficulties and unclarities, it should certainly be pursued further. (15)

26.6. The line of reasoning that underlies the discussion of 126.5 can be generalized beyond substitution techniques. Given any distributional property \( \Phi \) of words (or words and phrases), we can attempt to set up syntactic categories on the basis of \( \Phi \) by defining the distance between words (or the cohesion of a set of words), the cluster value of a category of words, and the joint cluster value of a set of categories, all in terms of \( \Phi \). In the preceding remarks, we have taken the total distribution of a word in the corpus (unweighted by frequencies) to be the relevant distributional characteristic \( \Phi \), and have defined cohesion as the logical product of distributions divided by their logical sum. But there are many other distributional characteristics which are reasonable candidates for this type of category analysis, and which might be easier to work with. For instance, we might take
as \( \Phi \) the frequency with which a word occurs as the first word in a sentence, or the frequency distribution of its occurrence as the \( i \)th word in a sentence. Or we might take its distribution (in the sense of \( \text{(26.5)} \)) weighted by frequency of occurrence in various contexts. Or, following a suggestion of V. Yngve, (16) we can define the gap between words \( x \) and \( y \) as the number of words occurring between \( x \) and \( y \) (\( x \) and \( y \) being here word tokens), and we can study the frequency distribution of gaps throughout the corpus for each pair of words (here, word types) \( x_1, x_2 \). This will be a certain function \( F_{i1} \). Fitting such a study into our framework, we note that for each word \( x_1 \) there is a set of such frequency distributions, \( \Phi \), one for each word \( x_1 \), \( \ldots x_n \) in a corpus of \( n \) distinct word-types. We can take this set of distributions \( \Phi \) as the characteristic \( \Phi \), so that with each word \( x_1 \) we have associated the set of functions \( F_i = \{ F_{i1} \mid 1 \leq i \leq n \} \), \( \Phi \), \( F_i \).

We can then attempt to define distance between words \( x_1, x_2 \) in terms of the pairwise similarity between the associated sets \( F_i, F_i \).

There are many other distributionally defined properties that deserve consideration here. Thus we might consider morphological criteria as being particularly crucial, or we might assign a special status to words of higher frequency like "the", "of", etc. There is no logical necessity to limit ourselves to one criterion. Some combination of these or other distributional properties may be selected as \( \Phi \). (17)
The correct choice of a distributional characteristic \( \varphi \) as the basic datum for category analysis will be determined, ultimately, by the empirical consequences of that choice. But whatever choice is made, it seems that the line of reasoning sketched in (i)-(iii) above, or something quite similar, will have to be followed, in order to determine the empirical consequences. The investigation of \( \varphi \) alone is of limited interest, in this connection, until we state how a set of categories is to be derived from it.

26.7. The purpose of the rather unorganized remarks about substitution techniques in \( \S 26.1-5 \) has been to indicate briefly the kind of problem that must be faced by such procedures, and to suggest that there are a large variety of ways of attacking these problems that have not been sufficiently studied. However, it is not surprising that an effective substitution technique should be so elusive and difficult to obtain, because if we could formulate such a technique, it would meet the strongest requirement that we considered above (\( \S 2.3 \)) for correspondence between the general theory and particular grammars. A substitution technique would be procedural in the sense that it would lead from the data directly to the correct grammar, that is, it would offer a practical and mechanical discovery procedure for grammars. It would tell us how to actually go about building the classes. Each of the elaborations discussed above has been designed with that end in view. Each of them began with the determination of a certain distributional characteristic of words (or word sequences)
and went on to develop a notion of syntactic category as a set of words maximally similar, in some sense, in terms of this characteristic.

In view of the difficulties involved in constructing such a powerful device, I think it can prove interesting to lower our aims to the weaker correspondence between theory and particular grammars, and to try to construct a definition of syntactic category that begins not with a distributional characteristic of words, but with a certain measurable characteristic of completed syntactic solutions; that is, a definition that merely enables us to assign a value, say a number, to each proposed analysis, and thus to decide mechanically between two proposed analyses, as to which is the better, with no concern as to how, in fact, these analyses were constructed. In accordance with this weaker aim, I would like to sketch a conception of syntactic category that seems to undercut many of the difficulties cited.

27.1. Suppose that we consider once again the method given in \(^{92}\) for generating sentences once we have a set of categories of a certain order.\(^{18}\) We rewrite each sentence of the corpus as a sequence of categories, replacing each word by the category to which it belongs. This gives a set of sentence forms, and we may now generate all sentences of these forms. This gives a great many new sentences, since along with each original sentence we now have all sentences of the same form as this original sentence, whether or not they appeared in the corpus.
Let us suppose now, for the sake of simplicity of exposition that all the sentences of the corpus are of the same length. We also assume, for the moment, that we are discussing only a given fixed order of syntactic categories, and that the number of categories of this order is fixed and preassigned. We return to the latter assumption in §27.2 and to the former in §27.3.

Suppose that the categories of a proposed syntactic analysis are set up on the basis of complete mutual substitutability. That is, two words are members of the same category only if in the original sample, each word occurs in every position in which the other occurs. It is clear that for such a syntactic analysis, no sentence can be generated if it did not already appear in the original sample. Thus no new sentences are generated.

At the other extreme, suppose we had a syntactic analysis in which words are members of the same class just in case they share no contexts. In this case a great many new sentences will be generated. In general, more new sentences will be generated to the extent that the distributions of the elements within a single syntactic category differ. The number of sentences generated by an analysis thus gives some measure of the extent to which elements in one class have similar distributions, and it seems reasonable to measure the value of a syntactic analysis by the number of sentences that it generates, fewer sentences being generated by a better analysis. Thus we evaluate an analysis by seeing how good an approximation it gives to
the original corpus, how few sentences it generates beyond those of the original corpus.

The foremost problem faced by a substitution technique was seen to be the difficulty of deciding how many contexts must be shared for two elements to be in the same category. Such questions are avoided here, since this technique does not build categories step by step, but rather provides a procedure for evaluating a completed solution. Elements may be in the same category even if they share no context. This property is important, since in actual linguistic material, the selectional restrictions on distribution are extremely heavy, and literal substitutivity is distinctly the exception rather than the rule. Nevertheless different nouns do substitute for each other in the sense that they all occur with some verb, though rarely with the same verb. Similarly, individual verbs are substitutable in contexts defined by the categories Noun, Adjective, etc., though rarely in the context of particular nouns and adjectives. This technique, as distinct from a substitution technique, permits us to use this fact by, as it were, setting up these classes simultaneously.

Consider now the problem of homonyms. This is essentially the problem of when to put a word into two or more of the categories of the syntactic analysis. If a word is put into more than one category, there is always a loss in the value of the analysis in one respect. To see this, notice that each time a certain category occurs in
a sentence form, a set of sentences is generated for each word in that category. Hence the more words in a category, the more sentences are generated, and the lower is the value of the syntactic analysis. When a word is put into a second category, this second category now has an extra member, and it follows that more sentences are generated wherever this second category occurs, with a corresponding drop in the value of the analysis.

On the other hand, if the element is a real homonym, there may be a compensating saving in the following way. Consider an English homonym like /tuw/ ("to", "two", "too"). If this word is put only into the category of prepositions, then, since /tuw/ can appear in sentences like: "there are two books on the table", it follows that all prepositions will occur in the numeral position in the generated language. But if /tuw/ is put in both the preposition and numeral categories, then a given occurrence of the word can be classed either as a numeral or as a preposition. Since the occurrence of numerals will occur anyway in "there are two books on the table", no new sentence forms are generated if this occurrence of /tuw/ is classed as a numeral, and there is consequently a considerable saving in the number of generated sentences. We see that assignment of a word to several categories may increase the value of the analysis. Thus we have a way of deciding when to consider a word as being in fact several homonyms (or course, it must be shown that the decision is the correct one, in terms of presystematic criteria). We do this if the
loss incurred in assigning it to several categories is more than offset by the gain; and there is always a numerical answer to this.

We return to the question of the adequacy of this conception of homonymity below in §33, and in the appendix to this chapter.

27.2. We can now determine for each $n$, the best analysis in terms of $n$ categories. As $n$ increases, the $n$-level classes will be smaller, and projection will be more limited and selective. Thus the degree of grammaticality of the projected sentences will be higher. Where $n = \text{the number of words in the corpus}$, the set of sentence forms is exactly the corpus itself, and no new sentences are generated. Where $n=1$, there is only one sentence form, but every possible sequence is generated in terms of it. The number of sentences generated is thus a non-increasing function of the number of categories.

However, we still have the problem of selecting a system $C$ from this set of analyses. This is the problem of determining for which $n$ we actually set up the $n$-category analysis as an order of the system $C$, i.e., a set $C^n = \{ \sum_{i=1}^{n} a^n_i \}$. We have noted this problem above in the discussion of cluster analysis (§26.5). Our aim here is to select a certain $n$ such that the $n$-category analysis compares very favorably with the $n-1$ category analysis, but is not much worse than the $n+1$ category analysis, that is, such that there is a large drop in the number of sentences generated when we move from the $n-1$ category analysis to the $n$-category analysis, but only a small drop in moving
from the $n$-category analysis to the $n+1$ category analysis. In other words we are interested in minimizing a certain function of $n$ and the number of sentences generated by the $n$-category analysis. At this point we can only speculate about which function should be chosen for minimization. There are several possible candidates, and at this point there seems to be no compelling reason for making a choice one way or another. This decision turns upon the empirical consequences of the various choices, and we simply haven't the requisite data at this stage of our knowledge.\(^{(19)}\) But it seems reasonable to hope that this is no defect in principle, and that the proper kind of empirical investigation may lead directly to a decision, thus filling in the remaining gap.

Whatever function we do minimize, it must be remembered that we are interested in relative minima, not just the absolute minimum, since we will in general be interested in finding several admissible $n$'s, i.e., several orders of analysis, each order providing a more detailed analysis than the orders with fewer (and larger) categories. We will see below (\(\xi, \lambda\)) that the absolute minimum, and the dichotomous partition that it imposes on sequences, may, however, have a special significance.

27.3. We have seen how, given a set of sentences of a fixed length \(\lambda\), we can determine, for each $n$, the best analysis into $n$ categories of the words of which these sentences are composed. If we make the additional assumption that in any language, the optimal $n$-category analysis for
sentences of length $\lambda_1$ will be the same as the optimal $\alpha$-category analysis for sentences of length $\lambda_1$, for any $\alpha$, $\lambda_1$, $\lambda_1$, then the theoretical development of syntactic categories need be carried no further. In order to determine the system $C$, it will be sufficient to choose a single sentence length $\lambda$, and to investigate only sentences of length $\lambda$.

But it might be argued that almost any word can a occur as a one word 'sentence'. Clearly the shortest utterances must be excluded from the syntactic analysis. We would naturally not choose $\lambda=1$, and attempt to determine the system $C$ from investigation of one word sentences, since no grammatical structure appears here. Thus a more adequate form of this assumption would be that there is a fairly small $\lambda_0$, fixed in advance for all languages, such that for all $\alpha$, $\lambda_1$, $\lambda_1$ ($\lambda_1$, $\lambda_1 > \lambda_0$), the optimal $\alpha$-category analyses for $\lambda_1$ and $\lambda_1$ are identical.

Some such assumption seems fairly reasonable. We would hardly expect the basic principles of sentence construction to vary markedly from one sentence length to another.

If we do not wish to make this assumption, there are several courses available. (20) The most reasonable approach seems to be to assume that any sequence of sentence forms, with sentence break marked, can constitute a 'discourse form', and to measure the value of an analysis in terms of the number of discourses it generates. We thus add the break between sentences as a 'word' belonging to a separate category $\#$, no matter what the analysis. (21) Thus a discourse
form is simply a sequence of categories, including the category # at various places. Generation of discourses from discourse forms is exactly like generation of sentences from sentence forms.

Suppose we let \( S_\lambda \) be the number of grammatical discourses of length \( \lambda \) generated by a given analysis into \( n \) categories from all grammatical discourse forms of length \( \lambda \). We are interested in determining the limit of some function of \( S_\lambda \) as \( \lambda \) approaches infinity, i.e., as discourses become longer and longer. But it might be the case that for infinitely many values of \( \lambda \), \( S_\lambda = 0 \) (for instance, if all sentences are exactly ten words long). It is thus preferable to investigate not \( S_\lambda \) but \( S'_\lambda \), where \( S'_\lambda \) is defined in terms of a more general notion than discourse form. We define a general discourse form as the initial part of some discourse form. Thus a general discourse form is a sequence of categories formed by stringing together zero or more sentence forms (with sentence break properly marked), followed by the initial (perhaps null) part of a sentence form. \( S'_\lambda \) is the number of sequences of length \( \lambda \) generated by a given analysis into \( n \) categories from all grammatical general discourse forms, i.e., the number of generated beginnings of grammatical discourses (where of course a discourse is the beginning of some discourse). Since there is a finite upper bound to sentence length in our present discussion, it turns out that in the limiting case, we are still measuring only the contribution of complete sentences, not sentence
fragments, just as we require. We can now define the value of a given analysis \( A^1 \) into \( n \) categories as

\[
\text{Val}(A^1) = \lim_{n \to \infty} \frac{\log S^1}{\lambda}.
\]

and we can choose as the best analysis into \( n \) categories, that analysis \( A^1 \) for which \( \text{Val}(A^1) \) is minimal. (22)

27.4. This conception of syntactic analysis has an information-theoretic interpretation; in fact, it was initially suggested and motivated by this interpretation. The procedure of evaluation that has been suggested, in terms of generated discourses, can be interpreted as a procedure of determination of the information per word in the generated language. In fact, the definition of the value of an analysis \( A^1 \) given above in (16) is exactly the definition of information per word in the language generated from the general discourse forms constructed in terms of \( A^1 \). (23)

The best syntactic analysis into \( n \) categories has been defined to be the analysis that in fact minimizes the information per word in the generated language of grammatical discourses (sequences of sentences). We have to do here with a very special and elementary case of information, since the frequency of words and word sequences is nowhere considered.

An elaboration of this interpretation may prove illuminating. By the redundancy of language is meant, essentially, the restriction on the freedom of the choice of elements in discourse, and in our present context, it can be understood
as a measure of restrictions on the freedom of choice of words. We might picture this redundancy as being broken down into two factors, the first involving the restrictions provided by the grammatical structure of the language, and the second, the restrictions provided by all other factors, including the content of discourse and all its extra-grammatical concomitants. In other words, at every point in the stream of discourse the speaker must choose a particular single word, and it makes sense to ask to what extent his choice of a particular word was governed by the grammatical structure of the language, and to what extent it was governed by all other factors. The more rigid the grammatical structure, the fewer discourses are permissible altogether (for each length), and the larger is the share of the constraints contributed by the grammatical structure. Essentially, the conception of syntactic analysis given above has been designed in such a way as to minimize the number of possible discourses of each length, consistent in a special sense with the corpus, and thus to maximize the contribution of the formal grammatical structure to the total redundancy. As we move to lower degrees of grammaticality, this contribution decreases. Even for highest degree grammaticality (i.e., first order, highly selective categories), we should expect it to be relatively slight.

28.1 This interpretation for the proposed constructions focuses attention on a characteristic feature of the linguist's ordinary conception of grammar. I have in mind the sharp distinction maintained between grammatical and statistical structure. In view of the recent developments in statistical linguistics, it seems important to give a somewhat more
systematic statement of this distinction and its consequences, even at the cost of some repetition of earlier remarks.

Customarily, the linguist, in carrying out grammatical analysis, disregards all questions of the frequency of elements, and simply notes the occurrence or non-occurrence of each element in each context in his observed materials. Thus he will record the fact (if it is a fact, in his corpus) that in the context "I found the --", the elements "boy" and "toy" occur, but "of" and "go" do not, but he will not record the number of times that "boy" and "toy" occur. A consequence of this approach is that the resulting grammar sets up a sharp division between a class $\mathcal{G}$ of grammatical sentences, and a class $\mathcal{U}$ of ungrammatical sequences, between sentences that 'can occur' (to use a very bad and misleading, but common usage) and those that 'cannot'.

The formal properties of language might be studied in other ways. Instead of noting merely the occurrence or non-occurrence of each element in each context, we might develop a statistical analysis of the corpus in which the probability of occurrence of each element in each context is tabulated. Or we could present a complete picture of the statistical structure of the language by constructing a stochastic matrix, essentially, a statement of the conditional probability of occurrence of each element as the $n^{th}$ element of a sequence, given the first $n-1$ elements. And this information might be recast and utilized in many ways.
One consequence of such an approach would be an assignment of a certain probability of occurrence to each sequence.

The grammatical approach, with its establishment of a sharp dividing line between the two classes of sequences $G$ and $\overline{G}$, contrasts with a statistical approach that assigns a probability of occurrence to each sequence, and thus leads to an ordering of sequences from more to less probable, rather than a sharp division into two classes, within which no such gradations are marked. This literally correct statement of two different approaches can be misleading. It would be easy to picture the statistical approach just described as being a generalization of the grammatical approach, conceiving of the latter as imposing a rough approximation to the full statistical variation, with all sequences of higher than a certain probability being assigned to $G$, and all others to $\overline{G}$. But this would be a gross misconception. We have already noted that if our theory is to begin to satisfy the demands that led to its construction, then $G$ will have to include such sentences as (17) and (17')

(17) This is a round square
(17') Colorless green ideas sleep furiously

and (5), this chapter, but not (18) or (18')

(18) This are a round square
(18') Furiously sleep ideas green colorless

or (2), (11), this chapter. But clearly these sequences are not distinguished by their assigned probabilities. In any corpus that might serve in a model of the process of projection, this probability will be zero in all cases. Nor can they be distinguished, in some more sophisticated way,
in terms of the probability of their parts. The full statistical picture is not a direct generalization of the grammatical analysis with its simple yes-no system of constraints. There is no obvious tie-up between the two approaches. Sequences can be assigned to \( G \) no matter what their probability or the probability of their parts. The grammatical approach cannot be interpreted as giving a gross account of the full richness of the probabilistic picture, a schematized and simplified description of the real variety of the 'actual' language. Nor can the generalization to degrees of grammaticalness be understood as simply a closer approximation to this variety.

This is a simple but important point. The linguist uses such words as 'pattern' and 'structure' quite freely in describing his own activities. He says that he is interested in describing the structure of the language, the pattern to which its utterances conform. The distinction between two kinds of nonsense, grammatical nonsense like (17), and ungrammatical nonsense like (18), can serve as a simple illustration of the significance of this reference to pattern and structure. Here we have two sequences of words, neither of which, presumably, has ever been uttered before -- in fact, no part of either of these sequences has, presumably, ever appeared in connected discourse. Yet any speaker of English will recognize at once that (17) is an absurd English sentence \(^{25}\) while (18) is no English sentence at all, and he will consequently give the normal intonation pattern of an English sentence to (17),
but not to (18). Such examples as this give empirical content to the linguist's search for pattern and structure. The distinction between grammatical and ungrammatical nonsense cannot be explained by simply giving a more and more detailed description of the observed linguistic behavior, ultimately, let us say, a statistical tabulation of the probability of occurrence of each item in each context. In terms of such a description alone, both (17) and (18) will be excluded, on identical grounds, since none of their parts ever occur. This distinction can be made in the manner described above by developing a notion of sentence form, and demonstrating that (17) is an instance of the *Arabic* sentence form Adjective-Adjective-Noun-Verb-Adverb, which is grammatical by virtue of such sentences as (19) revolutionary new ideas appear infrequently that might well occur in normal English.

28.2. The custom of calling the class of 'possible' sentences, or those that 'can occur', is no doubt responsible for much confusion here. It is natural to understand 'possible' as 'highly probable', and 'impossible' as 'highly improbable'. When this interpretation is rejected, as it must be in this case, it becomes equally natural to take the next step of throwing out the notion 'possible sentence' as a mystical concept.

Actually, although the notion of grammaticalness is undoubtedly complex and difficult to reconstruct, it is by no means mystical, and we have good ideas as to how to go about reconstructing it. Given a corpus of sentences, we
define the set $G$ to be the set of sentences conforming to the rules established for describing this corpus, whether or not these sentences happen to occur in the corpus. The problem of constructing $G$, then, is the problem of determining how to construct a proper description of a fixed linguistic corpus — it is the problem of constructing a linguistic theory as we have several times described this project above. Linguistic theory must provide us with the system of formal structures that can be realized in language and with a procedure for evaluating any proposed realization of this system, based on a given corpus. Given such a theory, and given a corpus, we can select the highest-valued interpretation of the theory, for this corpus, and can determine $G$ as the class of sentences formed in accordance with the principles of construction laid down in this interpretation. To construct such a theory is no mean task, but it is important to recognize that there is no apparent difficulty in principle.

The system $C$ is one such structure that can be given an explicit interpretation, given an adequate corpus, and in §27 we have suggested one way in which any interpretation might be evaluated. Describing a corpus in terms of $C$ automatically produces a certain projection of the corpus. Further projection will be discussed below in terms of other structures. Whether or not any of these proposals prove ultimately to be adequate and effective, they do indicate that there is nothing mysterious about the project.
28-2. We see, then, that the linguist is led to the study of underlying form, and to the formulation of principles of classification in terms of substitutability, similarity of function, similarity of formal features, etc.

It would be easy to misconceive the linguist's search for structure as being motivated by a feeling that the 'reality' of language is too complex to be described completely, thus a feeling that he must content himself with a schematized version of this complex reality. But as we have seen, this emphasis on underlying structure does not arise from any desire to impose a rigid and simplified system on the actual variety of the real language. A structural analysis is not a schematic summary developed by sharpening the blurred edges in the full statistical picture. On the contrary, this emphasis is forced on the linguist by the nature of the behavior that he wishes to investigate. Such goals as that of distinguishing grammatical from ungrammatical nonsense serve as a principle of relevance for linguistic description. They determine the degree of detail with which it is necessary to study the corpus in order to determine grammatical structure.

We have frequently noted that the problems of projection and phonemic distinctness are twin aspects of the problem of determining the subject matter of the grammatical description. We can relate these remarks to the study of phonemic distinctness in chapter I by noting that the pair test offers a similar principle of relevance on the phonemic level. There is no limit to the detail with which it is
possible to study the phonetic characteristics of sounds, and such study may be perfectly proper. But it is also perfectly in order to draw the line just at the point where differences fail to be significant in the sense provided by the pair test. Phonemic theory is developed by drawing the line at just that point.

28.4. In §28.1 we noted that the linguist generally disregards frequency and simply records occurrence or non-occurrence. We have seen that this leads to a non-statistical and 'algebraic' conception of grammar, which seems well adapted to the solution of certain fundamental linguistic problems. But the validity of this conception of the form of grammar must be considered independently of the question of the validity of the particular non-statistical approach that happens to lead to it. It may turn out to be the case that statistical considerations are relevant to establishing, for instance, the distinction between G and G. As mentioned in §26.5, the relevant distributional characteristic Q may turn out to be statistical in nature. There is no a priori way to determine this. Or, now that in §27 we have lowered our aims to the establishment of an evaluation procedure, rather than a practical discovery procedure, we can consider the possibility of establishing an evaluation procedure for the system C (i.e., for an n-category analysis) on statistical grounds, to be compared, for adequacy in correctly reconstructing C, with the non-statistical evaluation procedure proposed in §27. For example, given a text and an n-category analysis, we might rewrite the text in terms of categories (i.e., construct the discourse form of the text) and determine the frequency of occurrence
of each category in the resulting discourse form. In terms of these frequencies, we can determine what the distribution of pairs, triples, etc. of categories would be if this discourse form were a random sequence of categories with these frequencies. If this category analysis is not a significant one (e.g., if it is based on alphabetical ordering), then we would expect the actual distribution of pairs, triples, etc. in the resulting discourse form to be approximately that of a random sequence of elements of these frequencies. If this category analysis is a significant one, there should be a departure in statistical properties from a sequence built up by independent choices. We might propose the best \( n \)-category analysis as the one in which the distribution of pairs, triples, etc., deviates most sharply (in a sense which must be specified) from the distribution in a discourse form with no structure. (26) Such an approach would have an advantage over the method of \( n \) in that it is more easily testable, and a disadvantage in that it loses its significance if classes are not disjoint (i.e., it offers no direct solution to the homonym problem). \[ \text{Appendix 2} \]

\[ 27.5 \] We may find that the methods of linguistic analysis and the structures of linguistic theory can most effectively be defined in probabilistic terms, or we may find that this is not the case. (27) The only way to determine this is to obtain a clear picture of the tasks at hand and actually to develop and apply various proposals for carrying these through.

The extra information utilized by the statistical
approach may prove essential to these tasks, or it may, on the other hand, prove detrimental, in that it blurs important distinctions with irrelevant detail. It is by no means the case that utilization of more information automatically yields a more powerful method. This truism scarcely requires demonstration, but to take an example of some independent interest that might turn out to shed some light on this, consider the problem of determining word boundaries in sequences of phonemes by estimating the freedom of occurrence of the next phoneme at each point in the sequence. There are several ways in which this freedom of occurrence can be measured. Suppose we know the sequences of phonemes that form grammatical sentences, and we know, in addition, all transition probabilities for the language. We might proceed by determining, for each sequence of \( n-1 \) phonemes \((n \geq 1)\), the absolute number of phonemes that can occur as the \( n \text{th} \) phoneme in this sequence, in some grammatical sentence. We can then place word boundaries at the points in a sequence where this number rises sharply, after a gradual fall following the preceding rise. This method has been developed and studied by Harris\(^{(28)}\) for determining morpheme boundaries, with interesting results, but his data seem to indicate even more impressive results for word boundaries. This, then, is one way to measure freedom of occurrence, and in terms of it, to give a tentative characterization of word boundaries.

A second approach would be to determine for each sequence of \( n-1 \) phonemes, not the number of phonemes that can occur
as the $n^{th}$ phoneme in the sequence, but the uncertainty at each position (i.e., the information associated with the state defined by this sequence of $n-1$ phonemes). Measuring freedom of occurrence in terms of uncertainty, we can place word boundaries at the points where this rises sharply. This method has been suggested by Hockett. (29) These methods might give identical results, but not necessarily so. Suppose that at a given point in a sequence 30 phonemes can occur, but one occurs 99% of the time. Then the number of possibilities is very high, but the uncertainty is very low. Insofar as these approaches yield different word boundaries, and insofar as we can determine on some independent grounds which analysis is correct, we are faced with an empirical problem and we can arrive at a decision as to which approach is more adequate. On a priori grounds, it is impossible to determine this.

The linguist's disregard of frequency of occurrence in grammatical work may or may not turn out to be significant. At the present stage of our knowledge we must surely keep an open mind on this matter. The purpose of these remarks has been to point out certain unclarities that sometimes seem to bar the way to a rational investigation of this important question.

29. The notion of level of grammaticalness has some further implications which might be explored with profit. If we drop a certain sentence from the corpus, and apply the analysis to the corpus minus this sentence, we would ordinarily expect that this sentence will be generated at
the highest degree of grammaticalness (i.e., by generation in terms of first order categories). But for certain sequences, this will not be the case. Suppose for instance that a certain sequence of the corpus is a slip of the tongue, or is an interrupted sentence, or the like. Then if it is struck out of the corpus, it will not be reintroduced by the process of generation at any level of grammaticalness at all, above the lowest. Or consider a sentence like (20) Misery loves company.

This may be the only sentence of the form Abstract Noun - Verb\textsubscript{k} - Abstract Noun, where Verb\textsubscript{k} is a certain class of verbs that occur otherwise only in such contexts as Proper Noun --- Abstract Noun, etc. If (24) is dropped out of the corpus, then, it will not be reintroduced at the highest level of grammaticalness, but only at some lower level, i.e., at the level at which 'misery' and 'John' are in the same class, since 'John loves company' will surely be generated at the highest level. This suggests that we need not consider all occurring sentences as of the highest degree of grammaticalness just because they occur. Just as before we developed a way to consider certain non-occurring sequences as being of the highest degree of grammaticalness, now we have a way of considering certain occurring sequences as being of some intermediate degree of grammaticalness. The method is to strike them out of the corpus, redo the analysis on the reduced corpus, and see at what point the eliminated sentences are reintroduced, if ever. More generally, if a certain
sentence form is inadequately represented, in some sense that must be defined precisely, we can drop it and investigate the level at which its instances are regenerated. Though this account is oversimplified, it points out the possibility that certain idioms or metaphors might be characterizable as extending sentences which occur, but are not of the highest degree of grammaticalness, and that mistakes might be characterizable as occurring sentences of the lowest degree of grammaticalness. In this way we may be able to develop a method of projection of the kind originally discussed in §23.

We see that in terms of the system $\mathcal{C}$, such sentences as (20) have a special and exceptional status. They belong to sentence forms that are quite inadequately represented. $\mathcal{C}$ is just one of the systems in terms of which we describe linguistic structure. We will see below ( ) that (20) has an exceptional status in the light of higher level structures as well. Cf. §35 for further discussion in this connection.

30. When the proposed analysis of the notion of syntactic category was introduced in §26.7, it was asserted that this exemplifying approach has a more modest aim than what has been called a 'procedural' formulation, since it provides a method for evaluating a proposed analysis, but not for arriving at the correct analysis directly. But this is not literally correct. Given a finite corpus, there are a finite, but astronomical number of ways in which the words can be arranged into $n$ classes, and the procedure of systematically running through these, evaluating each one, and choosing the
best is in fact a mechanical, terminating procedure. But it is clear that this is not the sense in which we speak of a procedural definition or technique (cf. §2.2). However, the difference is not easy to characterize. It is not the difference between finite and infinite, but the more elusive difference between too large and not too large.

Clearly the definition we have constructed can never in fact provide a procedure for discovering the correct grammar, though it can provide a practical procedure for evaluating a given proposal. In this respect, it is much like the measure of simplicity discussed in chap. III. These constructions meet the requirements for linguistic theory as these were laid down in §2. Nevertheless, it is interesting to investigate the possibility of actually constructing a stronger theory, i.e., of converting this evaluation procedure into a practical discovery procedure. A combination of the ideas of §§26.4, 26.6 and 27 might be useful to this end. The procedures discussed in §§26.3-4 can be applied directly to data to provide a complex system of classes from which a system $C$ can be selected in various ways. Applying the evaluation procedure to these various proposals, we can select the best of them. Even though the substitution procedure will not lead directly to the system $C$, it may reduce significantly the number of alternative analyses that have to be evaluated. Hence if we do have an effective evaluation procedure, it becomes quite important to develop substitution procedures (or other procedures of the general type outlined in §26.6) even if
these prove to be only partially effective in themselves.

The relation between the system described in \( \text{§} 27 \) and those discussed in \( \text{§} 26 \) might be investigated further with profit. The criterion of \( \text{§} 27 \) can be translated into a clustering criterion. It might be interesting to see what form it would take in these terms. Cluster analysis (including as a special case, the method of \( \text{§} 27 \)) can be used to develop the system \( C \) directly, or just to eliminate illegitimate contexts, and to avoid the problem of homonymity, thus establishing the basis for application of a substitution technique such as that of \( \text{§} 26.2-4 \). (cf. \( \text{§} 26.5 \)). We have already noted that these approaches might lead to different results, above the first order categories.

Since each of these approaches seems reasonable, it might be interesting to try to determine formal conditions on language under which these approaches will in fact coincide. Such questions cannot really be put seriously until the methods of analysis in question are characterized quite precisely. At any rate, there seems to be an area of formal investigation here that might ultimately prove to be of considerable significance for linguistics.

Can the method of \( \text{§} 27 \) guarantee uniqueness of the best analysis in terms of \( n \) categories in principle, even if it cannot arrive at this best analysis in a practical way? The answer to this is of course that it cannot, unless we lay down certain formal restrictions on the sets of symbol sequences that are eligible for consideration as a linguistic corpus. We can easily invent 'languages' in which, for instance, for every \( n \), every possible analysis into \( n \) categories
is equally good. A language where each sentence is one word in length would satisfy this condition. The problem of stating the formal requirements on symbolic systems for qualification as 'language' is an important one. In a sense, it is the goal of linguistic theory to solve exactly this problem.

31. It is interesting to note that the generality of the technique of classification of §27 makes it available on other levels of linguistic analysis as well. In particular, we can consider the possibility of setting up phonological classes like Consonant and Vowel, etc., on a distributional basis, (30) by asking how can the elements (here phonemes) occurring in phoneme sequences be put into n classes in such a way as to lead to a minimal projection of the actually occurring phoneme sequences. The case of homonymy might have a formal analogue here in such problems as that of classifying semi-vowels, or syllabic consonants, on the level of a two class analysis. A certain amount of experimentation in this direction has shown some promise. Cf. Appendix 2.

32. Once we have constructed a system C for a given language, we can give a relative sense to the expression 'X and Y have the same grammatical form', where X and Y are sequences of words of the same length. In this case, we can say that X and Y have the same n-order grammatical form if the lowest order of categories in terms of which they are instances of the same sentence form is the-order n. Thus any two sentences of the same length have the same grammatical
form on some order, since at least they are instances of
the same sentence form in terms of the one-category analysis
$C_1^N$ (cf. §25.2).

But it will also be necessary to give an absolute sense
to the expression 'same grammatical form'. The reasons
for this are already evident from our discussion of
grammatical and ungrammatical nonsense in §28.1. And further
reasons will appear below, in succeeding chapters. Thus
it is necessary here to fix an absolute or optimal level
of grammaticality, a set of optimal categories.

In §27.2 we suggested that the set of $a$'s for which
an $a$-category analysis is to be constructed as an order $C_1$
of $C$, be selected by determining the relative minima of
a certain function $f$ (which we left unspecified, pending
further empirical investigation) of the number of categories
and the number of sentences generated (i.e., in terms of
§27.3-4, the information per word in the generated set of
sentences). We might then take the absolute minimum (31) of
the function $f$ as defining the absolute order of grammaticality
and the absolute categories, as we will refer to them below.

Since we have neither specified nor investigated the
function $f$, we can do no more than speculate here about the
properties of the absolute minimum. Speculating, then,
it would seem reasonable to suppose that a proper choice
of $f$ will give as the absolute order a set of fairly large
classes, so that the absolute order will not correspond to
one of the highest degrees of grammaticality. The
absolute analysis embodies the major grammatical restrictions.
Presumably these will be stated in terms of such classes as Noun, Verb, Preposition, etc. There will be many further grammatical restrictions that have to do with limited and special contexts, and that will, presumably, be reflected in superior degrees of grammaticalness (i.e., smaller, lower order categories). These further restrictions correspond in part to what Harris, following Bloomfield, has called selection. Thus selectional restrictions can be defined as those which refer to an account of grammaticalness which is more detailed and specific than that provided by the absolute analysis, i.e., those which refer to orders of grammaticalness below the order $n_A$ of the absolute analysis. Although Preposition may well turn out to be a class of the absolute analysis, there will be subclasses of prepositions that occur with different nouns and verbs, etc., and at the first order, we may even find that although many of the categories are still quite large (e.g., proper nouns), the categories of prepositions may be extremely small, even unit classes.

Below we will find it necessary to make assumptions about absolute categories which are not warranted by any empirical evidence. This is unfortunate, and naturally we will try to reduce them to a minimum. But we cannot drop all further investigation into linguistic structure until all problems connected with grammaticalness are completely resolved. One reason why this is impossible is that the problems of grammaticalness are not independent of later considerations. The analysis of grammaticalness must be
carried out in such a way as to tie in correctly with more advanced constructions, and we cannot know how extensive the description of this first step in determining grammaticalness must be until we have some idea about what can be accomplished on higher levels of analysis (cf. Introduction). Whenever we make assumptions about the absolute analysis, these can be understood as conditions on the function $f$. That is, not only will $f$ have to be designed so as to give the correct relative minima, but also so as to give the correct absolute minimum. As is likely with any assumptions, investigation may reveal that these assumptions are not valid, i.e., that these conditions on $f$ should be removed. This must be kept in mind when we suggest reliance on absolute categories below. Linguistic theory is a complex system with many interconnections between its parts. It is apparently necessary at this early stage in its development, to let speculation outrun available evidence at many points in the theory, so that the data attainable at other points may fit into some reasonable conceptual framework -- this data then supplying conditions of adequacy which the underlying conceptual framework must meet.

33.1. The definition of syntactic categories outlined in §27 leaves us free to put conditions on the $C^m$ in various ways. The most interesting question, perhaps, is whether the categories of a given order should be required to be disjoint. This is again the question of how to treat homonyms. If we allow classes to overlap, then homonyms can be defined very simply as elements which appear in the overlap of two
or more categories of a given order. Given a set of \( n \) overlapping categories, we can convert it into a set of not more than \( 2^n - 1 \) non-overlapping categories by regarding all overlaps as separate categories. Thus if non-overlapping categories are required for any purpose, they can readily be constructed.

To require that the categories be disjoint is essentially to require that homonyms be set up as separate categories. Since such a word as /red/ ("read", "red") has certain contexts distinct from those of past tense verbs, and others distinct from those of adjective, then in terms of the procedure of \( \S 27 \), and under the requirement of disjointness, there will presumably be a sufficient gain in setting up /red/ as a separate category to warrant this step.

If we allow categories to overlap, we have an obvious explanation for the fact that intuitively, /red/ is not a separate and quite unique sort of element, but that rather we have here a case of a (past tense) verb and an adjective that happen to have the same phonemic shape. We can explain this by showing that the analysis is superior, on the level of \( n \) categories (for a certain \( n \) for which an order is constructed) if we assign /red/ to both categories, than if we assign it to either one or the other (cf. \( \S 27.1 \)). We will see in the next chapter that this solution, in a

obvious way, establishes the homonymy of /red/ as a special case of the very general phenomenon of constructional homonymity (cf. \( \S 4.3,140 \)). If on the other hand, we require disjointness, we may have grounds for setting up /red/
as an element of a separate category, but we will have no explanation for the fact that this is a category of homonyms, that intuitively /red/ can be interpreted as a verb or an adjective which happen to conform. The category containing /red/ will simply be a category like any other.

To explicate the homonymity of /red/, it will be necessary to develop an additional and unique construction. The general lines of this construction are clear. We have defined the distribution of a word as the class of its contexts. We define the distribution of a category $C^n_i$ as the logical sum of the distributions of the elements of $C^n_i$. Then we say that a category $C^n_i$ is a category of homonyms of the classes $C^n_{i1}, \ldots, C^n_{im}$ if the distribution of $C^n_i$ is approximately the same as the logical sum of the distributions of $C^n_{i1}, \ldots, C^n_{im}$.

The trouble here is with the word 'approximately'. We cannot expect the distribution of the category containing /red/ to be exactly the sum of the distributions of the categories of adjective and (past tense) verb. If, for instance, /red/ were the only member of this category, these distributions would be quite dissimilar. Thus the account of homonymity may prove quite difficult. In fact, we seem to be facing a familiar kind of clustering problem, for which the only clear solution seems to be to consider, once again, the effect of assigning certain occurrences of /red/ to Verb, and certain to Adjective.

If we permit overlap of categories, then we have a natural solution to the problem of homonymity as an automatic
consequence of the construction of $C$, and as a special case of a general phenomenon. If we insist on disjointness, then to explicate this homonymity, we will have to bring in complex considerations which may in fact involve the setting up of overlapping categories as an ad hoc device for solving this problem. A simpler way of describing the situation is this. From the overlapping analysis, we can automatically derive the non-overlapping analysis by simply considering the overlaps to be categories. But the converse is not the case. To construct the overlapping analysis from the non-overlapping analysis we must determine which categories form the overlap of several other categories, and to determine this it may be necessary to retrace the same steps that were necessary to set up the overlapping analysis directly. Since the analysis with overlaps, or something quite similar, is apparently needed in order to explicate homonymity, it seems most reasonable to set up overlapping categories in the first place, as long as we can find no clear reason against this.

22.2. Notice that the analysis $A$ with overlapping categories and the analysis $A^*$ derived from it by taking overlaps as separate categories generate exactly the same set of sentences. Hence it might be thought that the choice between permitting and rejecting overlaps is irrelevant to the question of determining grammaticalness. But this would be a mistake. If $A$ has $n$ categories, $A^*$ has $n^*$ categories, where $n \leq n^* < 2^n$. Thus $A$ is to be compared with other $n$-category analyses, and $A^*$ with other $n^*$-category
analyses. A may be the best \( n \)-category analysis in the system which permits overlapping, but \( A^* \) not the best \( n^* \)-category analysis in the system which prohibits it (or vice-versa). \( n \) may be a point at which the function \( f \) (§27.2) has a relative minimum in the system with overlaps, but \( n^* \) may not provide a relative minimum in the system which requires disjointness (or v.v.). Hence the decision as to whether or not categories may overlap may affect the account of grammaticalness. Thus we may ultimately be able to make this decision on grounds other than systematic efficiency. But we are not now in a position to do so, and on the only grounds now available, it seems preferable to permit overlapping in \( C \).

23.3 Let us investigate the consequences of this decision a little further. We know that the number of sentences generated (or the information per word, under the interpretation of §27.3-4) by the best analysis into \( n \) categories is a non-increasing function of \( n \). It is always possible to find an analysis into \( n+1 \) categories that is at least as selective as the best analysis into \( n \) categories. Suppose that we have in an \( n \)-category analysis, the categories \( A_i \) and \( A_j \) where we designate the overlap \( A_i \cap A_j \) of the two categories. If the distribution of \( O \) is distinct from the logical sum of the distributions of \( A_i \) and \( A_j \) (in the sense of §23.10), then there is always a gain in constructing an \( n+1 \) category analysis by taking \( A_i, A_j, \) and \( O \) as three separate categories. Whether this gain is sufficient to set up an order of categories at \( n+1 \) instead of at \( n \) depends on how much of a gain this is, i.e., on how
distinct is the distribution of 0 from that of \( \bar{A}_1 \). This will determine whether 0 contains 'real' homonyms (if an order of categories is established at \( n \), with \( \cap A_i \), or whether there is no homonymity, and the \( n \)-category analysis merely reflects an insignificant and chance similarity between the distribution of one category and the sum of distributions of two other categories (in the case that an order is established at \( n+1 \) (not \( n \)) with \( A_1 \), \( A_1 \), and 0 three distinct categories).

Suppose now that the distribution of 0 is exactly the sum of the distributions of \( A_1 \) and \( A_1 \). Then no matter what the relative sizes of \( A_1 \), \( A_1 \), and 0, there will be no gain in constructing an \( n+1 \) category analysis by setting up \( A_1 \), \( A_1 \), and 0 as separate categories (again under the assumption of fn.32). There is no alternative then to the analysis of the 0's as homonyms. If we insist on disjointness of categories, this will not be the case. 0 will have to be assigned either to \( A_1 \) or \( A_1 \), and there will always be a considerable gain if we extract it as a separate category. The larger, proportionately, is 0, the greater is the gain.

As an instance where these considerations operate, consider as \( A_1 \), English intransitive verbs, and as \( A_1 \), transitive verbs. The overlap 0 is very large -- a very high proportion of verbs can be used either transitively or intransitively. It seems to correct to say that the distribution of 0 will, in an adequate corpus, be approximately equal to the sum of the distributions of \( A_1 \) and \( A_1 \). Thus if we permit overlapping, 0 will be considered as a class of
homonyms.

It is difficult to judge whether or not this is an acceptable solution. On the one hand, it seems unreasonable to consider "eat" to be a homonym by virtue of "they eat" and "they eat lunch". However, it does not seem unreasonable to consider "feed" a homonym, given "they feed on grass" and "they feed the animals." Hence intuition is of little assistance. We will see in the discussion of phrase structure that it is preferable there to set up \( O \) as the overlap of \( A_1 \) and \( A_1 \), i.e., to accept the homonym solution. But it is not clear that this, even if correct, is relevant here.

Speculating, then, it seems likely that the decision to reject overlapping will lead to the non-homonym analysis for \( O \), and that the decision to permit overlapping will lead to an analysis of the \( O \)'s as homonyms. We know from \( \text{423-2} \) that the alternative solutions may lead to a different account of grammaticalness, and this may provide a clear answer to the problem. But until we have sufficient data of this sort, this situation remains a crucial and unresolved test case. \( \text{34} \)

To cite one more case of this nature, \( \text{35} \) let \( A_1 \) be the class of nouns and \( A_2 \) the class of verbs. Again, the overlap \( O \) is quite heavy, and again it seems likely that permitting overlap will lead to analysis of the \( O \)'s as homonyms, while requiring disjointness will lead to a non-homonym analysis of \( O \) as a separate category. Words like /riyd/ ("read", "reed") favor the former analysis, -- words like "walk", the latter. But
here I think there is a good case to be made for the homonym analysis. In chapter IX, we will see that there are good reasons for analyzing the noun "walk" as the verb "walk" plus a zero alternant of a nominalizing suffix, and it seems reasonable to suppose that a similar analysis will apply to all those members of O which, like "walk", seem to be instances of the same word, rather than real homonyms. This will leave only instances like /riyd/, and the analysis of such cases as homonyms certainly matches intuition. Thus if we choose the theory of syntactic analysis that leads to the establishment of O as a class of homonyms, we will have a formal explanation for the homonymity of /riyd/ in the normal way, and we will have a higher level explanation of the apparent non-homonymity of "walk".

Throughout this discussion we have been assuming that projection is determined by rules of combination framed in terms of words. But this is certainly a debatable assumption. There is nothing in the formulation of syntactic analysis that explicitly requires that words and not morphemes be considered as the fundamental units for this kind of analysis. However, the choice of words seems to me to be the correct one.

Before any description can be undertaken, the object to be described must be at least partially specified. In the case at hand, the set of grammatical sentences must be at least partially specified before the description of these sentences (i.e., the construction of the rest of the grammar)
is undertaken. This prior specification need not be total. Later considerations may play a certain role in determining where the boundaries of the set of grammatical sentences should be drawn. If the prior specification yields a system with complexities and irregularities, and if we find that slight adjustment of the boundaries makes possible a much simpler specification, then this adjustment may be permissible. But this is a delicate operation -- it is easy to fall into circularity, and the danger of formulating our methodology in such a way that the grammar at which we arrive is the set of all sentences, \( \text{transformational grammar} \). To reduce the danger of emptiness and circularity, we would like this prior specification to be as far-reaching and complete as possible. In the present connection, this means that the elements categorized in forming the system \( C \) must be the elements least likely to be revised and altered in the light of later syntactic considerations. It is obvious that this consideration rules in favor of words rather than morphemes. Morphemes may be continuous or discontinuous, they may undergo heavy and complex alternations. Decisions as to morphemic analysis may be heavily dependent on higher level syntactic considerations. (36) Furthermore, investigation of Harris' results on procedural isolation of tentative morphemic segments (37) seems to show that word boundaries are much more clearly indicated than morpheme boundaries, and that it may be possible to determine words directly from the phonemic record. The possibilities for morphemes seem much more doubtful. Mandelbrot's work (38) also supports the choice of words as the basic units.
for syntactic analysis, from quite a different point of view. Finally, we want the categorized elements to be as independent as possible in the stream of discourse. Clearly, the many special restrictions on sequence of morphemes will lead to serious difficulties for the type of approach we have suggested.

25.1. The method of §27 cannot furnish a complete answer to the problem of projecting the corpus to a set of grammatical sentences. For one thing, this method does not generate sentences longer than those of the original corpus. But the generated class of grammatical sentences is infinite — there is no longest sentence. There are many other grammatical possibilities and restrictions that clearly cannot be adequately characterized by investigation of syntactic categories. At best then, the proposed account of grammaticality represents only the first stage of projection. But an analysis of this account brings out the general character of each stage in the construction of the set of grammatical sentences.

At each stage of this construction we are presented with a set of sentences, and we are required to project this new set to a new set, where 'projection' is to be understood in the broad sense of fn.1. The process of projection is intimately bound up with the notion of linguistic level. We can picture each new level as being constructed in order to simplify the description of the sentences already generated in terms of the preceding levels. (cf. §26, §4, chap.I). At each stage of linguistic analysis, we find that the description
of the presented grammatical sentences can be simplified if we project this set to a new set, adding many new sentences and perhaps dropping certain sentences. Thus the generation of new sentences becomes an automatic consequence of the process of describing already given sentences in terms of the descriptive machinery available in the new level. Putting this a little differently, we might say that each level provides a certain point of view from which to investigate the structure of the set \( S \) of sentences already generated. Investigating \( S \) from the new vantage point offered by this higher level, we discover that the structure thus presented as underlying the set \( S \) is only partially realized in this set — there are many gaps, and certain exceptions. By filling in the gaps with new sentences, and dropping the exceptions, we project \( S \) to a new set \( S^* \), which serves as the presented set \( S \) for description in terms of the next level.

In the foregoing discussion of the first stage of projection, the set \( S \) of presented sentences was the corpus itself. Introducing the level of syntactic categories (the system \( \mathcal{C} \)), we find that the corpus can be studied as a set of instances of a relatively small number of sentence forms. But the set of sentence forms is imperfectly realized in the corpus. We discard certain inadequately represented forms (in the manner of \( \text{22} \)), and we form new sentences conforming to the adequately represented forms, thus projecting the corpus to a set \( S^* \) of sentences of the highest degree (first order) of grammaticalness (and incidentally, we presumably unearth a good deal of further information about
lower degree, partial grammaticality and absolute grammaticality). This set $S^*$ serves as the basis for study in terms of higher levels. We need not be concerned about the fact that partially grammatical sentences are not discussed on other levels. They have not been totally excluded from the grammar, and once we have the system $C$, they can be derived from the set of highest degree grammatical sentences.

In subsequent chapters, we will investigate this set $S^*$ of highest degree grammatical sentences from the point of view of phrase structure and transformational structure. In each case, we see that the process of projection outlined above is repeated in terms of the descriptive potentialities of these higher linguistic levels.

35.2. This account of projection emphasizes the implications for the order of descriptive operations of the point of view we have adopted. It suggests that after phonemic analysis, the first step in grammar construction is the placing of word boundaries, and the second step is the limited projection provided by the method of $C^*$ or something similar, i.e., by description in terms of the system $C$ of syntactic categories of words. We then find that the relation between words and phonemes, as well as between sequences of words and phrase sequences, is simplified tremendously by morphological analysis of words. In this view, then, morphology appears as a higher level of analysis. Morphological representations are provided for grammatical sequences of words in such a way as to simplify the description of those sequences and the statement of their relations to phoneme and phrase sequences. A full-fledged theory of linguistic structure
will also investigate the implications of the morphological representation for projection. The next step is constituent analysis and the study of phrase structure, and below we will suggest an even further stage of transformational analysis.

This outline of an order of descriptive operations should not, however, be taken literally. We are not attempting to present procedures for the grammatical analysis of languages. Cf. above, chap. I, § 2.2, for further discussion of this point.

35.2. To determine the adequacy of any proposed account of the first stage of projection will be an enterprise of tremendous scope, but also of considerable importance, extending beyond linguistics, if we accept the motivation for the study of grammaticalness suggested in § 24.2.

In carrying further the study of grammaticalness and grammatical structure, it is necessary to make certain assumptions about this first stage of grammaticalness. Some such assumptions have such strong intuitive support that we can take their verification to be a criterion of adequacy for any proposed analysis of grammaticalness, that these assumptions be verified. As far as possible, we will try to develop our analysis of English in subsequent chapters on the basis of such assumptions.
Appendix I.

36.1. The decision as to the validity of a particular construction is ultimately an empirical one. In the case of a proposed evaluation procedure for the system \( \mathbb{C} \), we must determine whether the set of wider and wider projections given by the proposed procedure matches the actual projections of native speakers, and whether the systematic consequences of application of this procedure for other levels prove to be valid. An important step towards this goal would be a demonstration that the procedure in question leads to a successful solution for the homonym problem on some order of analysis, that is, that it classifies words in accordance with our intuitive categories. We will give a brief summary of a partial result of this kind for the construction proposed in 427. It serves not only as a first step towards validating this approach to grammaticalness, but also as a sketch of one kind of empirical study which can be used to test any proposed solution to the problem of projection.

Given a corpus of sentences (considering now only the simplest case, where all are of the same length), and a classification of the words in the corpus into \( n \) categories, we evaluate this syntactic analysis by choosing a set of grammatical sentence forms in these categories that covers the corpus, in the sense that every sentence of the corpus is generated by one of these sentence forms. In the case of an analysis into non-overlapping categories (i.e., a non-homonym analysis), there is only one covering set. But if these categories overlap, a covering analysis can be chosen in many ways, in general. We want to find the minimal covering, i.e., the one that generates the fewest number of sentences. But there is no procedure (in the vague sense in which we have been using this term) for making this decision. That is, we must exhaust all possibilities in order to discover the best. The problem is to which category to assign a given occurrence of a word that happens to fall in the overlap of two
categories. But we cannot know in advance the systematic consequences of an assignment one way or another. Any choice of a covering set gives an upper bound to the number of sentences generated by the analysis, so that we really have a only a partial evaluation procedure, given an n-category analysis with overlapping categories. We can arrive at a lower bound by forming a disjoint analysis by considering each overlap to be a separate category, and then evaluating this $2^{n-1}$ category disjoint analysis. This lower bound will be the actual value whenever the distribution (in terms of categories) of the overlap of any k categories is identical with the sum of the distributions of the non-overlap parts of these k categories.

Given a covering set of sentence forms, we evaluate this the analysis by determining the number of sentences generated. In the case of a disjoint analysis, this is straightforward. For each sentence form $F_i = C_{i_1} C_{i_2} \ldots C_{i_n}$, where $C_{i_1}$ is a category with $a_{i_1}$ members, we determine the product $a_{i_1} a_{i_2} \ldots a_{i_n}$. The sum of these products, for all $F_i$, is the value of the analysis. But in the case of an overlapping analysis, the problem is slightly more complicated, since a given sentence may be generated by distinct sentence forms; and in evaluating the analysis we are interested in determining the absolute number of sentences generated, so that we must be careful not to generate any sentence twice. This can be avoided by a mechanical, though tedious process. It raises difficulties for an attempt to evaluate a category analysis by sampling.

26.2. The analysis into Nouns, Verbs, and Adjectives is a fundamental one which reappears in almost any syntactic treatment of English. We can evaluate it as an analysis into four categories, N, V, A, and X (everything else), with heavy overlaps. The first step is to determine the size of the categories, and the size of the overlaps. In practice,
this means that we choose a set of 'diagnostic contexts' for each class, and that we sample the words in the corpus in terms of these contexts. These diagnostic contexts thus, in practice, determine the analysis, although the particular conception of syntactic categories that we have considered gives them no special significance in theory. We then fix a covering set of forms and proceed as above.

Systematic pursuit of this course with a proper experimental procedure is a large scale project. Before actually proceeding with the systematic investigation of any proposed procedure of evaluation, it is worthwhile to attempt, in various ways, to get estimates as to the reasonableness of this proposal, and the likelihood of its adequacy. In an attempt to get such a prior estimate, I took as a vocabulary the word list compiled by Eldridge, and as a corpus the entire stock of four word sentences (this being about the upper limit for what can be evaluated exhaustively without mechanical assistance) made up of these words and normally pronounced with a preassigned intonation, namely, the normal assertion intonation of declaratives. This means making certain somewhat arbitrary decisions as to which forms to include. E.g., should we include "My friend John came", "That morning he arrived", "they said he came", "They said: come early", etc. Among these, only the last was excluded, as being beyond the intonational limits, in its normal pronunciation. A decision to exclude more border-line cases would have had the effect of slightly sharpening the results obtained, but not to any significant degree. It is important that the decision as to where to draw the limits of the corpus be made quite independently of any of the analyses investigated, and intonation can be taken as a neutral feature.

Each investigated analysis was some variation of the \( N, V, A, X \) analysis, which presumably is basically the best four category analysis of English. With this small number of categories, a great many grammatical distinctions can not be recognized, and this has the consequence that the
syntactic analysis can be improved by making certain very unnatural assignments of words to categories. For example, words of the form *ing*, like "running", "eating", etc., can always be assigned either to N (as in "running is a good sport") or to A (as in "this is running water"). Even in sentences like "he is eating lunch", "eating" can be assigned to A rather than V, with some improvement in the analysis. Similarly, words of the form *en* (e.g., "given", "taken") can always be treated as A, even in "he was given a prize" (which on this level of analysis, can be regarded as of the same form as "he was an old employee"). There are many other cases where we are led to a perhaps surprising analysis, because of the limited amount of grammatical structure recognizable with so few categories. Thus *auxiliary verbs* in their use as auxiliaries, are best treated as members of X, and words like "let", "make", etc., that can occur in N—NV or —NXX (e.g., "John let him escape", "make him come here"), etc., are best analyzed as V, X homonyms in the homonym analysis, and as X in the non-homonym analysis. Many other instances could be cited.

One feature of our analysis that is quite significant, however, is the treatment of compound nouns such as "cowboy", "skyscraper", "lighthouse", "air raid", "income tax collector", etc. (in the same category are "blackbird", "easy chair", etc.). The question here is whether or not these should be regarded as single words. This decision must precede syntactic analysis, and must be made on the basis of lower level considerations, primarily (cf. §24). Our decision was to treat them as single words. The grounds for this are debatable -- basically, the decision turns on complex questions of how to describe stress and intonation which are beyond the scope of this study. The decision is of some importance, because N is the largest class, and if these are treated as single words there is a heavy restriction on NN sequences. It seems to me that both on precedent grounds, and on the grounds of the type of selectional restriction that holds for these compounds (this being more of the character of morphology
than syntax), the decision to treat them as single words is the correct one.

As diagnostic contexts for the categories, I used the obvious and standard ones, e.g., contexts stated in terms of the most frequent words ("the", "of", "a", etc.). But the category assignment was modified in accordance with the possibilities for simpler analysis discussed in §26.2. Note that the question of how the categories are formed is of no consequence for our purposes.

Given the categories N, V, A, X, with arbitrary diagnostic contexts, we have the possibility of 15 different assignments for each word. It can either be an N, a V, an A, or an X, or it can belong to one of the overlaps NV, NA, NVA, etc. Only eight of these possibilities had instances in a sample of 300 words from Eldridge's list(41) and their relative sizes were as follows:

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<tr>
<td>N</td>
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Note that more than one-half of the sampled vocabulary elements are syntactic homonyms (in fact, 10% are triple homonyms) on this level of analysis. Thus there is a very large expansion in the size of the categories when we set up a full homonym analysis, and we might expect that this would be so detrimental to this analysis (cf. §27.1) that some analysis with no homonyms would be more highly valued, contrary to our intuitive classification. But this was not the case, in the analyses that were investigated.

Among a large number of investigated non-homonym analyses,
the best was the one that kept all pure nouns, pure verbs, and pure adjectives separate, and threw all other elements into the fourth category. Thus, where N, V, A are the non-homonym parts of Noun, Verb, Adjective, as in (21), this analysis is

\[(22) \quad C^4_{\text{non-hom}} = N \ (22), \ V \ (14), \ A \ (7), \ X^* \ (57),\]

where the parenthesized numbers give the relative size of the categories. I.e., X* contains all elements in any overlap of the analysis with homonyms.

This syntactic analysis generated 84% of the possible four word sentences. Sequences.

All other non-homonym analyses investigated generated more than this. The fact that the best disjoint analysis is so closely related to the intuitively correct homonym analysis is itself of some significance.

The intuitively correct, full-homonym analysis is

\[(23) \quad C^4_{\text{hom}} = N, V, A, X,\]

where

\[
\begin{align*}
N &= N+NV+NA+NVA \quad (67) \\
V &= V+NV+NVA+VA \quad (47) \\
A &= A+NA+NVA+VA \quad (45) \\
X &= X \quad (4)
\end{align*}
\]

where N, V, A, X are as in (21), and the parenthesized numbers give the relative sizes.

Evaluating \(C^4_{\text{hom}}\) in the manner outlined in \(4.36.1\), it was found that this syntactic analysis generated only 63% of the possible four word sentences. As we saw in \(4.36.1\), this is only an upper bound on the number of sentences generated, though in fact, this is almost certainly very close to this number, since I used my intuitive knowledge of the grammar in selecting a covering set. But this is unimportant. Since clearly the upper bound for the homonym
analysis is considerably below the value for the non-homonym analysis, we see that the former and intuitively correct analysis is in fact given a superior value by this evaluation procedure.

This limited result suggests that it may be worthwhile to go ahead with a more extensive and systematic investigation of the empirical consequences of the theoretical constructions of §27.

37. Appendix 2. To be included here are several studies of the empirical consequences of the methods of §27 and §28.4 for classification of phonemes, as suggested in §31.
Footnotes - Chapter IV

(p. 109)

(1) We will use the word 'project' in a broad sense, it being understood that the projection of a class to a new class may drop some of the members of the original class. In the particular kind of projection that we are discussing in this chapter, the deletions will necessarily be of a minor and peripheral character. But in §32 we suggest that the projection being discussed here is only a special case, a first stage in a series of projections. And in later stages (e.g., on the morphological level), the projection may be primarily deletion. Cf. §32.

(2)(p. 110) The particular theory that we develop will determine what qualifies as a significant class.

(3)(p.111) It might be argued that even if contexts like (3) lose their discriminatory significance, we can still distinguish abstract from proper nouns on the basis of such near-paradigmatic contexts as "the --...". This argument would be based on a theory that gave such contexts a special significance, on some grounds (perhaps the frequency of these elements). But this would be difficult to maintain. For one thing, the mass of contexts that differentiate abstract from proper nouns will have been struck out from K, and any two words differ somewhat in distribution. Secondly, "the sincerity" rarely occurs alone as a noun phrase, generally occurring with a modifying prepositional phrase or relative clause (e.g., "the sincerity of his remarks", "the sincerity that he revealed."). But in such contexts we also have proper nouns, e.g., "he is the Jones from Philadelphia (not from New York)", "he is the Jones who won the tournament", etc. If such sentences are somehow barred, we will lose the ability, on these new grounds, to distinguish "Jones" from "golf". A further difficulty for such an approach would result from carrying the argument ever to colloquial German, where the article appears regularly with proper nouns. Further arguments and counter arguments can be given. As mentioned
above, we are not attempting to provide a counter argument for each possible position, but only to suggest a form for such counter argument. The examples in the text are especially relevant to the kind of theory discussed below, where the mass of contexts are approached in a neutral unweighted manner.

(4)(p.112) We might try to argue this on the grounds that the correct pairing minimizes the number of consonant changes. But, on the other hand, we might argue that the incorrect pairing gives the parallelism break-brought, take-taught (having teach-took as the remaining pair). As Wells has pointed out in another connection (Immediate Constituents), wrong analyses often tend to support each other. It is difficult to see what general grounds one of these arguments would be given greater weight.

(5)(p.113) We will try to attach more content to such modes of expression below.

(6)(p.115) And we have also noted, incidentally, in §7.1, that statistical order of approximation, although an interesting notion (as seen by its behavioral correlates), also does not correspond to grammaticalness (nor to meaningfulness, as Miller and Selfridge point out). Cf. §28.

(7)(p.118) When we fit this discussion directly into our system of levels, this stipulation will mean that w is a prime of a certain level W.


(9)(p.124) Alternatively, we could dispense with the model of §5.2 and attempt to define degree of grammaticalness directly from the system of classes given by this procedure.

(10)(p. 125) For a detailed introductory study of the topics touched upon in the last two paragraphs, see Chomsky, Syntactic Structures, Journal of Symbolic Logic, 1955.

(11)(p.125) Cf. Wells, op. cit., for discussion of this problem. Also Harris, Methods, chap. 16.
(12) (p. 125) Cf. Harris, Methods, for discussion of this problem.

(13) (p. 127) In making this precise, we must specify how large these classes must be in relation to the number of steps already taken in constructing the analysis. There is an analogous problem in the method proposed for the analysis of homonyms.

(14) (p. 128) There may be other grounds for rejecting certain applications besides the infrequency of the resulting analysis. Thus if an improper context occurs too early for rejection by our stipulation, it will often be the case that the whole analysis collapses rapidly into a single class, and we may be able to formulate conditions which exclude this possibility.

(15) (p. 131) A special case of this problem, the case of clustering of points on a line, has been investigated by A.W. Holt in an unpublished MIT master's thesis.

(16) In an unpublished paper,

(17) (p. 132) A word of caution seems in place here. There is a tendency to criticize a certain approach to the definition of some linguistic notion because it fails to take into account 'all criteria' relevant to establishing the element in question. Such arguments in favor of a multiplicity of criteria carry no weight, however, unless we are shown exactly how all relevant criteria are to be simultaneously applied. In the case of most suggestions of this nature, this poses a constructional task of considerable magnitude, which is usually disregarded.

(18) (p. 134) The central ideas of this section were developed jointly by Peter Elias and myself.

(19) Perhaps it is worth emphasizing once again that the fact that no serious data is available is not a reason for judging such theorizing as we have been engaged in to be pointless. We cannot know what kind of data to collect until we have a theory that offers some hope of solving the difficulties that we know in advance to exist. The purpose of these remarks, of course, is to suggest such
a theory (cf. Introduction).

(20)(p.140) It is important to bear in mind throughout this discussion, that we are dealing with a fixed and finite corpus, with an upper bound on sentence length. The question of projecting to longer sentences only arises later. We have not gone into the question of adequacy of the corpus, but we do make certain obvious assumptions throughout, e.g., we assume that no sentence length is considered unless a sufficient number (in a sense that must ultimately be defined) of sentences of this length occur in the corpus, where the minimum sufficient number of course increases with increasing length. Whenever we speak of all sentence lengths, or all sentence forms, we mean all sentence lengths or forms having a sufficient number of instances, and  longer than $\lambda_0$.

(21)(p.140) Our approach is limited in that we are not considering suprasegmental features. A more complete theory that included these would no doubt identify sentence breaks with intonation morphemes belonging to a special category.

(22)(p.142) We take the log in (16) to ensure the existence of a limit.


(24) Conceiving of the language as a Markov chain with a finite number of states.

(25)(p.146), More properly, an absurd semi-English sentence, when we have set up degrees of grammaticalness.

(26)(p.151) Note that we must determine the frequencies of the categories first. We could not define the best $n$-category analysis as the one which deviates most sharply from a random sequence of $n$ elements, or the best analysis would be obtained by assigning each of the $n-1$ most infrequent elements to a separate category, and all other elements to the remaining category.

(26)(p. 151) See [reference]. For a study of phrase classification in [language], which can be interpreted in this way.

(27)(p. 151) Notice that there is no question being raised here as to the legitimacy of the probabilistic approach, just as the legitimacy of the study of meaning was in no way brought.
into question when we pointed out (§5.2) that projection cannot be defined in semantic terms. Whether or not the statistical study of language can contribute to grammar, it surely can be justified on quite independent grounds. These three approaches to language (grammatical, semantic, statistical) are independently important. In particular, none of them requires for its justification that it lead to solutions for problems which arise from pursuing one of the other approaches. Nevertheless, these three approaches are in some way related. The object of investigation is ultimately the same, and ultimately, we might expect them to fall into place in some larger semiotic theory.


(31)(p.159) Further specification is necessary if we do not assume uniqueness of this.

(32)(p.164) This is true if we use A in the maximally efficient way in constructing the set of sentence forms, i.e., if for each occurrence of a homonym H in the corpus, we make the best decision as to the assignment of this occurrence to one of the categories to which H belongs. On some of the difficulties involved in making this decision, cf. Appendix I. These difficulties might be used as an argument in favor of requiring disjointness, since it might be argued that we do not have a practical evaluation procedure for analyses such as A.

(33)(p.165) We assume now that this gives the best n+1 category analysis, and we neglect other possibilities, e.g., that part of 0 can be set up as a separate category.

(34)(p.167) In fact, neither of the available solutions seems particularly acceptable. Intuitively, "feed" is quite a different sort of homonym from "will" or /riyd/. This suggests that we have in this case failed to discover the real basis
for solving this problem. This should certainly be
looked into further.


(36)(p.169) Cf. Hockett, Problems of Morphemic Analysis, Language,
1947, and Harris, Methods, chapters 12-3, for discussion
of these problems.

(37)(p.169)cf. fn.28. It is important for our purposes
to show that Harris' results can be obtained from a
sufficiently large corpus, and that they do not require
a knowledge of all grammatical sentences, since we are
suggesting now that these results be used in constructing
the notion of grammaticalness. These investigations are
at an early stage, but it seems correct to assume that
circularity can be avoided here.

(38)(p.169)Cf. fn 51, chap.I.

(39)(p.3AP) Six Thousand Common English Words

(40)(p.3AP) Cf. Harris, Methods, chap. 6, and many other studies.
Thus we include imperatives, but not interrogatives.

(41)But note that for the determination
of the covering set of sentence forms we must consider all
overlaps that have some member, even if their relative size
is so small that no sentence containing these elements need
be considered. This is very important if these essentially
null overlap categories happen to appear in some contexts
not shared by other elements in the syntactic category of
which they are a part. In this case a sentence form must
be added to cover this sentence, and even though the number
of sentences covered is essentially zero, the number of
sentences generated by this form may be quite large.

(42)(p.6AP) The same was true for a sample of a corpus
of seven word sentences.

(43)(p.6AP) With the qualifications of 436.2. But these are
hardly serious, since it is fairly clear that slightly
more detailed category analysis would make the artificial
classifications of V-ing and V-en impossible.
Chapter V - Lower Levels of Grammatical Structure

27. Most of the methodological work in modern linguistics has been devoted to the problems of phonemic and morphological analysis. We have nothing to add to this here, and the discussion in this chapter is limited to a brief description of the formal structure of the statement of the existence of levels of phonemes, morphemes, words, and syntactic categories, with no detailed examination of the character of these levels beyond some minor investigation of their logical status, in terms of the notions that have been introduced above.\(^{(1)}\) The purpose of this statement is to present a framework into which we can fit the higher levels of phrase structure and transformational structure, which provide the real substance for our investigations. In the appendix to this chapter, we begin the task of relating the point of view that has been developed to actual language material, with a detailed morphophonemic study.

To characterize a linguistic level \(L\) we must present the set \(I\) of primes of \(L\) and the set \(\mu\) of \(L\)-markers, and we must describe relations among elements of \(L\), the mapping \(\Phi^L\) of \(L\)-markers into grammatical utterances, and the relations of \(L\) to other levels. We begin by considering a corpus of utterance tokens upon which conformity has been defined by the pair test.\(^{(2)}\) Conformity can be defined on parts of utterances in the manner of fn. 41, chap. I, if our phonemic theory requires this. In general, in describing \(L\) we need not characterize \(\Phi^L\) as a mapping of elements of \(L\)
directly into the set of utterance-tokens. It is sufficient to define the mapping \( \Phi^L \) into a set of elements in a level \( L' \) which are mapped into utterances by \( \Phi^L \), or which are in turn mapped into elements of a level \( L'' \) which are mapped by \( \Phi \) into utterances, etc.

\[ \textit{IX} \]

\[ \textit{IX}. \] The characterization of the linguistically significant levels may begin with \( \Phi \) the establishment of a lowest level of representation whose elements are given a physical description by means of the mapping \( \Phi \) on this level. Suppose then that we establish a level \( P_n \) of phones as the lowest level of representation. This can be identified with the first level of representation discussed in \( \Phi^6.2 \).

The set \( P_n \) of primes of \( P_n \) is the set of phones (cf. \( \Phi^6.2 \)), and \( \Phi \) is a set of strings of phones whose membership is determined partly by constructions on higher levels, \( \Phi \) in the manner discussed in the preceding chapter. \( \Phi \) will associate these strings of phones with specific utterances. That is, \( \Phi \) will provide a physical specification of strings in \( P_n \); if \( \bar{X} \) is a string of phones, then by "\( \Phi \) (\( \bar{X} \))" we will designate the set of utterance tokens (actual physical events) which are represented by \( \bar{X} \) on the level \( P_n \). \( \Phi \) must have the following properties.

(i) All phones are in the domain of \( \Phi \).

(ii) With each prime \( p_i \) of \( P_n \) (each phone) there is associated a certain set \( S_i \) of defining physical properties. That is, \( \Phi \) maps \( p_i \) into a set of phone tokens (cf. \( \Phi^6.1 \)) having the properties \( S_i \). If we have established conformity among phone tokens, \( \Phi \) in the manner of fn. \( \Phi^6.1 \). chap. I, then \( \Phi \) associates to each phone a maximal set of conforming phone tokens. If not, then the detail of specification, on this pre-phonemic level, is arbitrary.

It depends on the detail in which we wish to describe free and
contextual variation. Either way, we can think of the application of \( \varepsilon^P_n \) as a process of setting up phone types in the corpus from phone tokens. Each phone type is a set of phone tokens having a set of defining physical properties. We can consider each phone type sequence as having associated with it the corresponding sequence of these sets of physical properties.

(iii) If \( \bar{x} \) and \( \bar{y} \) are strings in \( P_n \), the \( \varepsilon^P_n(\bar{x}, \bar{y}) \) is the set of phone token sequences formed by concatenating one of the sequences from \( \varepsilon^P_n(\bar{x}) \) and one of the sequences from \( \varepsilon^P_n(\bar{y}) \), in that order. That is, if \( S_{\bar{x}} \) and \( S_{\bar{y}} \) are the sequences of physical specifications associated with \( \varepsilon^P_n(\bar{x}) \) and \( \varepsilon^P_n(\bar{y}) \), respectively, then the physical specification of \( \varepsilon^P_n(\bar{x}, \bar{y}) \) is the sequence \( S_{\bar{x}}, S_{\bar{y}} \).

(iv) Two utterances of the corpus conform if and only if they have the same representation in terms of phones, i.e., the same \( P_n \)-marker.

Thus \( \varepsilon^P_n \) gives a physical specification of each string in \( P_n \) in terms of the sets of physical properties associated with phones \( P_n \). \( \varepsilon^P_n \) can be understood as a denotation or naming relation holding between elements of this lowest level of representation and segments of the language itself, whether in the corpus or not. Thus we may have a string \( \bar{x} \) in \( P_n \) denoting utterances of the corpus (in this case, all of these utterances conform, and any utterance conforming to one of these is denoted by \( \bar{x} \)), and certain strings may denote grammatical or non-grammatical utterances which do not happen to be in the corpus. Each such string is essentially a phonetic description of some utterance, recorded or unrecorded, grammatical or ungrammatical. Every utterance built up out of phone types of the corpus has a unique phonetic representation.
Hence it is sufficient to relate all higher levels to $P_n$.

29.1. The first really significant linguistic level is the level $P_m$ of phonemes. The essence of a phonemic theory is contained in the definition of the mapping $\Phi^P_m$ which carries strings of phonemic phonemes into strings of phones (thus $\Phi^P_m(\underline{x})$ will designate a string of phones, where $\underline{x}$ is a string of phonemes). The principles by which phones are 'assigned' to phonemes in the procedures of linguistic analysis will thus appear here as conditions on $\Phi^P_m$. Let us consider now some of these conditions that $\Phi^P_m$ must meet.
unrecorded, grammatical or ungrammatical. Every utterance built up out of the phone types of the corpus has a unique phonetic representation. Hence it is sufficient to relate all higher levels to $P_m$.

29.4. The least significant linguistic level is the level $P_m$ of phonemes. The essence of a phonemic theory is contained in the definition of the mapping $\phi^{P_m}$ of strings of phonemes into strings of phones. Let us consider now some of the conditions that $\phi^{P_m}$ must meet.

From these general characterizations of levels it follows that

$P_m$

(1) $\phi^{P_m}$ is single-valued,

that is, the reading of a phonemic representation is unambiguous. Moreover, we will surely want $\phi^{P_m}$ to meet the analogue in $P_m$ of (iii), (28). That is, we want it to be the case that the reading of the string $\bar{X}Y$ is just the reading of $\bar{X}$ followed by the reading of $Y$.

(2) If $X$ and $Y$ are strings in $P_m$, then $\phi^{P_m} (X Y) = \phi^{P_m} (X) \phi^{P_m} (Y)$.

But the phonetic shape of a phoneme (i.e., the phone associated with it) may depend on context. Thus if (2) is to be satisfied, $\phi^{P_m}$ can not in general be defined on phonemes, but only on strings of phonemes that are invariant in their phonetic representation. Let $I$ be the set containing all such strings. Then we require that $I$ be closed under concatenation; i.e., any string formed by concatenating strings in $I$, $I$ is in $I$.

Many of the minimal elements in $I$ will be syllables, but some may be syllable sequences. We require in particular that the phonemic representation of every utterance in the corpus be contained in $I$. It follows from the general characterization of levels (chap. II) that $\phi^{P_m}$ (the set of phonemic representation of grammatical utterances) must be included in $I$. From (1) above and (iv), it follows that phonemes must meet the fundamental condition that
(3) two utterance tokens of the corpus conform if they have the same phonemic representation.

Some would add the further condition that no utterance token have more than one phonemic representation, so that phonemic representation is bi-unique, but there seems to be no particular motivation for this requirement, in our terms, and there is some question as to its correctness.

(3′) See Harris, Methods, p. for discussion of this condition. Also stress paper for considerations in favor of its rejection.

Let us make the assumption that each string $\mathbf{X}$ of $I$ which is $n$ phonemes in length is carried into a string $\mathbf{Y}$ which is $n$ phones in length. Then we can define

(3′′) This assumption that length is preserved under application of $\phi^p_m$ is too strong, but we will make it to simplify the discussion. The argument and constructions must actually be elaborated for the more general case.

a derived relation between individual phonemes and their allophones of the level $P_n$ first in $\zeta$ relative, then in an absolute sense. We will say that a phone $\mathbf{X}$ is an allophone of a phoneme $\mathbf{X}$ with respect to the phonemic context $\mathbf{w}$ if $\phi^p_m$ carries $\mathbf{X}$ into $\mathbf{X}$, where $\mathbf{w}$ is the counterpart in $\mathbf{X}$ of $\mathbf{w}$. Then $\mathbf{y}$ is an allophone of $\mathbf{x}$ if there is any context $\mathbf{w}$ with respect to which it is an allophone of $\mathbf{x}$. More explicitly,

Def. 1. Suppose that $\mathbf{x}$ is a prime, $\mathbf{X}$ is a string of $n$ phonemes, and $\mathbf{y}$ is a string of $m$ phonemes.

Then $\mathbf{y}$ is an allophone of $\mathbf{x}$ with respect to the phonemic context $\mathbf{A}_n$ if and only if there is a string $\mathbf{A}_n$ of $n$ phones and a string $\mathbf{B}_m$ of $m$ phones such that

$$C (\mathbf{A}_n \mathbf{X} \mathbf{B}_m) = \mathbf{A}_n \mathbf{Y} \mathbf{B}_m$$
Def. 2. \( Y \) is an allophone of \( X \) if and only if there is some context with respect to which \( Y \) is an allophone of \( X \).

For example, suppose that the phoneme sequence /spil/ is carried by \( \tilde{p}^{\text{Pn}} \) into the phone sequence \( \tilde{\text{p}}^{\text{Pm}} \). Then \( p^{m} \) is an allophone of /p/.

(3'\text{r}) We will henceforth follow the usual practice of giving phoneme transcriptions within slant lines, and phonetic transcriptions within brackets.

with respect to the phoneme context /\( \text{a}\-\text{i}\-\text{l} /. Hence, by definition, it is an allophone of /p/.

Def. 2. Let Allo(\( X \)) be the set of phones \( Y \) such that \( Y \) is an allophone of \( X \).

Let Allo*(\( X \)) be the set of phone tokens represented by allophones of \( X \).

The members of Allo*(\( X \)) will also be called allophones of \( X \).

Thus \( \text{Allo}^* = \tilde{p}^{\text{Pn}}(\text{Allo}) \); Allo* is a mapping that carries phonemes directly to phone tokens, which are called allophones of these phonemes.

We say that two phones \( a, b \) are in contrasting distribution in a set \( K \) of strings if there are two strings of \( K \) which differ only in these phones. Otherwise \( a \) and \( b \) are said to be in complementary distribution in \( K \).
Def. 7. a and b are in contrasting distribution (contrast, form an opposition) in the set \( K \) of strings if and only if there are strings \( x, X \) and \( y, Y \) in \( K \) such that \( K \) contains both \( X^a Y \) and \( X^b Y \). Otherwise, a and b are in complementary distribution in \( K \).

\[ \text{Ref.} \]

39.2. At this point we could proceed to develop an explicit phonemic theory in one of several different ways. There are many divergent approaches to phonemic analysis.

(4) See Harris, Methods, for discussion of many considerations that can be brought to bear in phonemic analysis. Cf. also Bloch, Postulates, for a careful exposition of one approach. The following comments have also been influenced to a considerable degree by the approach developed in Jakobson, Fant, and Halle, Preliminaries, and particularly, in Halle, Strategy.

and there will remain room for disagreement until some objective theory has been shown to meet clear criteria of adequacy. Instead of attempting to construct an explicitly explicit theory, we will investigate some of the fundamental requirements that are usually put forward for phonemic analysis. In addition to (1)-(3) familiar above (with (3) possibly extended to bi-uniqueness), almost every/conception of the phoneme requires that

(4) allophones of phoneme must be

(a) in complementary distribution

(b) phonetically similar.

(4') or in free variation. But discussion of free variation belongs with the discussion of phonemic distinctness, see chap. 2. Free variation is a different type of notion than complementary distribution, since the amount of free variation that one chooses to recognize is quite arbitrary. In a sense, every two utterance-tokens display free variation.
Consider now requirement (4a). We have **mix** defined complementary distribution, **mixed** to a degree of detail which will **mix** be adequate for our present purposes, and we have defined two senses for the term "allophone". Hence we **mix** can consider various ways of understanding (4a). It appears that **mix** first of all that (4a) is too strong a condition if we interpret it, in the usual manner, as referring to allophones in the absolute sense (of Def.2). This becomes clear when we recognize that the real motivation for the requirement of complementary distribution is **mix** to **mix** ensure non-ambiguity of phonemic representation. It is true that if allophones of a single phoneme never contrast, then phonemic representation will be unambiguous (i.e., (1) will be preserved), but we **mix** complementary distribution of allophones is not a necessary condition for non-ambiguity. In fact, (4a) in this stronger (but usual) sense may fail even if there is a bi-unique relation between phonemic representations and phone type sequences. This might be the case if there is a phoneme A with allophones **mix** and **mix**, where **mix** is at the same time an allophone of B and **mix** an allophone of **mix**. Then **mix** (as an allophone of B) and **mix** may contrast, even though the strongest relation (biuniqueness) holds between phonemic and phonetic representation. Suppose, for instance, that **mix** in a given **mix** linguistic description is the relation defined by (5) where capitals stand for phonemes and small letters for phones.

\[
(5) \quad \text{If } Fm \quad \text{then } A \rightarrow B \rightarrow \ \text{mix} \rightarrow \ \text{mix} \rightarrow Z
\]

\[
A \rightarrow E \rightarrow \ \text{mix} \rightarrow \ \text{mix}
\]

\[
B \rightarrow F \rightarrow \ \text{mix} \rightarrow \ \text{mix}
\]

That is, using the notations of chapter II (cf. Def.1, \( \gamma \), 12.2), we have

\[
(6) \quad A \rightarrow \ \text{mix} \quad \text{in env.}
\]

\[
A \rightarrow \ \text{mix}
\]

\[
B \rightarrow \ \text{mix}
\]

\[
A \rightarrow \ \text{mix}
\]
\( (5) \Phi^Pm: \ A^C \rightarrow x^2 \\
\ A^D \rightarrow y^x \\
\ D^P \rightarrow x^y \)

That is, using the notations of chapter II (cf. Def.1, 192), we have

\( (6) (a) A \rightarrow x \text{ in env. } -- C \)
\( (b) A \rightarrow x' \text{ } -- D \)
\( (c) B \rightarrow x' \text{ } -- D \)

But in this set \( K \) of strings, the relation \( \Phi^Pm \) is one-one, even though the allophones of \( A \) are not in complementary distribution with respect to \( K \). Furthermore, this is quite a familiar situation in linguistic work. For example, if we take the phoneme /\( b \)\( /\) as \( A \), the phoneme /\( d\)\( /\) as \( B \), the phone [\( t\)\( ]\) as \( x \), and the phone [\( d\)\( ]\) as \( y \), the so-called 'strong' position instead of \( C \), and the 'weak' position instead of \( D \)

phoneme /\( t\)\( /\) as \( A \), the phoneme /\( d\)\( /\) as \( B \), the phone [\( d\)\( ]\) as \( x \), the phone [\( t\)\( ]\) as \( x \), the so-called 'strong' position as \( D \), and the so-called 'weak' position as \( D \), then we have precisely a situation in Danish phonology that has been discussed by Jakobson.

\( (4'1) \) See Preliminaries, pp.4-5. Also quoted in Harris, Methods, pp. 143-9.

In the strong position, /\( t\)\( /\) has the allophone \( R[\{d\}]} \) ((6a)); in the strong position, /\( t\)\( /\) has the allophone [\( t\)\( ]}\) ((6b)) and /\( d\)\( /\) the allophone [\( d\)\( ]}\) ((6c)). Thus in the strong position [\( t\)\( ]\) and [\( d\)\( ]\) contrast, even though they are both allophones of /\( t\)\( /\). But this is a perfectly satisfactory situation, and is even compatible with the strong one-one requirement sometimes posed for phonemic analysis. It appears, then, that (4a) is an arbitrary and unmotivated requirement, if "allophone" is understood in the absolute sense.
But in this set $K$ of strings, the relation $\leq_P$ is one-one, even though the allophones of $A$ are not in complementary distribution with respect to $K$. Since (5) is quite close to certain familiar cases of partial overlapping, we would presumably not want to exclude such possibilities by some general requirement on phonemic analysis. It appears, then, that (4a) is an arbitrary and unmotivated requirement, then, if "allophone" is understood in the absolute sense.

Suppose then that we weaken (4a), taking "allophone" in the relative sense of Def. 1. That is, we require that (7) be true.

(7) Let $K$ be the set of contexts with respect to which either of the distinct phones $\alpha$ or $\beta$ is an allophone of a given phoneme $A$. Then $\alpha$ and $\beta$ are in complementary distribution with respect to $K$.

(7) is a necessary and sufficient condition for the fundamental requirement (1) that $P_m$ be single-valued, i.e., that the reading of a phonemic transcription be unique. It has no special connection with the bi-uniqueness condition, but neither does (4a) in the stronger form. (7) then, seems to be the proper interpretation of (4a). Neither (4a) in the stronger form nor the bi-uniqueness condition seem particularly relevant or interesting, from our point of view.

(4') One can perhaps build a plausible case for these conditions, from the point of view of a procedural theory (cf. §2) whose purpose is to provide methods for actually arriving at a grammar. But we have set more modest goals here, and in our context, these conditions do not seem justified. Cf. also second reference cited, fn 3'.
29.2. Consider now requirement (4b). A phonemic theory is unthinkable without some such consideration. Furthermore, it appears that similarity in absolute terms cannot be what we want, since allophones of distinct phonemes may even be identical (cf. fn. 46, chap. I). Thus we cannot allophones of a single phoneme to be more similar to each other, in an absolute physical sense, than allophones of distinct phonemes; we cannot, for example, expect that alveolar replacement of one allophone of a given phoneme by another allophone of this phoneme will in general give the same word, with perhaps a strange pronunciation. It may give quite a different word.

Most accounts of phonetic similarity operate not in terms of absolute closeness of match, but rather in terms of possession of common properties from a given set of properties. Phonemes are thus said to be phonetically similar to the extent that they share phonetic qualities or distinctive features'. This allows much more latitude in the characterization of phonetic similarity since the distinctive features can be defined in relative terms (e.g., more front, more back, more or less aspirated, etc.) rather than in absolute terms (e.g., alveolar, alveopalatal, aspirated at such-and-such a degree, etc.), and the significant values of the distinctive feature can be different in different contexts. Thus suppose that phones A and B occur in one set of contexts and phones C and D in a distinct set, that A is more aspirated than B and C more aspirated than D, that B and C are in fact identical. Then if the distinctive feature in question is relativized to context, and if we are interested in relative rather than absolute value, that it will be the case that A and C (and B and D) are considered phonetically similar (rather than B and C), since and C are more aspirated than any other phone contrasting with them. This conception of phonetic similarity seems much more adequate and acceptable than any based on absolute match.

Suppose that we have such a set of distinctive features (we return below to the question of how these features are selected), physically defined in the general theory
and available for the construction of any phonemic system for any particular language. These features can be defined in two ways, either as physical properties or physical scales. In the first case, any phone token can either possess or fail to possess the property in question. If in a given language it is the case either that all phone tokens of a given set K possess this property, or all fail to possess this property, we say that the feature in question is singular with respect to this set K. If some tokens of K possess the property, and others do not, we say that the feature in question is binary with respect to the set K. Hence if features are defined as properties, which any phone either may or may not have, they are obviously either singular or binary, by definition. If on the other hand the features are taken to be physically defined (unidimensional) scales, then when we select a set K of phone tokens to be measured in terms of the scale in question, we will find that these phone tokens cluster into one or more distinct classes (in accordance with a criterion of clustering that must be established in the theory). If they fall into one class, we say that the feature is singular with respect to K. If they fall into two classes, we say (this time significantly), that the feature is binary. In general, if they fall into n classes, we say that the feature is n-ary with respect to K. The discussion of the binary character of distinctive features has occasionally been confused by the failure to distinguish between features that are binary (or by definition (i.e., properties) and those that may be binary significantly (physical scales).
and others do not, we say that the feature in question is binary with respect to the set \( K \). Thus if the features are taken to be physically defined (unidimensional) scales, then when we select a set \( K \) of phone tokens to be measured in terms of the scale in question, we will find that these phone tokens cluster into one or more distinct classes (in accordance with a criterion to be established in the theory). If they fall into one class, we say that the feature is singulary with respect to \( K \). If they fall into two classes, we say (this time significantly), that the feature is binary. In general, if they fall into \( n \) classes, we say that the feature is \( n \)-ary with respect to \( K \).

Given a class \( K \) of phone tokens, and a feature \( f \) which is \( n \)-ary with respect to \( K \), we can assign to each token the value 1, 2, ..., \( n \), depending on the class into which it falls (assuming an ordering of those classes) in an obvious way. As we have pointed out above, it is not necessary that the scales or properties be defined in absolute terms. Possession of a property or clustering along a scale can be defined in relative terms as well, and this is generally necessary in such analysis.

Underlying distinctive feature analysis is the notion of minimal set. A minimal set of utterances, in the simplest case, is a set of utterances each with \( n \) phonemes in length and differing from one another phonemically only in the \( i \)-th place (\( n, i \) fixed for the set). Call the \( i \)-th place the distinctive position for this set. We can define a minimal set of phone tokens as the set of phone tokens that occupy the distinctive position in utterances belonging to a minimal set. These minimal sets are the sets \( K \) with
respect to which the valuation in terms of features is carried out. If phone tokens are divided up into exclusive minimal sets (i.e., a partitioning of phone tokens), we can determine a set of features sufficient to distinguish all non-conforming pairs in each set, and we can assign two tokens from different sets to the same phoneme if in terms of these features, the values of these tokens in their respective sets are identical. This characterization of minimal sets covers only the simplest case. We may set up minimal sets of phone tokens even when the contexts of these tokens are not identical, but are only sufficiently similar in respects taken to be relevant. This possibility is allowed for below.

As a procedure this is vitiated by circularity, or by the necessity for an exhaustive classification of utterances into tentative minimal sets (much in the manner of 427, interpreted as a procedure, cf. 430). But we can set various conditions for the phonemic analysis of a language in terms of this stock of features, without circularity, understanding this approach as amx offering an evaluation procedure for phonemic analysis.

Given a phonemic analysis with the set A of phonemes \(a_1, \ldots, a_n\),

(i) The set of phone tokens is partitioned into sets \(K_1, \ldots, K_r\). Thus each phone token belongs to one and only one \(K_i\). These are called minimal sets. If two utterance tokens conform (in the sense given by the pair test), then
tokens occur in the same phonemic context, then they belong to the same minimal set. (It follows that if two utterance tokens conform, in the sense given by the pair test and its elaborations, then for each i, the i-th token of the first belongs to the same minimal set as the i-th token of the second) More precisely, suppose

(4.111) Note that a condition on segmentation is that conforming utterances must be segmented into the same number of phone tokens.

that X and Y are strings of phonemes, and that a and b are phonemes (primes of Pm). Then X^a Y and X^b Y are phonemic representations of utterances. Then if two phone tokens p and q are allophones of a and b, respectively, with respect to the phonemic context X -- Y, then p and q are members of the same minimal set. (4.111)

(4.111) Since we are speaking of p and q here as phone tokens, rather than *phones, this actually requires a slight extension of Def.1, replacing xpm with "q b Pm" replaced by "p Extended_Pm". 
for each \( i \), the \( i \)th token of the first belongs to the same minimal set as the \( i \)th token of the second (note that a condition on segmentation is that conforming utterances are segmented into the same number of phone tokens).

(ii) A certain subset \( F \) of features is selected from the stock of distinctive features. We order the features of \( F \) from \( f_1 \) to \( f_m \) so that for each phone token \( p_{i,j} \) we have a sequence of values \( v_{1,j}, \ldots, v_{m,j} \), \( v_{i,j} \) being the value of \( p_{i,j} \) in terms of the \( i \)th feature, evaluated with respect to the minimal set to which \( p_{i,j} \) belongs.

(iii) With each phoneme \( a_{i,j} \), we associate a sequence of values \( v_{1} = v_{1,j}, \ldots, v_{m} \). This sequence of values must exactly identify the set \( \text{All}^f(a_{i,j}) \) of all phones now being taken as tokens, in the sense that each allophone of \( a_{i,j} \) must have the sequence \( v_{i} \) associated with it in its minimal set, and any phone token having this sequence of values must be an allophone of \( a_{i,j} \). Thus each sequence \( v_{i} \) essentially is a statement of the phonetic similarity necessary and sufficient for two phone tokens to belong to the same phoneme. If features are defined in relative terms, closely matching tokens may be phonetically dissimilar, and physically quite different tokens may be phonetically similar, in the sense here defined.

We take \( \text{All}^f \) as a representing relation from phonemes to utterance tokens, so that a string of phonemes represents each utterance token made up of an allophone of the first phoneme followed by an allophone of the second, etc. Then

(iv) Two utterance tokens have the same phonemic representation just in case they conform.
Note that conformity of phone tokens need not be considered in this theory. That is, the level \( \text{Ph} \) need not be established. But if we wish to describe subphonemic contextual variation of allophones, then this intermediate level is required. The tie-up of \( \text{\textfrac{1}{2}}\) and \( \text{\textfrac{2}{3}} \) poses no problem, but it must be carefully noted that phone tokens may conform even if they do not have the same value sequence (since they may belong to different minimal sets, and valuation is relative to a minimal set), and they may have the same value sequence even if they do not conform.

Further conditions can be laid down (and must be, if we are to have a significant evaluation procedure) to limit the choice among analyses. We might require that the set \( P \), or the number of phonemes, be reduced to a minimum, consistent with other requirements, and that the distinctive function of the features be as fully exploited as possible.\(^{(6)}\) We may also state conditions of simplicity in terms of higher levels. That is, if we find that certain higher level analyses are simplified by a given phonemic analysis, then this fact can be used in support of the analysis. Thus we have conditions of 'compatibility', in the sense of \( \text{\textfrac{6}{5}} \).\(^{(7)}\)

41. The metaphor of higher and lower levels can be variously understood. If we understand a higher level as one whose primes are 'longer' (in the number of phonetic segments contained by their images under \( \Phi \)), then the next level above phonemics is the level \( M \) of morphemes. If we take a higher level to be one which is relatively independent of the construction of lower levels (i.e., we understand the
In the foregoing exposition we have noted certain unwarranted assumptions which served to simplify the exposition and to enable us to bring out more clearly certain more crucial points. These obviously require elaboration in a fuller phonemic theory. One assumption is crucially important; however, since the method chosen to elaborate it distinguishes two important schools of thought in regard to phonemic analysis. This is the assumption that the set F of distinctive features is given by the Beran model in advance of any particular phonemic analysis. (7')

(7') where by "in advance" I refer to the logic of the situation, not the history of development of linguistic theory. The positions described below are reinterpretations of the general framework which has been developed here, but I think this is a fair statement of their salient features.

Jakobson, in particular, has maintained the position that the distinctive features are in fact to be fixed in the general theory, or, are to constitute part of the definition of language, and he has attempted to show that a very small set of features will suffice for all languages. An alternative to this very strong position would be that some notion of "physical simplicity" is taken as a primitive term in the general theory (rather than taking a certain fixed set of physically defined features as primitives) and that in the case of any particular language, a set of features should be chosen which is most simple in this sense. The choice between these approaches to the origin of the features raises many difficult questions, and investigation of them would carry us far beyond the intended scope of the present discussion.

32.4. We find, upon further investigation, that the concept of the phoneme outlined above is not fully adequate, and that certain considerations of simplicity must be added to the requirements (1)-(4) of 32.1 and (i)-(iv) of 32.2 if we are to avoid certain very unsatisfactory phonemizations. Consider for example the phonetically distinct English words "writer" and "rider", which can be transcribed as

(a) [ˈruːtər]
(b) [ˈrɪər]

respectively, where [R] stands for the apico-alveolar tongue flap, and [aː] is the
single phone "long a". The methods if outlined above might enable us to identify [ inflamm ] as an allophone of some phoneme, perhaps /æ/, but we would still be forced to regard length in the second phoneme as the only distinctive feature distinguishing these utterances. We would thus have to set up the phoneme /æ:/ as a phoneme which occurs only in stressed position in the context --yd, and we would have to set up the distinctive feature of length as one of the phonemic features of this dialect, serving to distinguish utterances only in this very special case.

(711) We consider here a dialect which does not have the /æ:/ distinction sometimes found in "bomb", "balm", etc. Our other examples are also valid only for certain dialects, in particular, in my case.

But this is hardly an acceptable phonetic analysis. Investigating Inquiring into the matter further, we find that in the proposed phonetic system

(9) (i) /t/ never occurs in the context ey e

(ii) [a] and [a:] are contextually determined variants in the context

--yc, [a] appearing when C is unvoiced, [a:] appearing when C is voiced, except when C = /æ/, may occur. Thus we have


but [rayber] ("writer") - [rayber] ("writer").

(711)

But more generally than (i), we find that /t/ never occurs in any intervocalic position. That is, there is no distinction between "latter" and "ladder", "metal" and "medal", etc. A simple way to account for this fact (as with other, in a complete grammar) would be to say that /t/ can occur in this position, but that part of the characterization of C_Pm is given by the statement

(10) (a) [a] transfer in env. XxxxN e, where N is a 'syllabic nucleus', i.e.,
a vowel or semi-vowel.

(711)

We avoid here certain questions of phonemicization of "syllabic nucleus" and of dialectal differentiation which are interesting, but not relevant. Note also the /æ:/. The uniqueness condition discussed in (710) might be avoided, but need not concern us, since we found no motivation for this condition in our discussion.

That is, the mapping C_Pm carries both /t/ and /æ:/ into [R] in intervocalic post-stress position.
We avoid here certain questions about phonemicization of syllabic nuclei and dialectal differentiation which are interesting, but I think not relevant. Note also that the bi-uniqueness condition discussed in §32.1-2 is violated by (10). This could be avoided, but need not concern us, since we found no particular motivation anyway for this condition.

That is, the mapping \( \Phi^\text{Pm} \) carries both /t/ and /d/ into [R] in intervocalic post-stress position. But since (9) and (10) are needed anyway, irrespective of how we treat "writer"—"rider", we see that a very simple way to eliminate the phoneme /a:/ from the set of primes of \( \Phi^\text{Pm} \) (and the feature of length from the set of distinctive features) is to phonemicize "writer" as /ratær/ and "rider" as /raydær/. We can then drop in (9ii) the restriction "except when \( z-\tilde{a}/z \). Applying (11) and (10) (in that order -- cf. §32.2) to /ratær/ and /raydær/, we derive the phone sequences [ratær] and [raydær], respectively, just as we require.

We have in the process managed to eliminate one special condition in the characterization of \( \Phi^\text{Pm} \) (namely, that \( z-\tilde{a}/z \), in (9ii)), and to eliminate one phoneme and one distinctive feature of extremely limited and unique distribution. We also find, of course, that the new and simpler analysis correlates with independent morphological considerations.

However, this obviously superior analysis contradicts the requirements of §32.3, since the token [R] of "writer" and the token [R] of "rider" must belong to the same minimal set (by (1), §32.2, since they occur in the same phonemic context ray—y[r]). But since these tokens will not be distinguished by any physically defined distinctive feature in this minimal set, the words "writer" and "rider" will not be distinguished, contradicting the phonemic distinctness shown by the pair test. We might be able to carry through this phonemic analysis in a manner compatible with §32.3 if we were to weaken (1), §32.3, but only at the cost of establishing some extremely strange minimal sets. (7111) It seems much more reasonable,

(7111) Thus we might assign the [R] of "writer" to the same minimal set as the [d] of
"red", arguing that in this minimal set [R] is relatively unvoiced, hence an allophone of /t/; and we might assign the [R] of "rider" to the same minimal set as the [t] of "wet" arguing that in this minimal set [R] is relatively voiced, hence an allophone of /d/. This artificial analysis is ruled out by (1), §39.3, and it seems clear that it is proper to rule it out.

However, to regard the results of distinctive feature analysis as only a first approximation to a phonemic system, to be refined in terms of criteria of simplicity, as indicated above, for example. The procedures that Harris calls "rephonemicization" are also relevant to this second step in determining a phonemic system, as are the familiar considerations leading to the setting up of phonemic junctures. Thus we might consider distinctive feature analysis, as outlined in §39.2, to be at a level of representation which is intermediate between phones (PM) and phonemes (PM); or we might consider that the level PM contains among its primes both the elements set up in §39.3 and consonant combination on essentially phonetic grounds, and those set up in by the essentially distributional reanalysis hinted at by example in this section, with the name "phonemes" withheld to the latter elements. To complete the construction of the level PM, then, we would have to state precisely what sort of criteria of distribution and simplicity are to operate and how the elements set up are to be related to the elements set up by the considerations of §39.3. We will not go into this question, which is an important and difficult one, any further, but it does appear that even the rough analysis of simplicity discussed in chapter III may be fairly effective in this connection.
Once we recognize that \textit{FM} criteria of distributions and simplicity \textit{MK} must be applied if we are to arrive at \textit{MAXI} phonemic system, \textit{MAXI} we are naturally led to ask how great a burden can be placed on considerations of this sort. It might be argued that the notions of distinctive feature analysis that were introduced in \textsection 39.2 can be replaced completely by the consideration of the simplicity of characterization of the mapping \textit{PM} which \textit{MAXI} provides the phonetic "reading" of strings of phonemes. Some clarification of this proposal would be necessary before we could ascertain its relation to the \textit{MAXI} discussion of \textsection 39.2, but this may be a fruitful approach.

40. The metaphor of 'higher' and \textit{MK} 'lower' levels can be variously understood. If we understand a \textit{MAXI} higher level as one whose primes are 'longer' (in the number of phonetic segments contained by their images under \textit{MAXI} ), then the next level above phonemics is the level \textit{MK} of morphemes. If we take a higher level to be one which is relatively independent of the construction of lower levels (i.e., we understand the
metaphor as referring to the rough order of analytic procedures), then the next level above phonemes is the level $W$ of words. Following the latter course, we consider next the level $W$.

$\Phi^W$ will be a mapping of words and strings of words into strings of phonemes, meeting the analogue of (2). $W$ is a set of strings of words which will be defined on higher levels, in part, in terms of syntactic categories. It must be possible to determine the words from the corpus almost to uniqueness (cf. §7). The most important problem on the level $W$ concerns the method for accomplishing this. The reasonable suggestions that I know are those mentioned in §28.5. Harris' data seem to indicate that word boundaries can be placed quite effectively in this manner. The problem, for our purposes, is whether this can be done on the basis of a reasonably sized corpus. On the basis of the limited investigations that have been carried out, it seems reasonable to suppose that it can.

The mapping $\Phi^W$ will be a specification of the phonemic shape of words. This can be given as an immense list, but a good deal of simplification and organization is possible through the introduction of morphological and morphophonemic levels (cf. §16.2). In general, this mapping can be described most effectively and simply by assuming an amalgamation of the four levels involved, i.e., by the formation of a new system of concatenation with words, morphemes, morphophonemes, and phonemes as primes,
concatenation of these elements being freely permitted, and all relations and interconnections of these levels being carried over into this new system. This will require more careful restatement when these levels are constructed more seriously, but it is clear what has to be done. Then the mappings of words to phonemes can be given by a sequence of statements of the form \( \alpha \rightarrow \beta \), so that the derivations of phonemic sequences from word sequences can be constructed in the manner of chap. III. We can then study the relative simplicity of various proposed morphological and morphophonemic analyses, we can determine the hierarchy of statements within morphology and the degree to which conditions of optimality can be reached (cf. \( \text{121} \text{2} \)), etc. Cf. Appendix for detailed study of these matters with actual language material.

4\text{2}. Following the order of relative independence, the next higher level will be the level \( C \) of syntactic categories. This is the first level in which a string can represent a set of non-conforming utterance tokens. We form \( C \) by interpreting \( C \) of \( 325 \text{2} \) as a concatenation algebra, with the \( C^n_i \) as primes. Mirroring (14), chap. IV, we have the definition of the set \( C \) of primes of \( C \) as

\[
C = \{ C^n_i \}, \quad \text{where} \quad (i) \quad 1 \leq n \leq N \\
(ii) \quad 1 \leq i \leq a_N \\
(iii) \quad a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_N
\]

We have a relation \( g \) of generation, holding between strings in \( C \) and strings in \( W \), and meeting the condition
41.1. Following the order of relative independence, the next higher level will be the level $Q$ of syntactic categories. This is the first level in which a string can represent a set of non-conforming utterance tokens. We form $Q$ by interpreting the system of orders of categories, subcategories, etc. which was discussed in the last chapter as a concatenation algebra. The primes of the level $Q$ are thus the categories themselves, and any sequence of these categories (i.e., every sentence form, in the sense of 2424 §25) is a string in $Q$. In §25 we discussed various orders of categories.

There were to be a very large number of categories of order 1 (defining highest degree grammaticalness), a smaller number of categories of order 2, etc., and for some $N$, a very small number (perhaps one) of categories of order $N$, defining the lowest degree of grammaticalness considered in the grammar. Thus with each string of words we can associate a string in $Q$ of order 1, of order 2, ..., of order $N$. As a marker of a string $x$ of words, we can take the set of strings in $Q$, one of each order, that generate $x$ in the sense that each word in $x$ is a member of the first category, the second word is a member of the second category, etc. Thus a string of words will have two markers:

Suppose we have chosen markers for all utterances of the corpus. Then there will in general be a large number of strings of words not in the corpus but generated by these markers. This is the set of grammatical sentences of the highest degree of grammaticalness. We denote this set "Gr($W$)".

If grammar construction broke off at the level $Q$ we could identify the set of markers of grammatical utterances as $Gr(W)$. But as we have seen in §35, $Gr(W)$ is only a first approximation to $W$.

The most important contribution of the level $Q$ is the setting up of $Gr(W)$ as a first approximation to $W$. Ultimately, the purpose of a grammar is to provide a specification and description of the grammatical
utterances of the language, that is, the set of utterances represented by members of \( \mathcal{W} \) on the level \( \mathcal{W} \). But in we have pointed out that this description must be approached in stages of which the establishment of \( \text{Gr}(\mathcal{W}) \) is the first. We have seen in Chap. IV that at best \( \text{Gr}(\mathcal{W}) \) is a close approximation to the set of grammatical utterances of less than some fixed length, this being the length of the longest sentence form (adequately represented -- cf. §29) in the corpus. \( \text{Gr}(\mathcal{W}) \) is thus a finite projection of the corpus. \( \text{Gr}(\mathcal{W}) \) plays the same role for higher levels of analysis that the corpus itself played for the lower levels. That is, we construct higher levels in such a way that the simplest description of \( \text{Gr}(\mathcal{W}) \), in terms of the machinery available in these levels, leads to a projection of \( \text{Gr}(\mathcal{W}) \) (ultimately, an infinite projection), with perhaps some readjustment of the boundaries of \( \text{Gr}(\mathcal{W}) \) itself.

In this way we hope ultimately, when the investigation of all levels has been much advanced, to be able to define \( \mathcal{W}_L \) for some level \( \mathcal{W}_L \), presumably, the level \( \mathcal{W}_1 \). And with this we can obtain a characterization of the markers of each linguistic level, thus giving us a complete linguistic theory, and complete grammars.

In fact, we form \( \mathcal{C} \) by interpreting \( \mathcal{C} \) of §25 as a concatenation algebra, with the \( \mathcal{C}_n \) as primes. Mirroring (14), chap. IV, we have the characterization

\[
\text{definixxxx} \quad \text{of the set} \quad \mathcal{C} \quad \text{of primes of} \quad \mathcal{C} \quad \text{as}
\]

\[
(11) \quad \mathcal{C} = \left\{ \mathcal{C}_n \right\}, \text{where} \quad \begin{align*}
(i) & \quad 1 \leq n \leq N \\
(ii) & \quad 1 \leq i \leq a_n \\
(iii) & \quad a_1 > a_2 > \cdots > a_N
\end{align*}
\]

We have a relation \( g \) of generation holding between strings in \( \mathcal{C} \) and strings in \( \mathcal{W} \) and meeting the condition
(i) If $C_n^1 \in C$ and $g(C_n^1, X)$, then $X$ is a prime of $W$ (i.e., $X \in W$).

[That is, $g$ relates categories to single words. It is thus the converse of $\varepsilon$, in terms of $\text{§25.2.}$]

(ii) If $Y$ and $Z$ are strings in $C \ (Y, Z \not\in U)$, then

$g(Y, Z, X) \iff$ there are strings $W_1, W_2$ in $W$ such that

(a) $X = W_1 \wedge W_2$

(b) $g(Y, W_1)$ and $g(Z, W_2)$

[This is the analogue of (2) in $C$.]

(iii) For all $C_n^1 \in C$, there is an $X$ s.t. $g(C_n^1, X)$

(iv) For all $C_n^1, C_n^2 \in C$, if $i \neq j$, then there is an $X$ s.t. $g(C_n^1, X)$ but not $g(C_n^2, X)$

We define a grammatical sentence form of order $n$ as a string $X = C_n^1 \wedge C_n^2 \wedge \cdots \wedge C_n^{k_2}$ such that some $X$ generated by $X$ is in the corpus (omitting now, in this oversimplified sketch, the refinements of $\text{§22}$). We now define $Gr(W)$, the set of first order grammatical sentences in the sense of chap. IV.

**Def.** $Gr(W)$ is the set of $X$'s s.t. for some grammatical sentence form $Y$ of order 1, $g(Y, X)$

If the grammar construction broke off at the level $C$, we would identify $\mathcal{G}^W$, the class of $W$-markers, as $Gr(W)$.

But as we have seen in $\text{§35}$, $Gr(W)$ is just a first approximation to this class.

We can take the $C$-markers of a string $X$ in $\mathcal{G}^W$ as a set
We define $\mathcal{C}$-markers as follows:

**Def. 6.** $K$ is a $\mathcal{C}$-marker if and only if

\[ K = \left\{ C_1, \ldots, C_N \right\}, \]

where (i) $C_1$ is a grammatical sentence form of order $1$ 
(ii) If $\varepsilon(C_1, X)$, for $i > 1$, then $\varepsilon(C_{i-1}, X)$

It follows that $\bar{\varepsilon}^C$, the representing mapping which assigns $\mathcal{C}$-markers to grammatical utterances (represented in terms of words) is exactly $\varepsilon$, with domain limited to the first order sentence form of the $\mathcal{C}$-marker in question.

In stating $\mathcal{C}$ we also specify a certain $n_A, 1 \leq n_A \leq N$, which gives the absolute order of categories and a basic dichotomy between grammatical and non-grammatical. Cf. § 22.

In addition to these specifications, we of course require that $\mathcal{C}$ be related to the corpus in the manner discussed in chapter IV (see particularly § 27). The basic character of $\mathcal{C}$ is expressed in this set of relations.
Gr(W) is a close approximation to the set of grammatical sentences of less than some fixed length, this being the length of the longest adequately represented sentence of the corpus. Gr(W) is thus a finite projection of the corpus. Gr(W) plays the same role for higher levels of analysis that the corpus itself played for the lower levels. That is, we construct higher levels in such a way that the simplest description of Gr(W), in terms of the machinery available in these levels, leads to a projection of Gr(W) (ultimately, an infinite projection), with perhaps some readjustment of the boundaries of Gr(W) itself. In this way we hope ultimately, when the investigation of all levels has been much advanced, to be able to define \( \mu^L \) for some level \( L \), presumably, the level \( W \). And with this we can obtain a characterization of the markers of each linguistic level, thus filling in the major gap in the theoretical construction.

The level of morphophonemic representation is introduced to simplify the statement of the mapping \( \varphi^W \) from words to phonemes. The systematic motivation for introduction of a level \( \text{M} \) of morphological representation is primarily the resulting simplification in the derivation of word sequences from phrase sequences. Each string of words has a morphemic spelling, and the set \( \mu^M \) of \( \text{M} \)-markers will thus be a set of strings in \( \text{M} \) correlated by \( \varphi^M \) to \( \mu^W \). \( \varphi^M \) thus corresponds basically to an operation of putting in word boundaries in morpheme sequences. But this is an over-simplification. The correlation will not in general be order preserving. Morphemes can be discontinuous, a string of morphemes may correspond to a single word or
phoneme sequence even if parts of this morpheme sequence do not correspond to parts of the word or phoneme sequence. Since the systematic role of morphological analysis is to simplify the derivation of word sequences from phrase sequences, we can regard M as a level intermediate between the level W of words and the level P of phrase structure. This suggests that it might be useful to consider separately two classes of morphological elements, those that figure in the statement of phrase structure and those whose function is limited to the description of word structure. In the first class (call it $\overline{M}$), we have what we can call 'morphological heads' as well as those affixes that function syntactically (e.g., morphemic long components expressing agreement in gender and number, etc.). In the second class we have such elements as English ess (actress, etc.) which do not themselves enter into the description of phrase structure, but which enter into the formation of the minimal units that play some role in syntax. $\overline{M}$ can be pictured as embedded into the level $\overline{P}$. Derivations in $\overline{P}$ thus lead from the representation Sentence to strings in $\overline{M}$. These derivations are then extended through the levels $\overline{M}$ and $\overline{W}$ by first analyzing the morphological heads into strings of morphemes, and then placing word boundaries (i.e., applying $\Phi^M$).

$\overline{M}$ can be set up as a subalgebra of $\overline{M}$. The primes of $\overline{M}$ are then a set H of morphological heads and a set A of syntactically functioning affixes. $\overline{M}$ is the only part of M that need be considered on higher levels of syntax. We
will have several occasions below to return to the study of $\mathfrak{M}$, and its relations with $\mathfrak{M}$ and $\mathfrak{W}$.

The level $\mathfrak{M}$ will add an important contribution to the analysis of grammaticalness. On the one hand, it will surely be the case that the morphology can be simplified by cutting away at the boundaries of $\text{Gr}(\mathfrak{W})$ to some extent. On the other, there are productive morphological processes that can lead to a further extension of $\text{Gr}(\mathfrak{W})$ much in the manner discussed in chapter IV. But exactly how these processes of projection (in the broad sense) are to be characterized is a difficult and intricate problem to which we can offer no solution here.

44. Appendix: Morphophonemics of Modern Hebrew

This is a segment of a grammar, in the sense of chap. III, with an explicit procedure of generation of phoneme strings from word and morpheme strings represented morphophonemically. It appears that a great deal of compression and generalization is possible, using the devices of chapter III. Sample derivations are constructed, and it is shown that a partial ordering is imposed on the statements of the grammar by the criterion of simplicity, so that there is a hierarchy of morphological processes. This amounts to a partial validation of the grammar, in the one dimension of order (it must also be demonstrated that the elements chosen lead to the simplest grammar, etc.). The grammar apparently meets the conditions of optimality suggested in §21.3.

This is not included here.
(1) Cf. Hjé, Positional Algebras, for a more detailed analysis of certain of the questions touched on below.

(2) Conformity is taken here to be an equivalence relation, cf. fn. 6, chap. II.

(3) Within the limits imposed by (iv).

(4) There are many divergent approaches to phonemic analysis. The fundamental requirement for phonemic analysis is that there be a 1:1 relation between phonemic representations and equivalence classes of conforming utterances, or (in a form that is equivalent, under reasonable assumptions) that allophones of a single phoneme be in complementary distribution. Furthermore, phonemic analysis is unthinkable without some notion of phonetic similarity among allophones of a single phoneme, formulated in advance of any given phonemic analysis. But beyond this, there is much disagreement as to what constitutes a proper phonemic analysis. There will remain room for disagreement at least until some objective theory has been shown to meet clear criteria of adequacy. Cf. Harris, Method for discussion of many considerations that can be brought to bear in phonemic analysis. Cf. Also Bloch, Postulates, for a careful exposition of one approach.

For the approach discussed here, cf. Jakobson, Fant, Halle, Preliminaries, and Halle, Strategy. We follow the exposition in the latter.

(5) This set of conditions only covers the simplest case, where each feature has the same number of values with respect to each minimal set. Generalization is possible to cover the phenomenon called 'neutralization', and a generalized sense of neutralization, but this need not concern us here.

(6) I.e., that the redundancy of the description be minimized. Cf. Cherry, Halle, and Jakobson, Towards the Logical Description of Languages in their Phonemic Aspect, Language, 1953.

(7) Harris, Method, appendix to 2.4. Also Halle, How and Why We Study the Sounds of Speech, Georgetown University Monograph Series on Languages and Linguistics, 1954.
There is also a strong extra-systematic motivation for the construction of this level, namely, the desire to lay bare the formal basis for certain intuitions about form, i.e., about the placement of morpheme boundaries, identification of morpheme alternants, etc. Cf. §2.3, §7.

As in the case of what Hockett has called 'portmanteau' forms. Cf. Problems of Morphemic Analysis, for this and other discussions of morphology. Cf. also the relevant sections in Harris, *Methods*, and Mida, *Morphology*.

The distinction we are drawing is essentially that between the morphological processes of inflection and composition. Cf. Bloomfield, *Language*.
Chapter VI - Phrase Structure

The first level that we will consider in any detail is the level $P$ of phrase structure. The phrase is the next larger unit above the word. In accordance with the general program outlined in Chap. I, the following discussion will differ in certain fundamental respects from such treatments of constituent analysis as that of Harris,\(^1\) or of Wells.\(^2\) There will be no attempt here to actually furnish a practical procedure for discovering the constituents of a language. Instead, attention will be focused on the problem of stating the general underlying structure to which any proposed analysis must conform, and investigating the logical status and the interrelations of the notions which play a leading part in constituent analysis. The goal of this investigation, as on other levels, will be, ultimately, to provide a practical evaluation procedure for any description of constituent structure proposed for a given language.

The motivation behind the construction of the level $P$ has already been outlined, in \(15.4\) and \(16.2\). One further motive has appeared, incidentally, in the discussion of grammaticalness. There we were only partially successful in extending the set of observed sentences to a set of grammatical sentences. We were able to construct a technique of projection that enabled us to determine grammatical sentences of length $n$ for some $n$, but we were not able to account for the generation of longer and longer sentences—we could not account for the infinite extensibility of the
class of grammatical sentences.

When we turn to the level of phrase structure, we see that certain rules have a recursive character. Thus **Noun Phrase (NP)** might be defined in such a way that one of its components may be an NP. That is, we may have a statement \( \text{Noun Phrase} \rightarrow \text{Noun Phrase} \), and another statement \( \text{NP} \rightarrow \text{NP} ' \text{who}' \). If we have such statements, and if we can extend the notion of generation (cf. §21) to cover them significantly, then an infinite number of grammatical sentences will indeed be generated by that part of the grammar that deals with phrase structure. So the deficiency in generation which seemed insurmountable on the level of word may find a solution on this higher level, if we can construct an effective technique of constituent analysis.

How can this further (in fact, infinite) extension of the class of grammatical sentences be effected? The answer can be guided by the general and a description which we have found to be the case in the simplest possible manner:

"could" = "is" = simplicity of the laws whereby we describe what is.

We have already noted that the construction of a level of phrase structure was motivated by the desire to simplify the description of grammatical sentences, and we now have at least a tentative definition of simplicity. In accordance with the general approach outlined in §35, we will try to construct the level \( P \) in an abstract manner as a new point of view from which to discuss grammatical sentences with further descriptive potential. It should be the case that if the methods of chap. IV permit the construction of exactly the...
set of grammatical sentences of length $\leq n$ words, where $n$ is large enough, then the simplest description of this set of sentences in terms of the descriptive apparatus provided by $P$ will include a recursive characterization of certain elements. If we allow these recursions to run on freely, beyond the set of grammatical sentences already established, we will find that new and longer sentences are generated. And we can define the set of grammatical sentences in general as including the (now infinite) set of sentences generated by the simplest grammar constructed in terms of $P$ for the grammatical sentences of length $\leq n$, where these are determined by the methods of chap. IV.

In this way we may hope to remedy the inadequacy of the definition of grammaticalness afforded by $C$, though not without some reconsideration of the form of grammars. (cf. fn. 13, chap. III). Our program now is to construct the level of phrase structure and to investigate its adequacy in remedying the deficiencies that led to its construction, in particular, the complexity and intuitive inadequacy of a grammar that does not go beyond word structure, and the failure of such a grammar to account for the infinite extension of the class of grammatical sentences. This investigation will occupy us in the next two chapters. We will see that there is a large measure of success, though not complete success, in all of these respects. In chapter VIII we investigate the consequences of the remaining inadequacies of complexity and failure of intuitive correspondence. In $h$, 
chapter IX, we will reconsider the question of generation
and suggest a different source for the infinite projectibility
left unexplained in chapter IV.

The elements (or ‘vocabulary’) of the level $P$ will be,
for English, such representations of strings of words as
Sentence, Noun Phrase (NP), Verb Phrase (VP), Noun (N), Verb (V),
etc., as well as elements corresponding to individual
words and the grammatically functioning morphemes of $\tilde{M}$.
Thus when we actually interpret $P$ for English, we will
expect to find such strings in $P$ as $NP^{1}le^{2}NP$ and
Sentence$^{1}John^{2}ing^{3}VP$. The fact that the second string
represents nonsense will have to be brought out in the
grammar of English, but this fact does not prevent this
concatenation of elements from being a full-fledged string
in $P$ for English. There is no need to set up a level
distinction within $P$, such that only words can be
concatenated with words, only ‘first-level’ phrases with
other ‘first-level’ phrases, etc. This would lead to great
complexity with no apparent gain. It should be noted that,
just as in the case of the level $C$, the names chosen for
the elements of $P$ in the English interpretation (i.e., NP,
V, etc.) have no significance and are devoid of content.
That is, there will be no attempt here to define Noun
Phrase within general linguistic theory, but only to
define ‘constituent’ in such a way that Noun Phrases
and Verb Phrases, etc., will turn out to be constituents,
and in fact, different constituents. Another way of
saying the same thing is this. Although these constructions
will permit us to keep distinct the constituents within each
given language, they will not permit us to associate the
constituents of one language with the constituents of
another. It may be possible to extend these constructions
to include general definitions for Noun Phrase, etc., but
this is not within the scope of the present investigation.
It is not clear what sort of basis would be necessary for
such constructions. Cf. §11.

47.1. The level $P$ is based on a relation $f$ (read 'represents').
This is the relation which holds in English between NP
and the man, between Sentence and John came home,
between Sentence and NP$^\text{VP}$, between the latter and John came home,
or between N and John. The converse of $f$ can be read 'is a', i.e.,

\begin{equation}
(1) \quad f(\text{NP, the man}) \iff \text{the man is a NP.}
\end{equation}

$f$ is irreflexive, asymmetrical, transitive, and non-
connected. That is, it is a partial ordering of the elements
of $P$. In axiomatizing $P_m$, we will have to provide that $f$
is carried over under concatenation, i.e., if $A$ represents
$A$, and $B$ represents $B$, then $A^*B$ represents $A^*B$.

There will be a unique prime $P_0$ in $P$ such that if
$f(X, \ldots P_0 \ldots)$, then $X = P_0$. This will be the element Sentence.

There will be certain primes in $P$ which bear $f$ to no
element. Let us call the set of these primes and the
strings formed from them, the set $P$. Basically, $P$ can
be understood as the subalgebra $M$, which, as we have seen
in §43, contains everything in $M$ that is relevant to $P$, and
might thus be embedded in $P$. Actually it is more convenient
46.1 The level $P$ is based on a relation of 'representation' which we symbolize as $\mathcal{f}$. This is the relation which holds, in English, between NP and the 'old' man, between Sentence and John 'came home', between Sentence and NP VP, between the latter and John 'came home', or between $J$ and John. The fact that $X$ represents $Y$ is stated symbolically in the form "$\mathcal{f}(X,Y)$". Thus in English it will be the case that

(1) $\mathcal{f}(NP, \text{'old man'})$

$\mathcal{f}(\text{Sentence}, \text{John 'came home'})$

$\mathcal{f}(\text{Sentence}, NP VP)$

etc.

If $X$ exactly represents $Y$, we can say that $Y$ is an $X$. That is, the relation is a is the converse of $\mathcal{f}$. We can therefore read (1) as

(2) the 'old man' is a NP

John 'came home' is a Sentence

NP VP is a Sentence

etc.

The relation $\mathcal{f}$ has the properties that

(3) $\mathcal{f}$ (a) for no string $X$ is it the case that $\mathcal{f}(X,X)$ (irreflexivity)

(b) If $\mathcal{f}(X,Y)$, then it is not the case that $\mathcal{f}(Y,X)$ (asymmetry)

(c) If $\mathcal{f}(X,Y)$ and $\mathcal{f}(Y,Z)$ then $\mathcal{f}(X,Z)$ (transitivity)

$\mathcal{f}$ is not necessarily the case for each $X,Y$ such that $X,Y$.

$\mathcal{f}$ (non-connectedness)

We can summarize these properties by stating that $\mathcal{f}$ is a partial ordering of the elements of $P$. Looking at $\mathcal{f}$ as an ordering relation, there will be a unique element prime of $P$ which essentially stands 'first' in this ordering. This element represents 'Sentence', there is no $\mathcal{f}$ will be the element $\mathcal{f}$ Sentence. More correctly, we say that the element $\mathcal{f}$ Sentence has the property that if a string $X$ represents $\mathcal{f}$ Sentence $X$ (for any
Strings $X$ and $Y$ not both units, then $X$ is its itself the maxing prime sentence.

Again looking at $f$ as an ordering relation, there will be certain elements that are 'last' in this ordering. That is, there will be certain primes in containing just $P$ which bear the relation $f$ to no element. Let us call the set of these primes and the strings formed from them, the set $F'$. If $X$ is a string of $F'$, then there is no string $Y$ such that $f'(X,Y)$. Basically, $F'$ can be thought of as being the set of morphological heads and syntactically functioning affixes, and the strings formed from them. That is, $F'$ can be thought of as being essentially the subalgebra $\bar{M}$ constructed in $\mathfrak{S}_H$ of those elements of $M$ which are relevant to $P$. Actually, it is somewhat more convenient to take $F'$ not as $\bar{M}$ itself, but as a set of elements closely related to $\bar{M}$. In particular it turns out to be more convenient (or even necessary) to specify the exact morphemic shape of long components such as number, gender, etc. (i.e., elements whose domain extends over some syntactic unit longer than a word) not within the level $P$ itself, but by means of the mapping which relates $F'$ to lower levels. Thus we find it expedient to take $F'$ as the set containing morphological heads and 'bearers' of long components as its primes. Strings in $F'$ will then be mapped into actual strings of morphemes. There are several reasons for this decision, as we will see below. Cf., in particular,

In axiomatizing $F'$ we will have to provide that the relation $f'$ is carried over under concatenation. That is, if $X$ represents $\bar{X}$ and $Y$ represents $\bar{Y}$, then the compound string $\bar{X} \bar{Y}$ represents $\bar{X} \bar{Y}$. 
\( \mathbb{P} \) then is the set of strings which can be thought of as forming the

lowest level within \( \mathbb{P} \). \( \mathbb{P} \) is related to \( \mathbb{M} \) by the mapping \( \mathcal{P}^\mathbb{P} \) which

assigns a single morphological head from \( \mathbb{M} \) to certain primes of \( \mathbb{P} \) and

assigns any one of several 'inflectional' morphemes (the choice depending on

context) to the \( k \) other primes of \( \mathbb{P} \). Thus \( \mathcal{P}^\mathbb{P} \) will carry strings in \( \mathbb{P} \)

into strings in \( \mathbb{M} \). In applying \( \mathcal{P}^\mathbb{P} \) to a string in \( \mathbb{P} \) to form a string of

morphemes we may also find it necessary to rearrange the morphemes in various

ways. Hence \( \mathcal{P}^\mathbb{P} \) will not necessarily be order-preserving. Though \( \mathcal{P}^\mathbb{P} \) may

assign every string in \( \mathbb{P} \) to some string of \( \mathbb{M} \), only certain strings of \( \mathbb{M} \) will

be assigned to grammatical strings of morphemes assigned to them by the mapping \( \mathcal{P}^\mathbb{P} \).

This set of strings of \( \mathbb{P} \) — the only set that interests us — we will
denote "Gr(\( \mathbb{P} \))". Gr(\( \mathbb{P} \)) then contains all and only those strings of \( \mathbb{P} \) which

are assigned to strings of \( \mathbb{M} \). Sentence is the unique prime of \( \mathbb{P} \) that

beers to all the members of Gr(\( \mathbb{P} \)).

We have spoken of \( \mathcal{P}^\mathbb{P} \) as a mapping which carries Gr(\( \mathbb{P} \)) into the set of

grammatical strings in \( \mathbb{M} \) (these being carried into \( \mathcal{A}^\mathbb{M} \) by \( \mathcal{P}^\mathbb{M} \)). But this

requires qualification. We know that \( \mathcal{P}^\mathbb{P} \) must be the mapping that assigns a

\( \mathbb{P} \)-marker to each grammatical string of words as its phrase structure. As the

\( \mathbb{P} \)-marker of an utterance \( X \) we will have to choose some element on the level

\( \mathbb{P} \) which determines the entire phrase structure of \( X \). We will see that this

requirement leads us to take each \( \mathbb{P} \)-marker to be a certain set of strings in

\( \mathbb{P} \) (just as on the level \( \mathcal{A} \) -- see above, ). Thus the \( \mathbb{P} \)-marker of

"John is my brother" might include in particular the strings Sentence, \( \mathcal{W}^\mathbb{P} \),

and \( \mathcal{W}^\mathbb{P} \), as well as other strings. Each string of Gr(\( \mathbb{P} \)) will be

in at least one \( \mathbb{P} \)-marker. Gr(\( \mathbb{P} \)) can be defined as the set of strings in \( \mathbb{P} \) that

assign at least one \( \mathbb{P} \)-marker, these being assigned by \( \mathcal{P}^\mathbb{P} \) to grammatical strings in \( \mathbb{M} \).

(hence ultimately to strings of \( \mathcal{W}^\mathbb{M} \)). Furthermore, each \( \mathbb{P} \)-marker will contain one

and only one string of Gr(\( \mathbb{P} \)). Thus \( \mathcal{P}^\mathbb{P} \), in a derivative way, can be said to

relate Gr(\( \mathbb{P} \)) to the set of grammatical strings of \( \mathbb{M} \).
There are other conditions that $\mathcal{F}$ and $\text{Gr}(P)$ must meet. We will require that the relation $\mathcal{F}$ be carried over under concatenation. That is, if $X$ represents $\overline{X}$ and $Y$ represents $\overline{Y}$, then the compound string $\overline{X} \overline{X} \overline{Y}$ represents $\overline{X \overline{Y}}$. Furthermore, we will require that $\mathcal{F}$ never relate two strings vacuously, in the sense that the given relation between the two strings plays no part in the description of any string in $\text{Gr}(P)$. A string which does appear in the description of some string of $\overline{X}$ $\text{Gr}(P)$ will be called a normal string.

**Def. 1.** $X$ is a normal string if and only if one of the following is the case:

(a) $X = \text{Sentence}$

(b) $X$ is in $\text{Gr}(P)$

(c) $\mathcal{F}(\text{Sentence}, X)$ and there is a $\overline{Y}$ in $\text{Gr}(P) \cup \text{Gr}(P)$ such that $\mathcal{F}(X, \overline{Y})$.

We cannot require that every string represented by $\text{Sentence}$ be normal, as we will see below in studying the notion of restriction (cf. §42). Nor can we require that every string which represents a string of $\text{Gr}(P)$ be normal.

To see that this requirement would be too strong, consider the following case:

Suppose: $\mathcal{F}(A, \text{young}), \mathcal{F}(N, \text{men}), \mathcal{F}(V, \text{like}), \mathcal{F}(N, \text{sports})$.

Thus $N^V N$ represents $\text{men like sports}$, which is a string of $\text{Gr}(P)$.

Therefore $\mathcal{F}(\text{Sentence}, N^V N)$.

But since $\mathcal{F}$ is carried over under concatenation, $A^\text{Sentence}$ represents $\text{young men like sports}$, also a string of $\text{Gr}(P)$.

Thus $A^\text{Sentence}$ represents a string of $\text{Gr}(P)$; but certainly $A^\text{Sentence}$ is not to be considered a grammatical sentence in form. I.e., it is not the case that $\mathcal{F}(\text{Sentence}, A^\text{Sentence})$.

Thus normality is a significant and restrictive additional condition on strings.
Further conditions on \( s \) may be added in terms of the notion 'head' which will be discussed below (§2.2). Cf. also §42.2.

In the remainder of this chapter we will generally drop the superscript "P", it being understood that the mappings and classes in question belong to the level \( P \) unless otherwise specified.

We will also define a relation of direct representation (symbolized "\( S_1 \)") which holds between strings \( X \) and \( Y \) if \( X \) represents \( Y \), but there is no \( Z \) such that \( X \) represents \( Z \) and \( Z \) represents \( Y \). Thinking of \( S \) as an ordering relation, \( S_1 \) is the relation \texttt{greater-than-by-one}. Thus \( S_1(X, Y) \) if and only if either \( S_1(X, Z) \) or for some \( Z \), \( S_1(X, Z_1) \) and \( S_1(Z_1, Y) \), or for some \( Z \), \( Z_1, Z_2 \), \( S_1(X, Z_1) \) and \( S_1(Z_1, Z_2) \) and \( S_1(Z_2, Y) \), etc.; \( s \) is related to \( S_1 \) as the relation Ancestor is to the relation Parent, and technically, \( s \) is called the ancestral of \( S_1 \).

In the remainder of this chapter we will generally drop the superscript "P", it being understood throughout that the mappings and classes under discussion belong to the level \( P \) unless otherwise specified.

\( \text{§42.2.} \) To recapitulate, we are concerned with an algebra \( P \)

\[
(4) \quad P = \{ \emptyset, \wedge, \vee, \neg, \mathsf{Gr}(P), S, \cdot, \circ \},
\]

where \( \mathbb{P} = \{ P_0, \ldots, P_k \} \) is the set of primes, which we can suppose to be ordered in some way.

The axioms for \( P \) will include the following:
Ax. $\mathcal{P}$ is a partial ordering of strings in $\mathcal{P}$, i.e., is reflexive, transitive, anti-symmetric, and total.

Def. $\mathcal{P} = \{ X \mid \exists Y \text{ s.t. } \sigma(X, Y) \}$

Ax. $\mathcal{P} \cap \text{Gr}(\mathcal{P}) \neq \emptyset$

Ax. $\mathcal{P}$, if $P_{\alpha_1} \in \mathcal{P}$, then
$$\sigma(X_{\alpha_1}, P_{\alpha_1} \ldots P_{\alpha_n}) \iff \exists \mathcal{P} \text{ such that }$$

(i) $\sigma(X_{\alpha_1}, P_{\alpha_1} \ldots P_{\alpha_k})$ and $\sigma(X_{\alpha_1}, P_{\alpha_{k+1}} \ldots P_{\alpha_n})$

or

(ii) $\sigma(X_{\alpha_1}, P_{\alpha_1} \ldots P_{\alpha_k})$ and $X_{\alpha_1} = P_{\alpha_{k+1}} \ldots P_{\alpha_n}$

or

(iii) $X_{\alpha_1} = P_{\alpha_1} \ldots P_{\alpha_k}$ and $\sigma(X, P_{\alpha_{k+1}} \ldots P_{\alpha_n})$

Ax. There is a unique element $P_0 \in \mathcal{P}$ such that
$$X \in \text{Gr}(\mathcal{P}) \implies \sigma(P_0, X)$$

Def. $\sigma_1(X, Y) \iff \sigma(X, Y)$ and $\exists Z \text{ s.t. } \sigma(X, Z)$ and $\sigma(Z, X)$

Def. $X$ is normal if and only if one of the following is the case:

(i) $X = P_0$

(ii) $X \in \text{Gr}(\mathcal{P})$

(iii) $\sigma(P_0, X)$ and $\exists Y \text{ s.t. } \sigma(X, Y)$ and $Y \in \text{Gr}(\mathcal{P})$

Ax. If $P_{\alpha} \in \mathcal{P}$, then $\sigma(P_{\alpha}, X) \iff X_{\alpha}$, $k$, $l$ s.t.

(i) $Y_{\alpha}$ and $Y_{\alpha}$ are normal and distinct

(ii) $Y_{\alpha} = Y_{\alpha}(P_{\alpha} / X, Y_{\alpha})$ (6)

We cannot make the more stringent requirement that every string that represents a string of $\text{Gr}(\mathcal{P})$ be normal, i.e., be represented by (or $\sigma = P_0$. Similarly, we cannot require that
every string that is represented by \( p_0 \) be normal, i.e., represent a string in \( \text{Gr}(F) \), or be itself in \( \text{Gr}(F) \). That is, we cannot require, in addition to Axiom 4, the following:

(5) If \( x \not\in p_0 \), \( x \not\in \text{Gr}(F) \), then

\[
\exists (p_0, x) \quad \text{there is a } y \text{ such that } y \in \text{Gr}(F) \text{ and } y(x, y)
\]

The reason for not requiring the left-right condition (as well as the reason for not requiring the converse of Ax. 4, for \( x \in F \)) will appear below under the discussion of restriction (§42). We have discussed the unacceptability of the right-left condition above, at the end of §46.1.

Other conditions on \( f \) will be added below (§42, §53).
to take \( P \) not as \( W \), but as elements closely related to \( W \). The primes of \( W \), it will be recalled, are morphological heads and grammatically functioning affixes such as long components. It turns out to be much more convenient (or even necessary) to specify the exact morphemic shape of long components by mappings from \( P \) to lower levels, rather than within \( P \). Thus we find it expedient to take \( P \) as the set containing morphological heads and 'bearers' of long components. Strings in \( P \) are thus carried by mappings into actual strings of morphemes. There are several reasons for this decision which will appear below. Cf., in particular, (4.1).

We can discuss the relation between \( P \) and \( W \) by means of a relation \( \gamma \), somewhat analogous to \( g \) on the level \( g \). \( \gamma \) must meet the following condition.

**Condition (1):**

(i) If \( X \) is a prime in \( P \) and \( \gamma(X,Y) \), then \( Y \) is a prime of \( W \), or \( Y = W \) (i.e., \( Y \) is the unit of \( W \)).

In the latter case, we denote \( X \) by \( W_i \), for some \( i \) (cf. (4.3)). These cases are not exclusive. One value for \( X \) may be a prime and one a unit.

(ii) If \( X, Y \) are strings in \( P (X,Y \in W) \), then

\[ \gamma(X^0, Y, Z) \Leftrightarrow \text{there are strings } W_1, W_2 \text{ in } W \text{ such that} \]

(a) \( Z = W_1 \circ W_2 \)

(b) \( \gamma(X, W_1) \) and \( \gamma(X, W_2) \)

(iii) If \( X \in P \), then \( \exists Y \text{ s.t. } \gamma(X, Y) \)
(iv) If \( \gamma(\mathbf{x}, \mathbf{y}) \), and \( \mathbf{y} \) is a morphological head \( (\mathbf{y} \in \mathbf{H}, \text{cf. } \gamma_{42}) \),
then \( \mathbf{y} \) is the unique value of \( \gamma \) for \( \mathbf{x} \).

Thus only non-heads can be dependent on the context for their
morphological ‘realization’ (i.e., can belong to long components).

\( \gamma \) thus relates a string \( \mathbf{x} \) of \( \mathbf{P} \) to a string of \( \mathbf{H} \) (or to several
strings, since \( \gamma \) is not necessarily single-valued). But we have seen in
\( \gamma_{46.1} \) that \( \mathcal{F} \), in a derivative way, can be said to relate a string \( \mathbf{x} \) of \( \mathbf{P} \)
to a string in \( \mathbf{H} \) (this time a unique string, since \( \mathcal{F} \) is a single-valued mapping
of \( \mathbf{P} \)-markers). We will discuss the very close relation between \( \gamma \) and \( \mathcal{F} \)
below, after having developed the \textit{maximal} structure of \( \mathbf{P} \) somewhat further(cf. \( \gamma_{48} \)).

A major difference is that \( \mathcal{F} \) (considered derivatively as a relation between strings
in \( \mathbf{P} \) and strings in \( \mathbf{H} \)) need not be order-preserving, while \( \gamma \) is. Thus if
\( \mathbf{x} \in \text{Gr}(\mathbf{P}) \) and \( \gamma(\mathbf{x}, \mathbf{y}) \), it is not necessarily the case that \( \mathbf{y} \) is grammatical. It
may be a permutation of a grammatical string \( \mathbf{y} \) (it may \textit{maxx} also fail to be a
grammatical because \( \mathbf{mmmmxxvxxvxxvxvxxvxxvxxv} \) the element assigned as a value to
some non-morphological head of \( \mathbf{x} \) may not fit grammatically into its context in \( \mathbf{y} \)).

Axiom 6. This axiom gives the characterization of \( \mathcal{F} \). It cannot be stated until
the set \( \mathcal{F} \) of \( \mathbf{P} \)-markers is developed. Meanwhile, we consider condition
(1) as giving some content to the set \( \mathbf{P} \) and expressing a partial
relation between \( \mathbf{P} \) and \( \mathbf{H} \). \textbf{Axiom 6} is finally stated in \( \gamma_{47} \) \textit{v.6.}

47-\( \alpha \) There are several basic and interrelated notions that must be developed
within \( \mathbf{P} \).
(1) Given that $X \in \mathcal{G}(P)$ and $\lambda X \leq X$, we want to know whether $Y$ in some sense occurs 'significantly' in $X$. If this is the case, we will call $Y$ a constituent of $X$. And if $Y$ is a constituent, we want to know what kind of a constituent it is, i.e., which strings in $P$ represent it (is it a Noun Phrase, Verb Phrase, Adjective Noun, etc. (8)).

(2) We must develop a set of $P$-markers which characterize $\lambda X \leq X$ on the level $F$. Along with this, we will have to develop further the restricted relation between $\mathcal{G}(P)$ and $\mathcal{G}(P)$, since, as was asserted above, this is much narrower in general than $\mathfrak{S}$.

42.2. Perhaps the fundamental notion to be developed in $P$ is the relation between then the man and Noun Phrase, etc., i.e., the relation which might be read 'is a' (the man is a NP). This was proposed above as the reading for the converse of $\mathfrak{S}$, since $\mathfrak{S}(\text{NP, the man})$. But this must immediately be modified, and relativized to a given occurrence of the element in question in a given sentence. Thus "reading books" is a Noun Phrase in "reading books is his chief occupation," but not in "he was reading books on agriculture." Similarly, "his son in law" is not a Noun Phrase in "I met his son in law school," or to use an example where stress and juncture do not necessarily change, (9) "called up" is a Verb Phrase of some sort in "he called up the weather bureau", but not in "he called up the stairs." This leads immediately to the notion of constituent and the first part of question (1), the problem of determining which parts of sentences are significant constituents of them.

Given the sentence $X = "that young pianist will play a
Beethoven sonata" we would like to have warrant to say that, e.g., "that young pianist", "will play a Beethoven sonata", "a Beethoven sonata", "young", etc., are among its constituents, whereas "pianist will" and "play a Beethoven" are not. A fundamental condition on constituent analysis, without which it would quickly become chaotic, is that within a sentence constituents may not overlap. (10) But we are now faced with the further problem that in order to maintain this requirement, as I think we must, we must relativize the notion of constituent to a given "interpretation" of a sentence. To see this we push a bit further our search for examples of the type given above. The crucial cases now are those of constructional homonymity. (11) Sentences such as "I saw many old men and women", "they are flying planes", or "the children rolled up the rug" seem intuitively to have two interpretations, with conflicting constituent analyses. Among the constituents of the first can be either "old" and "men and women", or "old men" and "women," depending on whether the women are meant to be old or not. In the second, "flying planes" will be the constituent when the sentence is used to identify certain specks near the horizon, but the constituent will be "are flying" in answer to "What are John and Bill doing now?" In the third, the constituents will be "rolled up" and "the rug" in the context "...and carried it into the other room," but they could also be "rolled" and "up the rug" in the same sense as "the tanks rolled up the highway."

In order to reflect this ambiguity, it is necessary to relativize the notion of constituent to a given interpretation
of a sentence. This is the only way to retain non-overlap of constituents and still to reach the only intuitively valid description. (11)

In other words, the notion "X is a Y" will have to be doubly relativized, first to a given sentence in which X occurs, and second, to a given interpretation of that sentence.

"What exactly is meant by the phrase "a given interpretation of a sentence"? This question leads us directly to the construction of P-markers, for these elements will be the systematic analogues to the idea of an 'interpretation' on the level P, in accordance with the general conception of level from which we began.
48.1. To know whether \( X \) is a \( Y \) of \( Z \), we must certainly know which strings in fact represent \( X \). Suppose that \( X = Z \), and \( f(Y, X) \). Then clearly \( X \) is a \( Y \) in \( X \). More generally, suppose that \( Z = \overline{w}_1 \circ X \overline{w}_2 \), and suppose that there is a string \( Z' = \overline{w}_1 \circ Y \overline{w}_2 \) which represents \( Z \) (i.e., \( f(Z', Z) \)). Then in this case too we will say that with reference to \( Z' \), \( X \) is a \( Y \) in \( Z \). (This reduces to the preceding case if \( \overline{w}_1 = \overline{w}_2 = U \).) For example,

(6) let \( X = \text{are} \) flying

\[ Y = \text{Verb Phrase} \]
\[ \overline{w}_1 = \text{they} \]
\[ \overline{w}_2 = \text{planes} \]

Then \( Z = \overline{w}_1 \circ X \overline{w}_2 \) is they are \( \text{flying planes} \) and \( Z' = \overline{w}_1 \circ Y \overline{w}_2 \) is they \( \text{Verb Phrase planes} \). If \( f(Z', Z) \), then we will say that with reference to \( Z' \), \( \text{are} \) flying is a \( \text{Verb Phrase in they are flying planes} \) (\( X \) is a \( Y \) in \( Z \)).

Suppose, on the other hand, that

(7) \( \overline{X} = \text{flying planes} \)

\[ \overline{Y} = \text{Noun Phrase} \]
\[ \overline{w}_1 = \text{they are} \]
\[ \overline{w}_2 = U \]

Then \( Z = \overline{w}_1 \circ X \overline{w}_2 \) is they are \( \text{flying planes} \) \( \overline{X} = \text{flying planes} \) of (6) and \( \overline{X} \overline{Y} \overline{Z} \overline{Z} \) \( \overline{Y} \) is they are \( \text{Noun Phrase} \). If \( f(Z', Z) \), then we will say that with reference to \( Z' \), \( \text{flying planes} \) is a \( \text{Noun Phrase in they are flying planes} \) \( \overline{X} \) is a \( Y \) in \( Z \). Thus the \( \circ \) representing strings \( Z' \) and \( \overline{Z} \) give a conflicting (inconsistent) interpretation of \( Z \) \( \overline{Z} \), since the constituents of \( Z \) with respect to \( Z' \) and with respect to \( \overline{Z} \) overlap. All of this suggests that we define 'is a' in terms of a class of representing strings, chosen, in the interesting cases, so as to give a consistent interpretation of the constituent structure of \( Z \). If there is
a class \( K \) of strings containing \( Z \) and \( Z' \), where \( Z' \) differs from \( Z \) only in that it contains a substring \( Y \) replacing the substring \( X \) of \( Z \), and if furthermore \( f(Y,X) \) or \( X=Y \), then we say that the given occurrence of \( X \) is a \( Y \) of \( Z \) with respect to \( (wrt) \) \( K \). The expression "the occurrence \( W \) of \( X \) is a \( Y \) of \( Z \) wrt \( K \)" we write symbolically in the form "\( E_0(X,W,Y,Z,K) \)."

Def. 4. \( E_0(X,W,Y,Z,K) \) if and only if

(i) \( K \) is a class of strings

(ii) \( W \) is an occurrence of \( X \) in \( Z \)

(iii) \( Z \) and \( Z(Y/X,W) \) are in \( K \)

(iv) \( f(Y,X) \) or \( X=Y \)

This relation could be strengthened by...
adding an inductive step.

(a) $X = X_1 \cdot \cdots \cdot X_r$

(b) $Y = Y_1 \cdot \cdots \cdot Y_r$

(c) For each $i \geq 1$, $E_0(x_i, W^n \cdot X_1 \cdot \cdots \cdot X_i, Y_i, Z, K)$

(where $W = W^n X$)

I have not been able to determine whether this should be added or not. It is never necessary below, and so has been left out of the formal development. The decision turns on the consequences for particular grammars of accepting or rejecting this generalization.

The following immediate consequence of the definition of $E_0$ plays a certain role in subsequent discussions.

If there is any $x$ such that

$Th. 1. \quad \exists E_0(x, W, Y, Z, K) \implies E_0(U, W, U, Z, K)$
48.2. In chapter III we developed the notion of a derivation of a string $X$ as a sequence of strings beginning with Sentence and ending with $X$, and such that each string in the sequence follows from the preceding string by application of some rule of the grammar (cf. §21.1). We will now concern ourselves more particularly with that fragment of the derivation which appears within the level $P$. In this case the place of the rules of the grammar is taken by the statement of the relation $\mathcal{R}$ of representation within the level $P$. A $\mathcal{R}$-derivation of a string $Z$, then, will be a sequence of strings beginning with Sentence and ending with $Z$, and such that each string of the sequence is represented by the string that precedes it in the sequence. Furthermore, we require that any $\mathcal{R}$-derivation end with a string $Z$ in $P$.

More generally, suppose that we have a two-place relation $Q$ between strings which is irreflexive (i.e., for no string $X$ is it the case that $Q(X, X)$). Then we can define a "$Q$-derivation" as follows:

**Def. 5.** A sequence $D = A_1, \ldots, A_n$ is a $Q$-derivation of $Z$ if and only if

1. $A_1 = \text{Sentence}; A_n = Z$
2. $Q(A_i, A_{i+1})$
3. there is no $Y$ such that $Q(Z, Y)$.

If we take $Q$ as $\mathcal{R}$, we have $\mathcal{R}$-derivations in the sense described above. But we can also have $\mathcal{R}$-derivations (cf last paragraph of §46.1, Def. of §46.2), etc.

**Def. 6.** If $D$ is a $Q$-derivation of $Z$, then we call $Z$ the product of the derivation $D$ [in symbols, $Z = \text{Prod}(D)$]

**Def. 7.** We say that a $Q$-derivation is restricted just in case its product $Z$ is in the set $Gr(P)$ [see §46.1]

**Th. 2.** $Z$ is normal if and only if $Z$ is a step in some restricted $\mathcal{R}$-derivation.

If we present $Q$ as a system $\sum$ of rules $\alpha \rightarrow \beta$ (wherever $Q(\alpha, \beta)$),
then a \( Q \)-derivation becomes a derivation wrt \( \Sigma \) in the sense of \( \text{§} \text{21.1} \), with \( \Sigma \) playing the role of (27), \( \text{§} \text{21.1} \).

A \( Q \)-derivation is thus somewhat analogous formally to a proof of its product, with \( \text{§} \text{21.1} \) being the single "axiom", and \( Q \) playing the role of the consequence relation.

**Th. 2.** Every string in \( \text{Gr}(P) \) is the product of a \( f_1 \)-derivation. Therefore \( \text{Gr}(P) \) is exactly the set of products of restricted \( f_1 \)-derivations. Furthermore, the product of every \( f \)-derivation is in \( P \).

**4Q. 2.** We can now proceed to construct \( P \)-markers. A \( P \)-marker of \( Z \) must give us all information about the constituent structure of \( Z \), for a given interpretation. It must provide, with no ambiguity or overlaps, all information about \( E_0 \) for \( Z \). The natural approach would thus be to take a certain class of strings representing \( Z \) as its \( P \)-marker.

It is clear that \( P \)-markers will be closely related to \( f_1 \)-derivations. Since different interpretations of \( Z \) will certainly be related to different \( f_1 \)-derivations, we might consider taking the set of steps of a \( f_1 \)-derivation \( D \) as a \( P \)-marker (we cannot take \( D \) itself, since \( D \) is a sequence and a \( P \)-marker is to be a set). But the converse of this is not true; i.e., not all different derivations of \( Z \) correspond to different interpretations. Consider the \( f_1 \)-derivations \( D \) and \( D' \) of \( Z \).

\[
I(2) \quad D = A_1, A_2, A_3, A_4;
\]

where

\[
A_1 = \text{E}_1 A_2, A_3, A_4;
\]

\[
A_2 = b_1, b_2
\]

\[
A_3 = e_1, e_2, e_3, e_4
\]

\[
A_4 = c_1, c_2, c_3, c_4
\]

\[
Z = A_4 = c_1, c_2, c_3, c_4
\]
\( D^1 = A_1^i, A_2^i, A_3^i, A_4^i, \)

where \( A_1^i = b_1 \) and \( A_2^i = b_1 \)

\( A_3^i = b_1 \)

\( A_3^i = b_1 \)

\( Z = A_4^i = c_1 \)

Clearly these distinct derivations differ only in the order in which the constituents are developed, and thus inessentially from the viewpoint of constituent interpretation. They give equivalent interpretations of \( Z \) in a certain sense, whereas a different constituent interpretation of \( Z \) would correspond to \( D^1' \).

\( D^1' = A_1^{i'}, A_2^{i'}, A_3^{i'}, A_4^{i'}, \)

where \( A_1^{i'} = b_2 \)

\( A_2^{i'} = c_1 \)

\( A_3^{i'} = c_2 \)

\( A_3^{i'} = c_3 \)

\( Z = A_4^{i'} = b_2 \)

This may become clearer if we represent Q-derivations diagrammatically in an obvious way. Thus to \( D \), \( D' \), and \( D'' \) correspond the graphic representations of Fig. 1.

\[ \text{Fig. 1.} \]

\[ \text{Diagram of } D \]

\[ \text{Diagram of } D' \]

\[ \text{Diagram of } D'' \]
D and D' are clearly equivalent in a certain sense, while both differ from D''. This equivalence can be recorded graphically by collapsing the diagrammatic representation, dropping all mere repetitions.

Collapsed Diagram of D

\[ \begin{align*}
\text{Root} & \quad b_1 \quad b_2 \\
& \quad c_1 \quad c_2 \quad c_3 \quad c_4
\end{align*} \]

Collapsed Diagram of D'

\[ \begin{align*}
\text{Root} & \quad b_1 \quad b_2 \\
& \quad c_1 \quad c_2 \quad c_3 \quad c_4
\end{align*} \]

Collapsed Diagram of D''

\[ \begin{align*}
\text{Root} & \quad b_1 \quad b_2 \\
& \quad c_1 \quad c_2 \quad c_3 \quad c_4
\end{align*} \]

Fig. 2.

Thus D and D' reduce to the same collapsed diagram, while D'' reduces to a different diagram. These collapsed diagrams correspond to what we mean by different interpretations of \( D \).

48.4. It is clear that we could not define a \( P \)-marker as the set of 'steps' of a collapsed diagram, since all three collapsed diagrams have the same three steps, \( b_1 b_2 \), and \( c_1 c_2 c_3 c_4 \). We must require that the \( P \)-marker contain

\[ b_1 c_3 c_4 \] and \[ c_1 c_2 b_2 \] in the case corresponding to D and D',

\[ b_1 c_1 \] and \[ c_1 c_2 c_3 c_4 \] in the case corresponding to D'',

if we are to be able to state the constituent structure correctly.
in terms of the $\mathbb{P}$-marker. In other words, given a $\mathbb{P}$-marker $\Gamma$ of $\mathbb{Z}$, we will say that an occurrence $\mathbb{W}$ of $\mathbb{X}$ is a constituent of $\mathbb{Z}$ (wrt $\Gamma$) if $\Gamma$ contains $\mathbb{Z}(\mathbb{P}_1/\mathbb{X}, \mathbb{W})$, for some prime $\mathbb{P}_1$.

In this case the occurrence $\mathbb{W}$ of $\mathbb{X}$ is a $\mathbb{P}_1$ of $\mathbb{Z}$ wrt $\Gamma$.

Looking back at $\mathbb{D}$ and $\mathbb{D}'$ in terms of this requirement, we see that neither could serve alone as a $\mathbb{P}$-marker. In the case of $\mathbb{D}$, only for $\mathbb{X} = c_1^n c_2$ does $\mathbb{D}$ contain as a step $\mathbb{Z}(\mathbb{P}_1/\mathbb{X}, \mathbb{W})$ (in this case $\mathbb{P}_1 = \mathbb{P}_2$, $\mathbb{W} = \mathbb{Z}$). In the case of $\mathbb{D}'$, $\mathbb{Z}(\mathbb{P}_1/\mathbb{X}, \mathbb{W})$ is contained as a step only for $\mathbb{X} = c_1^n c_2$ (in this case $\mathbb{P}_1 = \mathbb{P}_1$, $\mathbb{W} = \mathbb{X}$). Thus neither of these derivations can serve alone as a $\mathbb{P}$-marker, though they can serve together, i.e., the class containing the steps of $\mathbb{D}$ and the steps of $\mathbb{D}'$ could serve as a $\mathbb{P}$-marker.

All of this suggests that we define $\mathbb{P}$-markers in terms of a class of equivalent $\mathcal{F}_1$-derivations, i.e., $\mathcal{F}_1$-derivations such as $\mathbb{D}$ and $\mathbb{D}'$ (but not $\mathbb{D}$ and $\mathbb{D}''$) which differ only in the order of development of their terms, and thus inessentially for constituent analysis. This equivalence, which has its graphic counterpart in identity of collapsed diagrams, we must now proceed to define carefully.

42.5. Before constructing $\mathbb{P}$-markers we may develop some of the further properties of $\mathcal{F}_1$-derivations. Each step in a $\mathcal{F}_1$-derivation is derived from the preceding step by developing a single occurrence of some prime which appears in this preceding step.
Th. 4. Suppose that $D = A_1, \ldots, A_n$ is a $s_1$-derivation of $Z$. Suppose further that for $i < n,$

$$A_i = P_{i1} \cdots P_{it_i}, \quad \left[ P_{i1} \in P \right]$$

Then there are $j, k$ such that (i) $j \neq i$

(ii) $A_{i+1} = A_i (X / P_{ij}, P_{i1} \cdots P_{it_i})$

(iii) $s_1 (P_{ij} X)$

Def. 8. In this case, $(P_{ij}, X)$ will be called the associated production of $A_j$ in $D$.

In symbols, $(P_{ij}, X) = AP_D (A_j)$

Thus $A_i$ and $A_{i+1}$ are minimally different, in the sense that all the primes of $A_i$ except one appear in $A_{i+1}$, in the same order, and this remaining prime is the $P_{ij}$ of the associated production. If there were more than one $P_{ij}$ developed, then $s_1$ could not hold between $A_i$ and $A_{i+1}$. In the grammar of the language, $AP_D (A_i)$ would be the rule used in carrying out the $i^{th}$ step in the derivation $D$ of $Z$.

Th. 5. $AP_D$ is a single valued function of $A_i$, but if $i \neq j,$ it is still possible that $AP_D (A_i) = AP_D (A_j)$.

(i.e., $AP_D$ is not necessarily 1-1).

Th. 6. Each prime $P_{ij}$ of $Z \in Gr(P)$ will be part of many constituents. The largest of these is $Z$, the smallest, $P_i$ itself. And our presystematic requirement is that for a given
occurrence of \( P_1 \), every two of these are either disjoint or one is wholly included in the other. Another way of putting this requirement is this. If parentheses were put around each constituent, the result of so marking all constituents would be a properly parenthesized expression, with an equal number of left and right parentheses between each left parenthesis and the matched right parenthesis (13).

We might then say that to each occurrence of \( P_1 \) there corresponds uniquely a sequence of nested strings, beginning with \( Z \) and ending with \( P_1 \) itself, and containing each segment of \( Z \) which contains this occurrence of \( P_1 \), and which is itself a constituent of \( Z \). E.g., to \( c_3 \) in derivation D, \( c_3 \) (fig. 1a,1b) corresponds the sequence

\[ a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot a_6 \cdot a_7 \cdot a_8 \cdot a_9 \cdot a_{10} \]

To this sequence of nested constituents corresponds uniquely (for a given occurrence of \( P_1 \) in \( Z \)) the sequence of primes which represent them (including finally \( P_1 \) itself). This sequence begins with \( P_1 \) representing \( Z \) and concludes with \( P_1 \). Thus to \( a_3 \) in the same example corresponds the sequence

\[ P_1 \cdot b_2 \cdot c_3 \]

This sequence of primes gives the constituent history of the given occurrence of \( P_1 \). It can be read off directly from the diagram by tracing the (unique) path from \( P_0 \) to the given occurrence of \( P_1 \). The string formed by concatenating the elements of this sequence in order, we will call the content trace (c-trace) of the given occurrence.
The idea behind this construction is simply this. If $AP_D(A_i) = (P_{i_1}, X_{i_1})$, then we make the $i$th element in the c-trace either $P_{i_1}$ or $X_{i_1}$, depending on whether or not this trace element is the element developed in the $i$th step. Since the $X_{i}$'s will drop out, the c-trace of an element will be the same for all equivalent derivations. Thus the c-trace will correspond to the path leading to the element in the collapsed diagram.

Although the idea is clear enough, the formal construction must be carried out with some care, since occurrences must be kept distinct, and since the associated productions may recur, and may be of many different kinds.

4.2. Preparatory to defining c-trace, we will define the auxiliary notion of sequential trace. This will be, essentially, the sequence of occurrences of primes that represent a given occurrence of $P_{i_1}$, with $X_{i}$ replacing repetitions of a representing prime as described above. The c-trace will then be the string corresponding to the sequence of these primes themselves (not their occurrences). Thus different occurrences of a prime may have the same c-trace, but not the same sequential trace, since in the latter, occurrences of the representing primes are kept distinct. In terms of sequential trace we will be able to define c-trace and further notions.

Suppose that $D = A_1, \ldots, A_n$ is a $^i$ derivation of $A$, where

(i) $A_i = P_{i_1}^\cdot \cdot \cdot P_{i_k}$, \hspace{1cm} ($P_{i_j} \in P$)
(ii) $A_{P_{\beta_{i}}}(A_{i}) = (P_{\beta_{i}}, X_{A_{i}}) \quad (\text{ign})$

(iii) $W_{1}, \ldots, W_{i-1}$ are strings such that

(a) $w_{i}^{\wedge}P_{\beta_{i}}$ is an occurrence of $P_{\beta_{i}}$ in $A_{i}$

(b) $W_{i}^{\wedge}X_{i}^{\wedge} \quad W_{i}^{\wedge}X_{i}^{\wedge} A_{i+1}$

Th. 5. The $W_{i}$'s exist and are uniquely determined by this condition.

Under the assumption of (iii), we define:

Def. 2. $(P_{\alpha_{1}}, X_{1}), \ldots, (P_{\alpha_{n}}, X_{n})$ is the sequential trace of $(P_{\beta_{1}}, P_{\beta_{i}}^{\wedge} \ldots P_{\beta_{i}}^{\wedge})$ in $Z$ wrt $D$ if and only if

(i) $i \leq t_{n}$ (14)

(ii) $P_{\alpha_{i}} = P_{\beta_{i}}$

(iii) $Y_{i}^{\wedge} = W_{i}$; $Y_{i}^{\wedge} = P_{\beta_{i}}^{\wedge} \ldots P_{\beta_{i}}^{\wedge}$ (or $Y_{i}^{\wedge}$, if $i=1$)

(iv) $\exists T_{1}, \ldots, T_{n-1}$ such that

(a) for all $i, i \leq j \leq n$, $\exists T_{i}^{j}$ such that $X_{i}^{j} = T_{i}^{j} P_{\beta_{i+1}}^{j} T_{i}^{j}$

(b) $T_{i}^{j} X_{i}^{j} \quad T_{i}^{j} X_{i}^{j}$

(c) If $Y_{i}^{\wedge} = W_{i}$, then it is the case that

(11) $Y_{i+1}^{\wedge} = X_{i}^{\wedge} T_{i}$

(12) $P_{\alpha_{i+1}} = P_{\beta_{i+1}}$

(d) If $Y_{i}^{\wedge} \neq W_{i}$, then it is the case that

(d1) $Y_{i+1}^{\wedge} = X_{i}^{\wedge} (X_{i}^{\wedge} / P_{\alpha_{i+1}}, W_{i}^{\wedge} P_{\beta_{i+1}})$

(d2) $P_{\alpha_{i+1}} = U + T_{i}$
Note that if \( Y_i \) is not an initial substring of \( X_i \), then
\[
Y_i(X_i/P_{\alpha_i}W_iP_{\alpha_i}^{-1})Y_i^{-1}.
\]
Thus if the development of \( A_i \) to \( A_{i+1} \) takes place to the right of the given occurrence of \( P_{\alpha_i} \) (i.e., after \( Y_iP_{\alpha_i} \)), then \( Y_{i+1}Y_i^{-1} \). If the development of \( A_i \) takes place to the left of the given occurrence of \( P_{\alpha_i} \) (i.e., within \( X_i \)), then \( Y_{i+1} \) differs from \( Y_i \) in that \( P_{\alpha_i} \) in \( Y_i \) is replaced by \( X_i \). If the given occurrence of \( P_{\alpha_i} \) is the prime of \( A_i \) which is developed in moving to \( A_{i+1} \), then we have case (ivc).

Th. 7. \( Y_i \) is an occurrence of \( P_{\alpha_i} \) in \( A_i \), for all \( i \).

Th. 8. The \( T_i \)'s are unique.

Th. 9. If \( j < t \), then the sequential trace exists and is unique.

Def.10. \( \Sigma \) is the c-trace of \( (P_{\alpha_i}, P_{\alpha_i}^{-1}, \ldots, P_{\alpha_i}^{-1}) \) in \( Z \) wrt \( D \) if and only if there exist \( P_{\alpha_1}, \ldots, P_{\alpha_t} \), \( X_1, \ldots, X_t \) such that

(i) \( (P_{\alpha_1}, Y_1), \ldots, (P_{\alpha_t}, Y_t) \) is the sequential trace of \( (P_{\alpha_1}, P_{\alpha_1}^{-1}, \ldots, P_{\alpha_t}^{-1}) \) in \( Z \) wrt \( D \).

(ii) \( \Sigma = P_{\alpha_1}^{-1} \cdots P_{\alpha_t}^{-1} \).

Th. 10. The c-trace of \( (P_{\alpha_1}, P_{\alpha_1}^{-1}, \ldots, P_{\alpha_t}^{-1}) \) in \( Z \) wrt \( D \) is unique.

As an example, in the derivation \( D \) of fig. 14, consider the c-trace of the second occurrence of \( c_1 \). In this case we have the following situation.
The first three columns, the first and fourth row of the fourth column, and the fourth row of the sixth column are given by assumption. And there is only one way to fill in the remaining blanks.

In the derivation \( D' \) represented in fig. 1b, we have, for the same occurrence of \( e_1 \), the following situation:

\[
\begin{align*}
A_1 &= \text{Sentence} \quad W_1 = U \\
A_2 &= b_3 \quad b_2 \quad W_2 = U \\
A_3 &= e_1 \quad e_2 \quad b_2 \\
A_4 &= e_1 \quad e_2 \quad e_3 \\
\end{align*}
\]

\[
\begin{align*}
P_{\beta_1} &= U \\
P_{\beta_2} &= b_1 \\
P_{\beta_3} &= e_2 \\
P_{\beta_4} &= e_3 \\
T_1 &= b_2 \\
T_2 &= b_3 \\
T_3 &= e_3 \\
T_4 &= e_3 \\
\end{align*}
\]

The \( c \)-trace of this occurrence of \( e_1 \) is \( P_{\beta_1}^{\alpha} P_{\beta_2}^{\alpha} P_{\beta_3}^{\alpha} P_{\beta_4}^{\alpha} \).

In the case of \( D \), this is \( U^{\alpha} b_2^{\alpha} e_1 = b_2^{\alpha} e_1 \). In the case of \( D' \), this is \( b_2^{\alpha} U^{\alpha} e_1 = b_2^{\alpha} e_1 \). Thus the two \( c \)-traces are identical, as are the traces of all other occurrences of primes of \( Z \) with respect to these two derivations. And these traces can in fact be read off from Fig. 2.

While the sequential traces differ for \( D \) and \( D' \), the \( c \)-traces are identical, term by term. If two derivations are equivalent
in the sense that their collapsed diagrams are identical, then term by term, the c-traces of their primes are identical. If the converse of this were true, we could define equivalence as term-by-term identity of c-traces. But it is in fact not true, and a counter-example to it is suggested by an investigation of \( D'' \).

Consider now \( D'' \) (fig. ic). \( D'' \) is not equivalent to \( D \) in the intended sense, and does not have the same collapsed diagram (cf. fig. 2). And in fact, although both occurrences of \( c_1 \) have the same c-trace as in \( D \) and \( D' \), \( c_2 \) does have a different c-trace, i.e.,

\[
(2) \quad D, \text{ c-trace}(c_3, c_2, c_2, c_2, c_2) = E_4 E_3 E_2 E_1 \quad \text{Subtree} \quad b_2, c_2, c_2 \quad E_1 E_2 E_3 E_4
\]

But suppose that \( b_2 \) were replaced in \( D, D' \), and \( D'' \) by \( b_1 \) i.e., in place of the derivations \( D, D' \), and \( D'' \) diagrammed in figs. 1 and 2, consider \( \Delta, \Delta', \) and \( \Delta'' \) diagrammed as follows:

\[
A_1 = \quad A_1' = \quad A_1'' =
\]

\[
A_2 = b_1 \quad A_2' = b_1 \quad A_2'' = b_1
\]

\[
A_3 = c_1 c_2 c_2 b_1 \quad A_3' = c_1 c_3 c_1 c_1 \quad A_3'' = c_1 c_2 c_3 c_1
\]

\[
Z = A_4 = c_1 c_2 c_3 c_1 \quad Z = A_4' = c_1 c_2 c_3 c_1 \quad Z = A_4'' = c_1 c_2 c_3 c_1
\]

3a. Diagram of \( \Delta \). 3b. Diagram of \( \Delta' \). 3c. Diagram of \( \Delta'' \).

Fig. 3.

Collapsing these diagrams, we have:
But now the c-trace of $c_3$ is $b_1 c_3$ with respect to $\Delta$, $\Delta'$, and $\Delta''$. Yet the derivations are related in the same way as are $D$, $D'$, and $D''$. This is too close to well-known cases (e.g., the nominal sentence) to be eliminated by further restrictive axioms. It is therefore apparent that a stronger condition than term-by-term identity of c-trace is needed to guarantee equivalence of derivations.

48.8. The problem is that c-traces do not specify uniquely how the string which is the product of a derivation to be broken up into constituents, although they do specify what sort of constituents these must be. Returning to the analogy of constituent analysis with parenthesized expressions, given the c-trace, we still do not know exactly where the parentheses are to be put in all cases, though we know what 'name' to associate with each parenthesized expression once the parentheses are put in. The problem can be solved by associating with each prime of $\Sigma_{\text{Gr}(P)}$ not only a c-trace, but also a position-trace (p-trace) which tells to which constituent (the first, second, etc) it belongs when the element representing the constituent to which it belongs is developed in the derivation. In other words, given $\Delta$, $c_3$ is
the first constituent (reading left to right) of the
second constituent of $D$ (which is the first constituent of
itself), so we might assign to it the p-trace $1, 2, 1$. But
with respect to $\Delta''$, the p-trace of $c_2$ will be $1, 1, 3$.
Similarly, the p-traces of the second occurrence of $c_1$ will
differ with $\Delta$ and $\Delta''$.

Two derivations whose products have the same p-traces term-
by-term will have the same form, though perhaps not the
same "representational content", in the sense that the same
primes represent their constituents. Thus $D, \Delta, D'$,
and $\Delta'$ will yield the same p-traces, and $D''$ and $\Delta''$ will yield the
same p-traces. The notion of equivalence that we have been
trying to construct has thus in a certain sense been broken
down into an identity of representational content (given
by c-traces and answering the question: what symbols
appear at the branch points of the diagrams) and an identity
of form (given by p-traces that indicate what is the form of
the diagram). Two derivations will then be equivalent if
they have the same underlying form and the same representational
content, i.e., if their products have the same c-traces and
the same p-traces term-by-term.

The notion p-trace can be defined readily in terms of
sequential trace. Given the same conditions as stated for
Def.9, namely (\text{\ref{def:sequential-trace}}), we define:
Def. II. \( \mathcal{Q} \) is the \( p \)-trace of \((p_{\alpha_i}, p_{\beta_1}^\wedge \cdots \wedge p_{\beta_j})\) in \( \mathbb{Z} \) wrt \( \mathcal{D} \) if and only if

\[
\exists a_0, \ldots, a_{n-1}, b_{\alpha_1}, \ldots, b_{\alpha_n}, b_{\beta_1}, \ldots, b_{\beta_j} \text{ such that}
\]

(i) \((b_{\alpha_i}, b_{\beta_1}), \ldots, (b_{\alpha_n}, b_{\beta_j})\) is the sequential trace of

\((p_{\alpha_i}, p_{\beta_1}^\wedge \cdots \wedge p_{\beta_j})\) in \( \mathbb{Z} \) wrt \( \mathcal{D} \).

(ii) \( a_0 = 1 \)

(iii) If \( p_{\alpha_i} = \emptyset \), then \( a_i = 0 \)

(iv) If \( p_{\alpha_i} \neq \emptyset \), then there is a \( T_{i_1} \) s.t. \( y_{i_1+1} \wedge y_{i_1} \wedge T_{i_1} \)

Then \( T_{i_1} = t_{i_1}^1 \wedge \cdots \wedge t_{i_1}^{a_{i_1}} \), where

(a) \( a_i > 1 \)

(b) \( t_{i_1} = \emptyset \)

(c) \( t_{i_1} \in p_{\alpha_i} \) for \( i \geq 1 \) (i.e., \( t_{i_1} \) is a prime)

(v) \( \mathcal{Q} = b_{\alpha_1}, \ldots, b_{\alpha_n} \) is the subsequence of \( a_0, \ldots, a_{n-1} \) containing all non-zero terms.

Th. II. The \( p \)-trace of \((p_{\alpha_i}, p_{\beta_1}^\wedge \cdots \wedge p_{\beta_j})\) in \( \mathbb{Z} \) wrt \( \mathcal{D} \) is unique, and the number of its terms is equal to the number of primes in the \( c \)-trace of \((p_{\alpha_i}, p_{\beta_1}^\wedge \cdots \wedge p_{\beta_j})\) in \( \mathbb{Z} \) wrt \( \mathcal{D} \).
Def. 12. $D \sim D'$ if and only if

(i) $D$ and $D'$ are $f_1$-derivations

(ii) $\text{Prod}(D) = \text{Prod}(D') = \mathbb{Z} = P_{n_1} \cdots \cdots \cdots P_{n_m} = (P_{n_i} \in \mathbb{P})$

(iii) For all $i$, if

\[ \Sigma_i \text{ is the } s \text{-trace of } P_{n_i} \text{ in } \mathbb{Z} \text{ wrt } D \]

\[ \Sigma_i \text{ is the } f \text{-trace of } P_{n_i} \text{ in } \mathbb{Z} \text{ wrt } D' \]

Then $\Sigma_i = \Sigma_i'$.

Th. 12. $\sim$ is an equivalence relation among $f_1$-derivations.
48.2 In this manner we can set up equivalence classes of derivations. Each equivalence class contains all derivations that maintain the same order of development of the constituents, pictorially, if and only if they have the same collapsed diagrams. Two equivalent derivations of course have the same product string of $\mathcal{P}$ as product.

Suppose that $D_1, \ldots, D_n$ constitute an equivalence class of derivations, with $Z$ as the common product. Suppose that a given occurrence $w$ of $X$ is a constituent of $Z$ in the presystematic sense of 'constituent' that we are trying to reconstruct. That is,

$$(13) \text{ For some } W_1, W_2, Z = W_1 \setminus X \uparrow W_2 \text{ and } W = W_1 \setminus X \text{ [where this occurrence of } X \text{ is, presystematically, a constituent of } Z].$$

Then there will be some derivation $D_{\alpha}$ in the given equivalence class such that in the derivation $D_{\alpha}$, the occurrence $w$ of $X$ is the last constituent to be developed. Thus $D_{\alpha}$ will be a sequence $A_1, \ldots, A_n$, where

$$(14) \quad A_1 = \text{Sentence}$$

$$A_2 = W_1 \setminus P_{\alpha_1} \setminus W_2 \quad \text{[where } P_{\alpha_1} \text{ is a prime]}$$

$$\ldots$$

$$A_n = W_1 \setminus X \setminus W_2 = Z$$

That is, at a certain step $A_{\alpha}$ of $D_{\alpha}$, everything will be reduced to primes of $\mathcal{P}$ except for a prime $P_{\alpha}$, which represents $X$; or in other words, for some prime $P_{\alpha}$, it will be the case that $A_{\alpha} = Z(P_{\alpha}/X.w)$. Suppose now that $\mathcal{T}$ is the set of strings containing just those strings which are steps of some derivation $H$ of the equivalence class under consideration. Then the steps $A_1, \ldots, A_n$ of $H$ the derivation $D_{\alpha}$ will be in $\mathcal{T}$, and in particular,
\( A_k \) will be in \( \Gamma \). It will thus be the case that the \( n \) occurrence \( w \) of \( x \) is a \( \mathfrak{p}_n \) of \( z \) with respect to \( \Gamma \) (i.e., \( E_0(x, w, \mathfrak{p}_n, z, \Gamma) \)), where \( \mathfrak{p}_n \) is a prime.

In such a case, we say that the occurrence \( w \) of \( x \) is a proper term of \( z \) with respect to \( \Gamma \).

**Def. 13.** The occurrence \( w \) of \( x \) is a proper term of \( z \) wrt \( k \) if and only if there is a prime \( \mathfrak{p}_n \) such that \( E_0(x, w, \mathfrak{p}_n, z, k) \).

We note that every occurrence which we would like to consider a proper term of \( z \) in the case discussed above is in fact a proper term of \( z \) wrt \( \Gamma \). Furthermore, every proper term is in fact what we mean, presystematically, by a 'constituent'.

In particular, no two proper terms overlap, unless one is included in the other.

Referring back to \( \gamma \cdot \theta \cdot \Delta \), we see that \( \Gamma \) can qualify as a \( \mathfrak{p}_n \)-marker in this case. In general, a \( \mathfrak{p}_n \)-marker of \( z \) will be a set of strings which contains all and only those strings which are steps in some \( j_1 \)-derivation of \( w \) \( z \) in a given equivalence class of \( j_1 \)-derivations.

**Def. 14.** \( \Gamma \) is a \( \mathfrak{p}_n \)-marker of \( z \) if and only if there is an equivalence class \( B \) of \( j_1 \)-derivations of \( z \) such that

1. \( B \) is the set \( D_1, \ldots, D_N \)

**Def. 14.** \( \Gamma \) is a \( \mathfrak{p}_n \)-marker of \( z \) if and only if there is an equivalence class \( B = \{ D_1, \ldots, D_N \} \) of \( j_1 \)-derivations of \( z \), where

1. \( D_i = \{ A_{i,1}, \ldots, A_{i,n} \} \)

2. \( \Gamma = \{ A_{i,1} \mid i \in I_1, i \leq n \} \)

**Th. 13.** There is a one-one correspondence between \( \mathfrak{p}_n \)-markers and equivalence classes of \( j_1 \)-derivations. Given a \( \mathfrak{p}_n \)-marker \( \Gamma \), there is a unique string \( w \) of which it is a \( \mathfrak{p}_n \)-marker, and \( w \) is a string of \( \mathfrak{p}_n \). \( w \) is the product of each derivation in the equivalence class corresponding to \( \Gamma \).
We extend the notion product so that in this case Z is the product of \( \Gamma \).

**Def. 15.** If \( \Gamma \) is a \( \mathcal{P} \)-marker of \( Z \), then \( Z = \text{Prod}(\Gamma) \).

We say that the occurrence \( W \) of \( X \) is a constituent of \( Z \) with respect to \( \Gamma \) if \( \Gamma \) is a \( \mathcal{P} \)-marker of \( Z \), and this occurrence is a proper term of \( Z \) wrt \( \Gamma \). (See Def.)

**Def. 16.** The occurrence \( W \) of \( X \) is a constituent of \( Z \) wrt \( \Gamma \) if and only if

(i) \( \Gamma \) is a \( \mathcal{P} \)-marker of \( Z \)

(ii) The occurrence \( W \) of \( X \) is a proper term of \( Z \) wrt \( \Gamma \).

Note that

**Th. 14.** If \( \mathcal{G}_0(X; W, P_1, Z; \Gamma) \) and \( \mathcal{G}_0(X; W, P_1', Z; \Gamma') \), where \( \Gamma \) is a \( \mathcal{P} \)-marker of \( Z \) and \( P_1 \), \( P_1' \) are primes, then either (i), (ii), or (iii) is the case:

(i) \( \not\in (P_1, P_1') \)

(ii) \( \not\in (P_1, P_1') \)

(iii) \( P_1 = P_1' \)

We have already defined "\( X \equiv Y \)" as meaning that the string \( X \) is a substring of the string \( Y \), i.e., we say that \( X \equiv Y \) in case either \( X = Y \) or \( X \subset Y \).

**Def. 17.** \( X \equiv Y \) if and only if either \( X = Y \) or \( X \subset Y \).

We say that \( K \) gives a consistent analysis of \( Z \) if proper terms overlap only if one is included in the other.

**Def. 18.** \( K \) gives a consistent analysis of \( Z \) if and only if for all \( X, W, X', W' \), if the occurrence \( W \) of \( X \) and the occurrence \( W' \) of \( X' \) are proper terms of \( Z \) wrt \( K \), and if \( W' \subset W \), then either (i) or (ii) is the case:

(i) \( W' \not\subset X' \)

(ii) \( W' \not\subset X' \)

If \( \Gamma \) is a \( \mathcal{P} \)-marker of \( Z \), then the constituents are exactly the proper terms, and we have the following result:
Th. 17. A $P$-marker gives a consistent analysis of its product.

We have thus met the fundamental non-overlap condition set in $\S 47.2$ and have essentially answered the question (1) posed in $\S 47.1$.

49.1. It has been pointed out above that not all derivations lead to grammatical strings, i.e., strings in $G(P)$, even though the product of every derivation must be a string in $\overline{P}$. We must now consider what is involved in the development of restricted derivations which always do lead to strings in $G(P)$ (see $\S 48.2$).

We say that the system of phrase structure $P$ is restricted just in case every $p$-derivation in $P$ does actually lead to a string in $G(P)$ as its product.
Def.19. $P$ is restricted if and only if every $\mathcal{J}$-derivation in $P$ is restricted.

Th.16. If $P$ is restricted then

(i) If $\mathcal{J}$(Sentence, $Z$) and $Z$ is in $\mathcal{F}$, then $Z$ is in $\text{Gr}(P)$

(ii) If $\mathcal{J}$(Sentence, $Z$), then $Z$ is normal

Th.17. For any language, there is a way of defining $\mathcal{J}$ such that $P$ will be restricted.

In particular, let $\mathcal{J}$ hold between Sentence and each string $Z$ of $\text{Gr}(P)$, and between nothing else. Then $P$ (the set of primes of $P$) contains just Sentence and the primes of $\mathcal{F}$, and $\mathcal{J}$ is exactly the same relation as $\mathcal{J}_1$, each $\mathcal{J}$-derivation containing exactly two steps.

Why should we not require that $P$ be restricted since we know that this requirement can be fulfilled? The reason is that this would give a conception of constituent analysis clearly at variance with the intended one, in the case of any sufficiently complex language. This is clearly the case when $P$ is restricted in the manner just described, but it must be the case for any restriction of $P$.

To see this, suppose that $P$ is restricted. Suppose that $\mathcal{J}$(Sentence, $P_1$, $P_k$), $\mathcal{J}_1(P_1, Z_1)$, and $\mathcal{J}_1(P_1, Z_k)$, where $P_1$ is a prime, and $\text{Context}$ is any context. Suppose furthermore that $P_1$ is the only prime which represents either $Z_1$ or $Z_k$.

Then for each string $W_1 = XZ_1Y$ there is a string $W_1 = XZ_2Y$ such that either both $W_1$ and $W_2$ are normal or neither is normal. In other words, $Z_1$ and $Z_2$ are completely mutually substitutable in normal strings. But this is much too strong a requirement to be placed in general on constituents. There will, for instance, be various strings which we would like to say are noun phrases, even though they do not all appear grammatically (with
first order grammaticality) as subjects of the same verb phrases, although each occurs grammatically with some verb phrase. This is the property of linguistic systems sometimes known as selection. (16)

Consider for example the case diagrammed in figure 5, and corresponding to (15).

\[ S(P_1, P_2 \neg P_3) \]
\[ S(P_2, P_{21}) \cup S(P_2, P_{22}) \]
\[ S(P_3, P_{31}) \cup S(P_3, P_{32}) \]
\[ P_{21} \land P_{31} \land P_{22} \land P_{32} \text{ (under) are strong at } C_r(P) \]
\[ P_{21} \land P_{32} \land P_{22} \land P_{31} \text{ } \\{ \text{Gr}(P) \}

![Diagram of a tree structure with nodes labeled P_1, P_2, P_3, P_{21}, P_{22}, P_{31}, P_{32}]

Fig. 5.

An interpretation in English is easily found.

Consider the interpretations \( M_1 \) and \( M_2 \).

\( M_1 \):  
\( P_1 = \text{Sentence} ; P_2 = \text{NP} ; P_3 = \text{VP} ; P_{21} = \text{John} ; P_{22} = \text{the rearming of Germany} ; P_{31} = \text{is at dinner} ; P_{32} = \text{is Western Policy} \)

\( \text{Gr}(P) (\text{John is at dinner, The rearming of Germany is Western policy}) \)

\( \text{Gr}(P) (\text{John is Western policy, the rearming of Germany is}) \)

\( \text{is at dinner} \).
For descriptive adequacy, then, we must have a non-restricted $P$ from which $P$-markers and constituent structure can be derived. Thus we will want to say that John and the rearming of Germany are NP's, that there is a general NP-VP (i.e., actor-action) form of sentence, etc. On the other hand, we want to construct a grammar of English that will provide derivations for exactly the members of $\mathcal{G}(P)$. And we want this grammar to be intrinsically related to $P$ in such a way that given $P$, we can derive the grammar, and given the grammar, we can determine the underlying $P$ on which it is based, and the constituent analysis that this underlying $P$ carries along with it.

The problem posed in the last paragraph can be construed as the problem of constructing a relation $f^T$ such that all $f^T$-derivations are restricted, and all restricted $f^T$-derivations are $f^T$-derivations. Then the grammar of the language will be constructed in terms of $f^T$. Of the two prerequisites for the level $P$ stated at the outset of $\text{4.2.1}$, $f$ will satisfy the first, and $f^T$ the second.

The relation between $P$ and the grammar stated in the last few sentences of $\text{4.2.1}$ has no content unless we
The problem posed in the last paragraph can be construed as the problem of constructing a relation \( \sigma^r \) such that all \( \sigma^r \)-derivations are restricted, and all restricted \( \sigma_1 \)-derivations are \( \sigma^r \)-derivations. We will define \( \sigma_1^r \) as the relation which holds between successive steps of restricted \( \sigma_1 \)-derivations. That is, suppose that \( A_1, \ldots, A_k \) is a restricted \( \sigma_1 \)-derivation. Then it will be the case that \( \sigma_1^r(A_1, A_2), \sigma_1^r(A_2, A_3), \ldots \) etc. \( \sigma^r \) will then be related to \( \sigma_1^r \) as \( \sigma \) is related to \( \sigma_1 \). That is, if \( A_1, \ldots, A_k \) is a restricted \( \sigma_1 \)-derivation, then it is the case that \( \sigma^r(A_1, A_2), \sigma^r(A_2, A_3), \ldots, \sigma^r(A_1, A_k) \), \( \sigma(\ldots, A_2, A_3), \ldots, \sigma^r(A_2, A_3), \ldots \), etc. Then the \( \sigma^r \)-derivations are exactly the restricted \( \sigma_1 \)-derivations, and the \( \sigma^r \)-derivations are exactly the restricted \( \sigma \)-derivations.

Given \( \mathcal{P} \), we can determine all restricted \( \sigma \)-derivations in \( \mathcal{P} \); hence we can determine \( \sigma^r \). We will see below that \( \mathcal{P} \) can be axiomatized in such a way that \( \sigma \) is reconstructible from \( \sigma^r \) as well. In the simplest case, we will determine \( \sigma \) from \( \sigma^r \) in the following manner: suppose that \( \sigma \) is a prime.

Then we know that there is some restricted \( \sigma_1 \)-derivation which contains the step \( A_1 \rightarrow A_2 \) and the step \( A_2 \rightarrow A_3 \) which is formed from \( A_2 \) by replacing \( P \) in \( A_2 \) by \( X \). In this case we will say that \( \sigma_1(P, X) \). We will see that, with some qualifications, we will be able to determine \( \sigma \) completely (that is, we will be able to determine for which strings \( X \) and \( \sigma \) \( Y \) it is the case that \( \sigma \) \((X,Y)\)) in this manner. This means that given the grammar of the language \((\text{constructed in terms of } \sigma^r \text{ and giving } \mathcal{X} \text{ us all } \sigma^r \text{-derivations, hence all restricted } \sigma \text{-derivations})\) we will be able to reconstruct completely the relation \( \sigma \) and the system of phrase structure \( \mathcal{P} \). The two prerequisites for the level \( \mathcal{P} \) stated at the outset of \( \sigma^r \) \((\sigma^r \text{-level})\), \( \sigma \) will satisfy the first and \( \sigma^r \) the second.

The relation between \( \mathcal{P} \) and the grammar as stated in the last few paragraphs has no content unless we
state what we mean by a grammar. We will go into this
below, arriving at a conception like that of chapter III,
except for an extension of the notion of generation.
Meanwhile, we assume that whatever a grammar is, it must
be related to the relation \( f^X \) in the sense that \( f^X \)
is uniquely recoverable from the grammar, and a grammar
can be constructed (perhaps in various ways) in terms
of \( f^X \). We then require that \( P_2 \) be related biuniquely to
\( f^X \) in the sense that \( P_2 \) be uniquely recoverable from
\( f^X \) and \( f^X \) be uniquely constructible from \( P_2 \).

\[ U_{412} \] By definition, the set of \( f \)-derivations is uniquely
determined by \( P_2 \). Thus if \( f^X \) is defined to uniqueness
in terms of \( f \), one part of this biunique relation will
hold. But the requirement that \( P_2 \) be recoverable from
\( f^X \) is more troublesome.

We want \( f^X \) to be a relation \( f \) such that
\[ (1) \]
the set of \( f \)-derivations is exactly the set of
restricted \( f \)-derivations.

From \[ (1) \] it follows that \( f \in f^X \). It also follows that
\[ (2) \]
For normal strings \( X \) and \( Y \), \( f(X,Y) \leftrightarrow f(X,Y) \).

From Axiom \( A_4 \) (cf. \( \text{cf. } A_4 \)), we know that \( f \) can be
recovered from the statement of \( f \) for the domain
of primes. Furthermore, \( f \) is simply the ancestral of \( f_1 \).
And it is in fact sufficient to know \( f_1 \), with the domain
limited to primes, in order to reconstruct \( f \). But Axiom \( A_4 \) \& lets
informs us that if \( f_1(P_4, X) \) (for \( P_4 \) prime), then there
are two normal strings \( Y_1 \) and \( Y_2 \) differing only in that
contains $f_1$, where $f_k$ contains $f$.

Then of course $S_1(Y_d, Y_k)$ and by $f_1(Y_d, Y_k)$.

In fact, $f_1(Y_d, Y_k)$, where $f_1$ is defined analogously to $f_1$ (cf. Def. 2).

But it is also the case that if $f_1(Y_d, Y_k)$ then $S_1(Y_d, Y_k)$. And we know from Theorem 4 (457) that this means that $Y_k$ differs from $Y_d$ only in that a single prime $P_1$ of $Y_d$ is replaced by some $X$ in $Y_k$. In this case, $S_1(P_1, X)$.

We see then that if for every two normal strings $Y_d, Y_k$, we know whether or not $f_1(Y_d, Y_k)$, and we always know in addition which prime of $Y_d$ is replaced by some $X$ in forming $Y_k$, then we can recover $f$ completely from $f_1$, hence from $f$, since $f_1$ is defined in terms of $f$.

Thus we are very close to being able to meet the second half of the biuniqueness condition. The difficulty is that we can not always tell, given $f_1(Y_d, Y_k)$, which prime of $Y_d$ is replaced by some $X$ in forming $Y_k$.

For instance, suppose it is the case that $P_1(P_1, P_2 P_3)$ and $S_1(P_1, P_2 P_3)$.

Then given $Y_d = P_1 P_2 P_3$, we can form $Y_k = P_1 P_2 P_3$ such that $S_1(Y_d, Y_k)$ by developing $P_1$ to $P_1 P_2 P_3$ or by developing $P_3$ to $P_2 P_3 P_3$.

Thus given $f_1(Y_d, Y_k)$, we could not tell which prime of $Y_d$ was developed in forming $Y_k$.

We can eliminate this difficulty by adding a further axiom to $P_1$, $P_2$, and $P_3$ are primes of $P$ (i.e., $P_1$, $P_2$, $P_3$).
Ax. 2. (i) If \( f_1(P_1^2, P_1^2 U) \) and \( f_1(P_1^2, P_1^2 U) \), then \( P_1 = P_2 \)

(ii) If \( f_1(P_1^2, P_1^2 U) \) and \( f_1(P_1^2, P_1^2 U) \), then \( P_1 = P_2 \)

(iii) If there is a normal string \( \ldots P_1^2 U \ldots \) then there are no strings \( X \) and \( Y \) such that \( f_1(P_1^2, P_1^2 U) \) and \( f_1(P_1^2, P_1^2 U) \)

**Axiom 2** limits the possibility of recursive statements. If there ever is any necessity to introduce pairs of grammatical rules excluded by \( \mathbf{A} \mathbf{x} . 2 \) into the grammar, then it is always possible to satisfy \( \mathbf{A} \mathbf{x} . 2 \) by adding extra intermediate primes, though only at the cost of some artificiality.

Given \( f \) meeting (16), we can now define \( f_1 \) in terms of \( f_1 \) in the following manner:

**Th. 16.** \( f_1(P_1^2, X) \) if and only if there are strings \( A_1 \ldots A_n A_{n+1} \ldots A_{n-1} A_{n-1} \ldots A_1 \)

\( A_{n-1} \ldots A_{n+1} \), \( W_1 \) and \( W_2 \) such that

(i) \( A_1 \ldots A_n \), \( A_{n+1} \ldots A_{n-1} \), \( A_1 \ldots A_n \), and \( A_1 \ldots A_n \), \( A_{n+1} \ldots A_{n-1} \), are \( f_1 \)-derivations.

(ii) \( A_2 = W_1^2 U_2 \)

(iii) \( A_{n+1} = W_1^2 U_2 \)

(iv) If \( X = Z U_1 \) and \( W_2 = Z U_2 \), then there is a string \( Z' \) and a prime \( P_2 \)

such that \( A_{n+1} = W_1^2 U_2 \) and \( P_2 \neq P_1 \)

(v) If \( X = Z U_1 \) and \( W_2 = Z U_2 \), then there is a string \( Z' \) and a prime \( P_2 \)

such that \( A_{n+1} = Y U_1 \) and \( P_2 \neq P_1 \).

**Th. 17** If conditions (i)-(v) are met in **Th. 16**, then \( \mathbf{P} (P_1, X) = \mathbf{A} \mathbf{P}_2 (A_2) \), where

\[ D = A_1, \ldots, A_n \]

[see **Def. 8.**, \( \mathbf{E} 1 \)]

In this way, if we define \( \mathbf{f} \) meeting condition (16) to uniqueness in terms of \( f \), we can establish the required biuniformness between \( \mathbf{P} \) and the grammar.
constructed in terms of $f^r$.

The obvious way to define $f^r$ is as the minimal relation meeting the condition (16).

**Def. 20.** $f^r(X,Y)$ if and only if $X$ and $Y$ are normal and $f(X,Y)$.

**Def. 21.** $f^r_1(X,Y)$ if and only if $f^r(X,Y)$ and there is no $Z$ such that $f^r(X,Z)$ and $f^r(Z,Y)$.

**Th. 20.** $f^r$ is the minimal relation meeting (16).

**Th. 21.** By Th. 18, $f$ is definable in terms of $f^r$. 
We have thus taken $f^p$ to be the relation that connects the steps of restricted $f$-derivations. It would be reasonable to inquire into the extent to which our choice was limited to this by condition (16). That is, we may ask how much $f^p$ can be extended, while still meeting this condition. We know that some of $f$ must be left out, under the assumption that $P$ is non-restricted.

We have seen from the examples that the crucial cases that require a limitation of $f$ are the cases of non-identical distributions of strings represented by a given prime $P_i$, where by the distribution of $X$ is meant the set of significant occurrences (i.e., occurrences as a constituent) of $X$ in normal strings.

More formally, we define the distribution of $X$ as a $P_i$.

**Def. 22.** Dist$(X, P_i) =$ the set of triples $\langle Y_1, Y_2 \rangle$ such that where $Z = Y_1^* X^* Y_2$

1. $Z$ and $Z(P_i/X, Y_1^* X)$ are normal
2. $P_i (P_i, X)$

Then the distribution of $X$ can be defined as

**Def. 23.** Dist$(X) =$ the set of triples $\langle Y_1, Y_2 \rangle$ such that for some $P_i$, $\langle Y_1, Y_2 \rangle \in$ Dist$(X, P_i)$.

Suppose we further define the following for any two place relation $f$.

**Def. 24.** The prime domain of $f$ = $Pd(f) = \left\{ P_i \mid P_i \in P \text{ and } \exists X \text{ s.t. } f(P_i, X) \right\}$

Thus the prime domain of $f$ is $P^p$ (the logical product
of \( P \) and the complement of \( \overline{P} \).

Although if \( P \) is restricted, \( f^R \) is properly included in \( f \), it still is possible, under certain special circumstances, that \( Pd(f) = Pd(f^R) \). \( Pd(f^R) \) can be extended to include a prime \( P_1 \) just in case the following condition is met:

(18) There is an \( X \) such that (i) \( f_1(P_1, X) \)

(ii) for all \( Y \) such that \( f_1(P_1, Y) \),

\[ \text{Dist}(X, P_1) \subseteq \text{Dist}(Y, P_1) \]

We can gain some further insight into \( f^R \) by asking to what extent the axioms for \( f \) (cf. (12)) can be satisfied by a relation \( f \) meeting condition (18) in a non-restricted algebra.

As defined, \( f^R \) automatically satisfies axioms (1), (2), (3), and (4), with \( f^R \) replacing throughout. There are special circumstances under which \( f^R \) could be extended to satisfy Axiom (3) from right to left. But \( f^R \) cannot satisfy Axiom (3) from left to right, for all \( P_1 \), in a non-restricted \( P \).

**Th. 22.** If \( P \) is non-restricted, and a relation \( f \) satisfies condition (18), then, left to right, cannot hold of \( f \) in the prime domain \( \mathfrak{A} \) in \( P \).

In other words, in a non-restricted \( P \) it will be the case that there are compound strings \( X \) and \( Y \) such that \( f^R(X, Y) \), but that this relation cannot be decomposed so that \( f^R \) holds term by term between each prime of \( X \) and some part of \( Y \). Thus \( f^R \) carries certain representations
into other representations only in certain contexts. This is the basic idea behind the restriction of $f$.

The notion of restriction leads directly to the consideration of long components. It might be possible to extend the formal systems which have been considered so as to permit these to be introduced in an effective way. The basic contribution of long components is that they offer a way of restricting $P$ without violating the prerequisite of descriptive adequacy, as would be the result if $P$ were restricted directly. This can be illustrated clearly by the example diagrammed in fig. 5 (§4.1).

Suppose that our notations were extended so that the following would be significant:

\[
\begin{align*}
\overset{(k\alpha)}{P_1} & \rightarrow \overset{(k\beta)}{P_2} \rightarrow \overset{(k\gamma)}{P_3} \\
\overset{(k\alpha)}{\sigma} & \rightarrow \overset{(k\beta)}{\alpha} \\
\overset{(k\alpha)}{\beta} & \rightarrow \overset{(k\beta)}{\beta}
\end{align*}
\]

And suppose further that a notational convention indicated that all identical superscripts assume the same value in derivations. Then we could interpret $P_2^a$ as $P_{21}$, $P_2^b$ as $P_{22}$, $P_3^a$ as $P_{31}$, and $P_3^b$ as $P_{32}$. The derivations would now work out exactly right, the algebra would be restricted, and the notions NP, VP, etc., would be retained with all essential generality.

This method might in fact be useful for i.e., in the case of agreement in number. But even if we did extend our formalism so as to admit this possibility,
we would still not want to apply it for \( M_1 \), because the vast selectional complex involved here would require a tremendous number of components, and a very inelegant characterization.

This is an important question, deserving a much fuller treatment, but it will quickly lead into areas where the present formal apparatus may be inadequate. The difficult question of discontinuity is one such problem. Discontinuities are handled in the present treatment by construction of permutational mappings from \( P \) to \( W \), but it may turn out that they must ultimately be incorporated somehow into \( P \) itself.

50.1. \( f^x \) is the relation between steps of a restricted \( f \)-derivation. Knowing it, we can construct all such derivations, and, by Th. 18, we can reconstruct \( P \) completely, in the manner sketched in \( \mathcal{C} \). However, the problem of actually bridging the gap between this theory and an actual grammar, or more generally, a fixed form of grammars for which the notion of simplicity has been given some meaning (cf. chap III), is still unsolved.

In chapter III, we presented a certain conception of a grammar as a sequence of statements "\( \alpha \rightarrow \beta \)" where \( \alpha \) and \( \beta \) are strings, and derivations are constructed mechanically by proceeding down the list of statements, interpreting "\( \alpha \rightarrow \beta \)" as the instruction "rewrite \( \alpha \) as \( \beta \)". We have also presented, in chap.V, Appendix, a detailed morphophonemic study constructed on that model. In chapter V,
we have seen how a grammar of this sort might be developed from levels lower than $\mathbb{M}$, so that derivations of grammatical utterances from strings in $\mathbb{M}$ can be mechanically constructed. We must now show how the statement of the structure $\mathbb{P}$ can be converted into a grammar of this form, so that we can derive (in the sense of chap. III) strings in $\mathbb{M}$ from the representation of grammars. This will complete (in outline) the statement of how the hierarchy of levels is related to the form of grammars. We know from the biuniqueness relation between $\mathbb{P}$ and $\mathbb{F}$ (cf. Def. 20, Th. 21) that it is sufficient to show how the statement of $\mathbb{F}$ can be converted in general into a sequence of statements of the form "$\alpha \rightarrow \beta"$, in such a way that $\mathbb{F}$ can be uniquely recovered from this sequence of conversions.

We can look at this in a slightly different way. One way to give a description of the level $\mathbb{P}$ is actually to present a list of pairs of strings $(\bar{x}, y)$ such that $\mathbb{F}(\bar{x}, y)$. But since we would like to make the description of $\mathbb{P}$ as simple as possible, we would like to discover an even more limited relation that $\mathbb{F}$ (hence, a shorter list) in terms of which $\mathbb{F}$ can be reconstructed. We will now consider various ways of reducing $\mathbb{F}$. The condition that all such reductions must meet is that the set of derivations (20) remain invariant, that any new technique of providing a description of phrase structure must lead to exactly the set of derivations obtained from $\mathbb{F}$. We will see that the attempt to reduce $\mathbb{F}$ in this manner leads to a sequence of statements of the form "$\alpha \rightarrow \beta"$, i.e., to a grammar in the sense of chapter III. In this development, we retrace some of the ground covered
in chapter III, but now building our case on examples drawn from the level $P$. 

\[ \text{Reduction 0.} \] First it is clear that $f^r_1$ is such a limited relation. Thus the grammar might consist of a complete statement of $f^r_1$, i.e., a list of all pairs $(\alpha, \beta)$ such that $f^r_1(\alpha, \beta)$. But even this list is "redundant" in the sense that a shorter list would provide all the information necessary to reconstruct $P$. Given a single member of each equivalence class of derivations, we can mechanically construct all derivations. But knowledge of $f^r_1$ enables us to construct all derivations directly, and thus contains an over-specification of $P$.

\[ \text{Reduction 1.} \] Construct $R_1$, a narrower relation than $f^r_1$, from which we can reconstruct one derivation from each equivalence class.

To see that $R_1$ may be narrower than $f^r_1$, consider the simple case we have often discussed before, diagrammed as follows:

\[ \text{Fig. 6.} \]
Given just this system of derivations, a complete account of $f_1^p$ would be the set of pairs (21)

$$\{(a, bc), (bc, defg), (defg, bfg)\}$$

whereas the set of pairs in the left hand column alone will suffice to give just $D_1$, and can thus be taken as $R_1$. Thus the grammar can be shortened (simplified) by dropping the right hand column, which can be recovered mechanically, given $F_1$.

Reduction 2. Drop from each pair in $R_1$ the terms which play no role in the derivation, i.e., those which are not developed and do not determine how the development takes place. The relation which consists of just the pairs which play a role in the formation of derivations can be called $R_2$.

Thus $R_1$ from (13) can be replaced by

$$\{(a, bc), (b, de), (c, fg)\}$$

which can be taken as $R_2$ in this instance. Notice that if we had performed reduction 2 directly on $f_1^p$ (i.e., in (19)), we would still have obtained (20). Thus reduction 1 was superfluous in this case. [Note further that $R_2$ is exactly $f_1^p$, limited to its prime domain. This will be the case whenever the system is restricted, as is the system of derivations given in Fig. 6.]

To take a more significant instance, consider the system $D_1, D_2, D_3$, where $D_1$ and $D_2$ are as in Fig. 5, and $D_3$ is
The system \( D_1, D_2, D_3 \) is now non-restricted. \( f_1^R \) for this system is \((14)\) plus 
\[(a, bh), (bh, s_e h)\]

Applying reduction 1 to \( f_1^R \) in this case, we obtain as \( R_1 \),
\[(a, bc) \quad (a, bh) \]
\[(bc, dec) \quad (bh, s_e h) \]
\[(dec, defg)\]

Applying reduction 2 to \( R_1 \), we obtain as \( R_2 \)
\[(a, bc) \quad (a, bh) \]
\[(bc, dec) \quad (bh, s_e h) \]
\[(a, fg)\]

Applying reduction 2 to \( f_1^R \) \((15)\) plus \((15)\) directly gives 
\[(a, bc) \quad (a, bh) \]
\[(bc, dec) \quad (bh, s_e h) \]
\[(a, fg)\]

Thus reductions 1 and 2 are independent.

Clearly \( R_2 \) is the simplest relation in terms of which \( P \) can be reconstructed (it contains fewest pairs, and each pair is as short as possible). Thus if we wish to construct an even more concise grammar than a listing of \( R_2 \), we cannot search for a simpler relation, but must consider features of the listing itself. That is, we must begin a metalinguistic
Reduction 2 provides a certain possibility of ambiguity which may make it impossible to reconstruct $P$ correctly from $R_2$. This results from the fact that $R_2$ relates parts of normal strings. Suppose that we have the derivation $D_4$ as in fig. 8 as our only derivation in $P$.

![Figure 8](image)

Then $R_1$ will be given by the set of pairs containing just $(a,ab)$ and $(ab,odb)$, as will $R_2$. But our general rule for reconstructing $R_1$ from $R_2$ will tell us that if $R_2(x,y)$ and $w_1^X w_2$ is a string in an ** derivation, then $w_1^Y w_2$ can be the next string in the derivation, and $R_1(w_1^X w_2, w_1^Y w_2)$. But in the case of $D_4$ as in fig. 8, this rule of reconstruction will allow us to reconstruct $R_1(ab,abb)$, since $R_1(a,ab)$ and $ab$ is a string in an derivation. Hence we can also have the derivation $D_5$ as in fig. 9, contrary to assumption.

![Figure 9](image)

The difficulty here is that we cannot distinguish between the case where a string $\alpha$ can be rewritten as $\beta$ in a derivation \textit{whenever} $\alpha$ is part of a line (i.e., in any context) and the case where the rewriting can only take place when $\alpha$ is the whole line (i.e., in the null context). We can best resolve this ambiguity by revising slightly our notion of derivation. We have considered a derivation to be a sequence of strings beginning with \textit{Sentence} and ending with a string in $P$. We now consider a derivation to be a sequence beginning with \textit{Sentence}' and ending with a string in $P$, where \textit{'} is a prime of $P$. 
indicating utterance initial and utterance final position (i.e., $\neq$ will be one of the zero elements $\emptyset_1$ of the level $F$). Then $D_4$ will be rewritten in the following manner:

```
  $\neq$ a $\neq$
    /     \
   # a    b #
      /  \  /
     # a   c b #
```

$R_1$ will be given by the set of pairs: $(\neq a \neq, \neq ab \neq), (\neq ab \neq, \neq ach \neq)$; and $R_2$ will be given by the set of pairs: $(a \neq, a\neq), (ab, abc)$. The reconstruction of $R_1 \times R_2$ now gives $\max D_4$ as the only derivation on the level $F$, as required in this case.
investigation of grammars. Thus we now investigate the possibility of utilizing the actual form of the presentation of the grammar in terms of \( R_2 \) to effect a further reduction. In this investigation we retrace much of the ground covered in \( \mathcal{R}12-21 \), but now with particular reference to the level \( P \).

We have seen that the system of phrase structure for a given language can be recovered from the relation \( R_2 \), that is, from a list of the pairs \((X, Y)\) such that \( R_2(X, Y) \). Suppose we have such a list. Then if we preserve the order of the listing, and we rewrite \( R_2(X, Y) \) as \( \langle X - Y \rangle \) we have a linear grammar in the sense of \( \mathcal{R}12.1 \). The linear grammar is a sequence of \( m \times n \) conversions statements \( S_1, \ldots, S_n \) where each \( S_i \) is of the form \( \langle X_i - Y_i \rangle \). We know that we can \( m \times n \) derivations from this linear grammar by applying the conversions \( S_i \) (interpreted as the instruction "rewrite \( X_i \) as \( Y_i \)"") in sequence. Among the \( S_i \)'s we distinguish between obligatory conversions that must be applied in the construction of every derivation, and permissible conversions that may or may not be applied. There are only a finite number of ways to run through the linear grammar, applying all obligatory and some permissible conversions, hence only a finite number of derivations can be constructed from the linear grammar \( S_1, \ldots, S_n \). This was not a difficulty on earlier levels, but we know that infinitely many sentences must be generated by some mechanism in the grammar. We can permit this infinite generation on the level \( P \) by allowing the possibility of running through the linear grammar \( S_1, \ldots, S_n \) an indefinite number of times \( \mathcal{R} \) in the construction of derivations. If the derivation formed by running through the sequence of conversions does not terminate with a string in \( P \), then we run through the sequence again. Thus we can understand the linear grammar to be the sequence of conversions \( S_1, \ldots, S_n, S_1, \ldots, S_n, S_1, \ldots, S_n, \ldots \). We then say that a derivation \( D \) is recursively produced from the linear grammar \( S_1, \ldots, S_n \). We define a proper grammar as a linear grammar which is so constructed that it is impossible to run through it over and over again vacuously. Our final reduction, reduction \( \mathcal{R}3 \), then, will be to construct a proper grammar from which \( R_2 \) can be completely reconstructed. That is, we construct a proper grammar which recursively produces just those derivations which are formed by the use of the relation \( R_2 \) (i.e., all \( S_0' \) derivations).
Investigation of grammars. Thus we now investigate the possibility of utilizing the actual form of the presentation of the grammar in terms of $R_2$ to effect a further reduction. We now proceed with a more careful development of the scheme sketched in the last paragraph. The grammar will be given by a list of statements of the form "$(\alpha, \beta)$", one statement for each $(\alpha, \beta)$ such that $R_2(\alpha, \beta)$. The significant formal feature of this list of statements is that it can be given in a linear order. It is natural to inquire whether a manipulation of this order can provide a simpler grammar. (22)

Suppose that we have a sequence $S_1, \ldots, S_n$, where $S_i$ is an ordered pair $(X_i, Y_i)$, $X_i$ and $Y_i$ being strings. In accordance with the conception of grammar in chapter III, we can interpret such a sequence (or the sequence of statements corresponding to it) as instructions for constructing derivations. That is, we might construct a definition something like this:

(15) D is produced from $S_1, \ldots, S_n$ if there is a sequence $\alpha_1, \ldots, \alpha_n$ such that:

(16) There is a sequence $\alpha_1, \ldots, \alpha_{n+1}$ such that:

(i) $\alpha_1 = X_1$

(ii) for $i \neq 1$, $\alpha_{i+1}$ is formed from $\alpha_i$ by replacing $X_i$ in $\alpha_i$ by $Y_i$ (thus $\alpha_{i+1} = \alpha_i$ if $X_i$ does not occur in $\alpha_i$).

(iii) D is formed from $\alpha_1, \ldots, \alpha_{n+1}$ by dropping each $\alpha_i$ which is identical with $\alpha_{i-1}$, i.e., all repetitions.
But this formulation is too rigid. First of all, $X_i$ may occur several times in $\alpha_i$, so that $S_i$ must apply several times in forming $\alpha_{i+1}$. Secondly, this formulation does not cover the case in which $S_i$ does not apply, even though $X_i$ appears in $\alpha_i$. That is, it leaves no room for a certain indeterminacy in the production of derivations that should be built into the grammar. In English, for instance, we may have $\gamma_i(NP, N)$, and $\gamma_i(NP, A^N)$. In listing $R_2$, then, we will have the pairs

$$(\alpha_i) \quad (NP, N)$$

$$(NP, A^N)$$

But if we defined production as in $(19)$, and if $(20)$ is part of the grammar, then ... $A^N$ ... will never occur in a derivation, since $NP$ will always be eliminated by the earlier statement. Thus we must distinguish between statements of the grammar which are obligatory and those which are optional. $(23)$ We can thus prefix to each statement either "O" (obligatory) or "P" (optional), and formulate the definition of "production" in terms of a sequence of statements of this form. Combining these two inadequacies, we see that we must also provide for the case in which $X_i$ occurs in several places in $\alpha_i$, but only certain of these occurrences are replaced by $Y_i$. We must define production, then, so that produced derivations may have any of these various properties.

Suppose in the following that we have a sequence $S=(S_1, \ldots, S_n)$ of ordered triples, where $S_i=(\alpha_i, X_i, Y_i)$, and $\alpha_i$ is either O or P, these being distinct elements of some
sort that we can construct in \( S_i \). Then \( S_i \) can be understood as the statement: "it is obligatory to rewrite \( X_i \) as \( Y_i \)", if \( S_i = B \), or "it is permitted to rewrite \( X_i \) as \( Y_i \)", if \( S_i = P \).

Preparatory to defining production, consider certain sequences \( t, \alpha, \) and \( Z(i) \) which jointly meet the following condition, given the sequence \( S = (s_1, \ldots, s_n) \), where \( s_i = (z_i, x_i, y_i) \) as above:

(2) (I) \( t \) is a sequence of numbers, \( t = (t_1, \ldots, t_{K+1}) \)

such that \( t_1 = 1 \), and \( t_1 < t_{i+1} \)

(II) \( \alpha \) is a sequence of strings, \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{K+1}) \)

(III) \( Z(i) \) \((1 \leq i \leq K)\) is a sequence of strings,

\[
Z(i) = z_{1}^{(i)}, \ldots, z_{m}^{(i)}
\]

such that where \( \alpha = \frac{t_{i+1}}{t_1} \), and

(i) \( z_1^{(i)} < z_2^{(i)} < \ldots < z_{m}^{(i)} \)

(ii) if \( X_i \not\subset \alpha \) and \( S_i = (B, X_i, Y_i) \), then

\[
\alpha_{i+1}^{(i)} \not\subset \alpha_i
\]

is an occurrence of \( X_i \) in \( \alpha \)

(iii) if \( z_1^{(i)} \) is not an occurrence of \( X_i \) in \( \alpha \), then

\[
0^{(i)} = \emptyset
\]

(IV) For each \( i \), \( t_{i+1} \cdot t_i \) is the number of distinct occurrences of \( X_i \) in \( \alpha \), if \( X_i \subset \alpha \)

and equality holds if \( S_i = (B, X_i, Y_i) \)
We then take $\alpha$ to be the derivation (when repetitions have been struck out). $\alpha_1$ will be $X_1$, and for $j \geq 1$, if $t_j < j_1 < t_{j_1} + 1$, the rule used in forming $\alpha_j$ will be $S_{j_1}$. The effect of this rule will be to replace an occurrence of $X_1$ in $\alpha_j$ by $Y_1$, whereas occurrence is essential.

Because of the way that occurrence has been defined, we must be sure that in the sequence of steps using rule $S_{j_1}$ to replace occurrences of $X_1$ by $Y_1$ in $\alpha_j$, occurrences to the right (literally, longer occurrences) are replaced first. (cf. \textsuperscript{13}2). We see that $S_{j_1}$ is applied $m_{j_1} = t_{j_1} + 1 - t_j$ times in constructing the derivation, where $m_{j_1}$ is the number of occurrences of $X_1$ in $\alpha_j$ if $S_{j_1}$ is obligatory. If $m_{j_1} = 1$, and $S_{j_1}$ is optional, then $Z_1^{(j)}$ may be either $U$ or an occurrence of $X_1$ in $\alpha_j$. $Z_1^{(j)}$ will always be $U$ when $X_1$ does not occur in $\alpha_j$. Note that no matter what $a, b, c, U$ are, $a(b/c, U) = a$. Thus in particular, if $m_{j_1} = 1$ and $Z_1^{(j)} = U$, then

$$\alpha_{t_1 + 1} = \alpha_{t_1 + 1} = \alpha_{t_1} (Y_1/X_1, Z_1^{(1)}) = \alpha_{t_1}$$

Def. 25. $D$ is produced from $S = (S_1, \ldots, S_n)$ if and only if there are sequences $t, \alpha$, and $Z^{(i)}$ (for $1 \leq i \leq k$) jointly meeting (27) and such that

(i) $\alpha_1 = X_1$

(ii) for $j$ s.t. $t_j < j_1 < t_{j_1} + 1$ (where $i \geq 1$),

$$\alpha_{j_1 + 1} = \alpha_{j_1} (Y_1/X_1, Z_1^{(i)})$$

(iii) $D$ is formed from $\alpha$ by dropping any $\alpha_j$ s.t.

(iv) $\alpha_{j+1}$ is a string in $\mathcal{F}$
Th. 23. In this case, for $i \geq 1$, 
\[
\alpha_{t_{i+1}} \equiv \alpha_i (z_{i+1}/x_{i+1} z_1) \cdots (z_1/x_1 z_{m_1})
\]

[cf. §13.2]

There is still a serious gap in this account of production of derivations from a grammar. Given any grammar $S_1, \ldots, S_n$, only a finite number of derivations can be produced from it in accordance with definition 25. But in fact, $F$ will typically contain infinitely many derivations. (24) Suppose for instance, that \( f_1(a, bc) \) and \( f_1(b, dac) \), in a certain system $F$. Then there will be infinitely many derivations in $F$, since we can have $a \rightarrow bc \rightarrow dac \rightarrow dbcc \rightarrow ddacc$, etc.

This is incidentally, not a far-fetched example. It occurs in English, for instance, with $a=$Sentence, $b=$Noun Phrase, $c=$Verb Phrase, $d=$that. To cope with this possibility it is necessary to reformulate Def. 25, adding a recursive step, so that infinitely many derivations can be produced by a finite grammar.

Def. 26. D is recursively produced from $S=(S_1, \ldots, S_n)$ if and only if, for some $m \geq 1$, D is produced in the sense of Def. 25 from $T_m=(T_1, \ldots, T_{mn})$, where, for $i \leq n$,
\[
T_i = T_{i+n} = T_{i+2n} = \cdots = T_{i+(m-1)n} = S_i
\]

Thus $T_1, \ldots, T_{mn} \equiv G_1, \ldots, G_m$, where $G_i = S_1, \ldots, S_n$. In other words, if we run through $S_1, \ldots, S_n$ and come out with a string $X$ which is not a string in $F$ (hence not in $Gr(F)$), then we run through the grammar again with $X$ as the initial term $\alpha_1$, and so on if the product is again not a string in
It is important to recognize that there is a mechanical and terminating procedure for determining whether a given string $X$ is a string in $P$, given $S_1, \ldots, S_n$, viz., we run through the finite list $S_1, \ldots, S_n$ and determine for each prime $X$ of $X$ whether or not $X$ is developed in some $S_i$ (i.e., whether there is an $S_i = (L_i, W_1^X W_2, W_3^X W_2)$). If no prime in $X$ is so developed, then $X$ is a string in $P$ (assuming that $S_1, \ldots, S_n$ is properly related to $P$, i.e., that all and only $\mathcal{J}$-derivations are produced from it).

Having defined recursive production, we must rephrase the optimality conditions of $\mathcal{J}$, replacing (i) by the requirement that every derivation can be recursively produced, i.e., produced by running through the grammar a finite number of times.

One further restriction should be added to the notion of a grammar $S_1, \ldots, S_n$ to eliminate the possibility of running through the grammar over and over again vacuously. This would be a possibility, for instance, if each statement were optional. To avoid this, we may define a proper grammar as

**Def. 27.** $S = (S_1, \ldots, S_n)$ is a proper grammar if each $S_i = (L_i, X_i, Y_i)$,

where $X_i$ and $Y_i$ are strings, and $L_i$ is either B or P,

and for each $X_i$ ($1 \leq i \leq n$), there is at least one $S_{b_i}$ in $S_1, \ldots, S_n$, such that $S_{b_i} = (B, X_i, Y_i)$.

Thus each $X_i$ must appear in one obligatory statement, not necessarily the final statement in which $X_i$ is the left-hand term. Given a proper grammar, we can run through it vacuously a second time only if the product of the first run-
through is a string in $F$, or contains none of the left-hand terms $Y_i$ from the grammar. The same effect could be achieved with a weaker but more complex condition.

It is worth noting one other point in the connection with the production of derivations. Suppose that we have a proper grammar $S$ and $T_1, T_2, \ldots$ as in Def. 25. If, for some $m$, $R$ is produced in the sense of Def. 25 by $T_m$ (hence is recursively produced by $S$), then since the last line of $R$ is a string in $F$ (cf. (iv), Def. 25), $R$ is also produced by $T_{m+1}$. But now suppose we introduce a notion of "weak production" which holds between $S=(S_1, \ldots, S_n)$ and a sequence $D$ which meets conditions (i)-(iii), Def. 25, but may or may not meet condition (iv). Thus if $R$ is produced by $S$ it is weakly produced by $S$, but if $R$ is weakly produced by $S$ it is not produced by $S$ unless the final string in $R$ is in $F$. We then say that

Def. 26. $R$ is a terminated derivation of $S$ if $R$ is weakly produced by $T_m$ and $T_{m+1}$, for some $m$.

Clearly every derivation produced from $S$ is a terminated derivation of $S$. But the converse is not true. Certain derivations may simply be "blocked" before they reach a string of $F$. They may terminate in strings which, though not in $F$, still do not contain any of the terms $Y_i$ which are converted into $A$ some $Y_i$ by some conversion $C_i$ of $S$. And in this case no further application of $C$ will carry them away the derivation any farther. However this need not trouble us, since given any terminated derivation $D$ of $S$ recursively we can immediately determine whether it is produced by $S$ and hence actually qualifies as a derivation by checking its primes against $S_1, \ldots, S_n$ in the manner indicated above. Hence given $S$ and a purported derivation $D$ we can mechanically determine whether $D$ is in fact a derivation. Given $S$, we can weakly produce terminated derivations and we can check to see whether or not they are real derivations.
50.4. We can now return to the problem of reduction. Given a listing of \( R_2 \) as in 350.2, we perform

**Reduction 3**: Construct a proper grammar \( S_1, \ldots, S_n \) which produces exactly those derivations constructible by means of \( R_2 \).

We have not yet shown that **Reduction 3** actually can give a simpler grammar (but cf. 221.2). To see how this may be the case, consider a system \( P \) consisting of the equivalent derivations \( D_1, D_2, D_3 \) as sketched in fig. 8.
R_2 will vary, depending on which derivation we choose to construct directly. Choosing D_1, we have, as R_2

\[ (22^\prime) \quad (a, bc), (bc, dec), (c, b^f), (b^f, g^f) \]

It is at once clear that this ordering of the grammar (22) provides an overspecification, since the following simpler grammar corresponds to (28), producing exactly D_1, if we require that the order of these 'grammatical rules' be preserved in constructing derivations.

\[ (29) \quad (a, bc), (b, de), (c, b^f), (b^f, gh) \]

Thus (29) can be taken as the grammar of this system. In this case all statements of the grammar are obligatory. Naturally, with the more complex cases that one finds in actual languages the reduction is far more significant, as was evident from the appendix to chap. V.

As we pointed out in the last paragraph of 4.22 and by the process of reduction we can convert a description of the system of phrase structure for a language into a linear grammar of the type studied in chapter III. It is also often the case that, with some minor modifications, the procedures in chapter III were developed. Thus the gap between the general theory and this model for
grammatical description is bridged completely, on the level $P$, and the characterization of simplicity developed in chapter III can be used to evaluate various models of $P$ proposed for a given language.

The recursive extension of $P$ might pose a problem for the determination of simplicity. Thus we may ask whether a grammar is simpler if it is necessary to run through it fewer times to set up derivations. There are several possible ways to place conditions on grammars that will eliminate a conflict between this consideration and the criterion of simplicity already established. Thus we might require that running through the grammar once produce a certain fixed finite set of derivations. E.g., we might require that for any $R_2 \subseteq R_1$ such that only finitely many derivations can be recursively produced from $R_2$, each such derivation must be produced by running through the grammar once. (We shall go into this question again further since we will find that there are strong reasons for dropping the notion of recursive production.)

Is every proper grammar a reduced form of some system $P$? The answer to this is in the negative. To determine whether a proper grammar $G$ does correspond to some $P$, we must investigate the derivations produced from $G$. There are two kinds of conditions that these derivations must meet.

In the first place, each derivation must have the proper internal construction. Each step must be constructed from the preceding step by developing exactly one prime into a string ($\sigma \gamma$). For another thing, no unbroken branch in the diagram of the derivation can pass from $P_1$ back to $P_1$. I.e., no piece of the diagram can be of the form
grammar once. We will not go into this question any further, since we will find below that there are strong reasons for dropping the notion of recursive production altogether. (See .)

50–51. Throughout our discussion of linguistic structure we have made a three-way distinction between what we may briefly call (i) language, (ii) structure of a language, and (iii) grammar of a language. By a "language" we mean here simply a set of utterances. By "the structure of a given language" we mean the particular system of levels which has been assigned to this set of utterances and which, we assert, underlies the set of utterances. By a "grammar" we mean, in general, any system having the properties described in chapter III, and above in §50. That is, a grammar is a 

\\[
\text{set of instructions "rewrite } \alpha \text{ as } \beta \" where } \alpha \text{ and } \beta \text{ are strings, and a "proper linear grammar" is a sequence of such instructions, with at least one obligatory instruction for each } \alpha \text{ which undergoes conversion in some rule.}
\]\n
By "the grammar of a given language (with a given structure assigned to it)" we mean, then, a description, written down on paper, of this structure, showing exactly how this structure is related to the language, i.e., showing how just this set of utterances can be generated in a mechanical manner in terms of the system of levels. On the level of phrase structure, the structure (ii) will be a particular model of the system \(P\), and the grammar will be a proper linear grammar of the form described above.

We have now reached the point where we can begin to consider certain fundamental questions about the limits of grammatical description, and the adequacy of particular conceptions of linguistic structure. We would like to know the answers to such questions as these: does there exist for every properly constructed grammar a linguistic structure which it describes? Can every linguistic structure be described by some properly constructed grammar? Does every language have a grammar? Is a certain abstract conception of linguistic structure rich enough so that for any language we can find a corresponding linguistic structure (i.e., an appropriate model of the system of levels) which can be described by a
grammar? Such questions have no clear meaning until we state what we mean by a linguistic structure (system of levels) and by a grammar. But we have done this on the level of phrase structure. We have given an abstract description of a certain system $P$ and have developed a notion of 'proper linear grammar', and we have shown how a particular model of $P$ may be reduced to a particular grammar which describes this model of $P$ in the sense that the model can be reconstructed in a mechanical way from the grammar. We can therefore ask the following questions:

(30) Is every proper linear grammar the description of some particular system of phrase structure? If not, how can we tell by investigating such a system whether or not it is reduced from some model of $P$.

(31) (a) Is there a system of phrase structure for every language?

(b) Is there a finite grammar in terms of $R_2$ for every language?

(c) Can every system of phrase structure be reduced to a proper linear grammar?

(d) Is there a proper linear grammar for every language? I.e., does every language have some underlying system of phrase structure which can be reduced to a linear grammar.

Negative answers to these questions will indicate actual and potential inadequacies in our theory, as we will see below.

50.5.2. Consider first question (30). We ask whether every proper linear grammar is a reduced form of some system $P$. The answer to this is clearly in the negative. To determine whether a proper grammar $G$ does correspond to some model of the system $P$ we must investigate the derivations produced from $G$. There are two kinds of conditions that these derivations must meet, if $G$ is reduced from a system of phrase structure.

In the first place, each derivation must have the proper internal construction. Each step must be constructed from the preceding step by
developing exactly one prime into a string \( \{W \} \). For another thing, no unbroken branch in the diagram of the derivation can pass from a prime \( P_i \) back to \( P_j \). I.e., no piece of the diagram can be of the form

\[
\begin{array}{c}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
P_i
\end{array}
\]

\( (32) \)

or the requirement of irreflexivity for \( \xi \) will be violated. There are several other conditions of this sort.

In the second place, the whole set of derivations must simultaneously meet certain conditions. Thus there can be no two derivations which \( \Delta \) together cause the \( \Delta \) asimmetry of \( \xi \) to be violated. More interesting is a certain condition of completeness required for the set of derivations corresponding to \( P \), a condition which shows how much more powerful and selective the notion of grammar can be in full generality than the notion of grammar reduced from \( P \). This condition can be stated as follows:

**Th. 24.** Suppose that we have a system \( P \), and suppose that \( D_1 \) are \( D_2 \) are \( \xi_1 \)-derivations in \( P \), where:

\[
D_1 = A_1, \ldots, A_i, A_{i+1}, \ldots, A_n
\]

\[
D_2 = B_1, \ldots, B_k, B_{k+1}, \ldots, B_n
\]

Suppose further that \( B_k = A_i \). Then there must also be a \( \xi_1 \)-derivation \( D_3 \) such that

\[
D_3 = A_1, \ldots, A_i, B_{k+1}, \ldots, B_n
\]

and \( D_3 \) is a \( \xi_1 \)-derivation in \( P \).

Clearly there is no necessity for a grammar \( G = S_1, \ldots, S_n \) to have the property that the set of produced derivations meets Th. 24. One of the conversions of \( G \) may be the statement \( S_i \) which gives the instruction "rewrite \( \alpha \) as \( \beta \)." Suppose that \( S_i \) is obligatory. Then, using the symbol "E" for obligatory conversions and the symbol "P" for permissible conversions, we would state \( S_i \) as
(33)  \[ B: \alpha \rightarrow \beta \]

But there may be another statement \( S_{i+k} \) which asserts that it is obligatory to rewrite \( \alpha \) as \( \gamma \), and is thus given in the form

(34)  \[ B: \lambda \rightarrow \gamma \]

\( S_1((33)) \) asserts that all \( \alpha \)'s appearing at this point in the construction of derivations are rewritten as \( \beta \), and \( S_{i+k}((34)) \) asserts that all \( \alpha \)'s appearing at this later point in the construction of derivations are to be rewritten as \( \gamma \), the \( \alpha \)'s appearing at the \( i+k \)th stage having been introduced.
after the change of \( \alpha \) to \( \beta \) at the \( i \)th stage. But this gramm may well violate \( \frac{T}{\alpha} \), since we may have derivations
\[ D_1 = A_1, \ldots, \alpha, \beta, \ldots, A_m \] and
\[ D_2 = B_1, \ldots, \gamma, \ldots, B_n, \] but no derivation
\[ D_2 = A_1, \ldots, \gamma, \ldots, B_n. \]

A historical analogy may clarify the point in question.

Our general conception of grammar is formally analogous to a model of historical change. Each statement "\( \alpha \rightarrow \beta \)" states that \( \alpha \) becomes \( \beta \). The analogy becomes clearer if we rewrite "\( \alpha \rightarrow \beta \)" using [Def. 1], chap. III (§10, 2) as $\text{RESX} \ "\beta \rightarrow \gamma^p \" \text{env:} a \rightarrow c \text{" where } a = \alpha, b = \beta, \gamma = \gamma^p. Thus \( a \rightarrow c \) can be understood as the condition, context for the change of \( b \) to \( b' \) (if \( a = e = U \), this would be analogous to the case of unconditioned sound change). In its full generality, the notion of grammar given in Def. 1 has the full power of a descriptive statement of historical change.

To return to the example of the preceding paragraph, suppose that at a certain historical period (the \( i \)th stage) every \( \alpha \) becomes \( \beta \) (i.e., where \( \gamma = a^\circ b^\circ \), every \( b \) in env: \( a \rightarrow c \) becomes \( b' \)). Suppose that after this change, \( \alpha \)'s are somehow reintroduced, and that at a later historical period (the \( i+k \)th stage) every \( \alpha \) becomes \( \gamma \). This is a perfectly possible historical statement. What happens at one period has no necessary connection with what happens at another.

But a grammar reduced from \( P \) does impose an organic connection between the changes introduced at one stage and those introduced at another. If \( \alpha \) becomes \( \beta \) at one point, then any later \( \alpha \) must also be transformable into \( \beta \). The specific case of grammars reduced from \( P \) corresponds, then, not to a model of historical change in full generality, but to a model of historical change (if this were possible) in terms...
of universal laws of development, or changes which are universal for a given language throughout its history.

These are certain necessary conditions for a grammar to be a reduced form of some \( P \). It would be possible and important to go on to investigate sufficient conditions. At any rate, we see that not every grammar is a reduced form of some system \( P \). In particular, the morphophonemic study presented in the appendix, chap. V, fails both types of tests outlined above, and thus could not be taken, even formally, as a description of some system of phrase structure. This is natural enough. \( P \) and \( M^{(26)} \) have certain similarities, and in particular, both are reducible to a single form of grammatical statement. A single grammar can be given incorporating both \( P \) and \( M \), so that sequences of phonemes can be generated directly from the representation \( \text{Sentence}^{(27)} \). But they are also formally different, e.g., in that the notion constituent has no significance when applied to morphophonemic representations. Another formal difference is that only finitely many derivations are produced by the grammar corresponding to \( M \), while infinitely many correspond to \( P \). This explains why it was possible to produce all derivations from the morphophonemic \( \text{statement} \) of chap. V by simply running through the sequence once. In this case produces is the same relation as recursively produces.

Since not every proper grammar is a reduced form of a system of phrase structure, it is necessary, when presenting a grammar, not only to show that the \( (\text{finite}) \) set of derivations of first order grammatical sentences is properly given, but
also to prove that the grammar is a reduced form of some model of $P$, i.e., that it actually describes a system of phrase structure. Only if this proof is given can we speak of an evaluation procedure for the grammar in question. This leaves an important theoretical gap, since we would like to have a mechanical evaluation procedure that can be applied directly to any proposed grammar, and there may not be a mechanical method for determining whether or not any given grammar is a reduced form of some system $P$. This leads us back to the second part of question (30) in 459.5.1. If we cannot provide a general mechanical method for determining whether, given any proper linear grammar, the structure derived from this grammar is actually a system of phrase structure, then our general program of constructing linguistic theory in such a way as to provide an evaluation procedure for any grammar will have failed. If the grammar produced only a finite number of derivations, this problem would not arise. Below we will find many other reasons to question the validity of the extension of the notion of production to recursive production, and we will finally arrive at the decision to limit production on the level $P$ to a finite set of derivations.

We turn now to the consideration of question (31), 50.5.1. This is essentially the question of whether our conceptions of phrase structure and of grammar are rich enough so that for any phrase structure we can construct a grammar which gives the phrase structure of this language.

A language is a set of finite strings of utterances. The phrase structure of a language is a particular model of the finite level $P$ which is associated with this language by a mapping $Q_P$ which maps $P$-markers into utterances. Each $P$-marker is a set of strings in $P$ containing exactly one 'lowest level' string of $Gr(P)$. Thus in a derivative way, $Q_P$ relates strings in $Gr(P)$ to utterances. We will simplify the study of question (31) by breaking it down into two parts. First we ask whether for any choice of $Gr(P)$ there is a phrase structure and a grammar.
means, essentially, that we consider the relation \( \mathcal{F} \) to be 1-1. Then in §59.5.4 we ask whether the mapping \( \mathcal{F} \) itself can be represented by a properly constructed grammar.

The only conditions on \( \text{Gr}(\mathcal{P}) \) is that it be a set, finite or infinite, of strings of finite length, and that only a finite number of symbols (primes of \( \mathcal{P} \)) be contained in the set \( \text{Gr}(\mathcal{P}) \). Since we are now taking the language to be just \( \text{Gr}(\mathcal{P}) \), question (31a) asks whether for any set \( \text{Gr}(\mathcal{P}) \) there is a system of phrase structure. The answer to this question is positive, trivially.

In discussing Th. 17, §42.1, we noted that if we allow \( \mathcal{F} \) to hold between Sentence and each string \( Z \) of \( \text{Gr}(\mathcal{P}) \), and between nothing else, then the axioms for \( \mathcal{P} \) are satisfied trivially. Hence if \( \text{Gr}(\mathcal{P}) \) is the set of strings \( Z_1, Z_2, \ldots, Z \), we can always assign to \( \text{Gr}(\mathcal{P}) \) the system of phrase structure which contains just the derivations (Sentence, \( Z_1 \)), (Sentence, \( Z_2 \)), ...

Question (31b) asks whether for any set \( \text{Gr}(\mathcal{P}) \) there is a finite set of instructions "rewrite \( \alpha \) as \( \beta \)" from which, for some associated system of phrase structure, we can generate just those derivations which are \( \mathcal{F} \)-derivations of strings in \( \text{Gr}(\mathcal{P}) \). I.e., does every language have a grammar in the weakest sense of grammar, where no order is imposed on the rules "\(-\)"? The answer to this question is negative, also trivially. It is easy to show that there are more languages than there are grammars; hence some languages cannot have grammars even in this weak sense. Note that this does not contradict our statement in §59.2 that any system of phase structure can be reduced (by Reduction 2) to a relation \( R_2 \). What we have stated now is that in certain cases the relation \( R_2 \) will be infinite in extension. In the same way we can show that the answers to (31c) and (31d) are negative; not every system of phrase structure can be reduced to a proper linear grammar, and not every language has some proper linear grammar.
What we have just asserted can be restated as follows:

(i) Reductions 1 and 2 can always be carried out, but the result may not be a finite grammar.

(ii) It is not always possible to carry out Reduction 3 (to a proper linear grammar), since a proper linear grammar must, by definition, be finite.

It is also the case that:

(iii) Even if Reductions 1 and 2 yield a finite grammar, when applied to a given system of phrase structure, it is not necessarily the case that Reduction 3 can be applied; i.e., this finite grammar may be so complex that it cannot be given as a proper linear grammar.

Our major effort, from this point on, will be devoted to exploring the various consequences of this situation. To anticipate a later development, we will see that while it is probable that Reductions 1 and 2 will always give a finite grammar in the case of actual languages, at the same time there is good reason to believe that this grammar will be so complex as to be almost useless, if we require that a constituent analysis (i.e., a P-marker) be provided directly for every grammatical utterance. Furthermore, it will appear below that English phrase structure, in particular, is not amenable to Reduction 3 (not reducible to a proper linear grammar) if the italicized condition is met. This state of affairs is indicative of a serious deficiency in our theory of linguistic structure. It will serve as one of many arguments for rejecting the requirement that a P-marker must be provided directly for every grammatical utterance. This failure to meet optimality requirements (cf. [4.2.1], 50.1 — Def. 261) will thus provide a systematic motivation for the development of a transformational analysis as a part of linguistic theory. We return to these questions in the Appendix to this chapter (7.4.7) and again in the following three chapters.

50.2.4. In the last section we asked whether there is a phrase structure and a grammar for any choice of Gr(P). Now we turn to the second part of question (31), as
this question was broken down in the second paragraph of §50.5.3. That is, we now ask whether the mapping \( \mathcal{P} \) which converts \( \mathcal{P} \)-markers into grammatical utterances (and hence, indirectly, converts strings of \( \text{Gr}(P) \) into grammatical utterances) can be represented by a proper linear grammar.

In §50.1 we posed the problem of developing a method for converting any given model of the structure \( \mathcal{P} \) into a grammar of the form described in chapter III, which can be evaluated in terms of its complexity. If this can be done, then we will be able to derive the strings of \( \text{Gr}(P) \) from the representation **sentence**. Assuming that we can derive such a grammar from lower levels, in the manner touched on in chapter V, we are also able to derive strings of phones from strings in \( \mathcal{M} \) (cf. §42). But there remains one gap to bridge before we can construct derivations of strings of phones from the representation **sentence**, thus completing the program of grammar construction laid down in chapter III. We must show that the strings of \( \text{Gr}(P) \) which are derived in \( \mathcal{P} \) can be converted mechanically by a sequence of conversions of the form \( \alpha \rightarrow \beta \) into the strings in \( \mathcal{M} \) from which lower level derivations begin (i.e., the set \( \mathcal{M} \)). This is essentially the problem of characterizing the mapping \( \mathcal{P} \). If we could define this mapping by a sequence of statements of the form \( \alpha \rightarrow \beta \), then this sequence of statements could be placed after the sequence of statements that forms the grammar of \( \mathcal{P} \), and before the sequence that gives the lower level grammar, and
the program would be completed. But we will find in investigating English syntax that this mapping cannot be so described unless we develop an inordinately complex system of phrase structure. If we try to develop a simple system of phrase structure for the language, we will find that the mapping $\mathcal{F}$ will have to perform certain reorderings of morphemes, and will have to assign specific values to certain non-heads $\mathcal{IX}(\mathcal{X})$ of a string $\mathcal{X}$ of $\text{Gr}(\mathcal{P})$, in a manner which requires knowledge not only of the actual shape of $\mathcal{X}$, but also of its past 'history', i.e., of the manner of its derivation from Sentence. As we have seen, this is equivalent to the requirement that we know the constituent analysis of $\mathcal{X}$ in order to convert it from a string of $\text{Gr}(\mathcal{P})$ to a sequence string of morphemes. And this is just a way of rephrasing the fact that the domain of $\mathcal{F}$ (i.e., the set of $\mathcal{P}$-workers) cannot be taken as strings of $\text{Gr}(\mathcal{P})$, but must be developed in the way we have described above. We will see, then, that if we are to achieve a reasonably simple grammar, then no sequence of rules of the form $\alpha \Rightarrow \beta$ will be sufficient to state the mapping $\mathcal{F}$, because these rules do not take into account the way in which $\mathcal{X}$ was developed from Sentence.

We see then that our formulation of the structure of a sentence in chapter III was incomplete, and that there remains at least the problem of describing $\mathcal{F}$, that is, of showing how a string of $\text{Gr}(\mathcal{P})$ with a given phrase structure can be converted into a grammatical string of morphemes. We discuss this somewhat further in \textsection50 below, but the real solution for this problem will appear as a special case in the development of transformations. (cf. )

50.6. To recapitulate, we have in \textsection50 developed a process of reduction whereby a model of the system $\mathcal{P}$ (assigned to a given language as its phrase structure) can be transformed into a grammar, which can be evaluated with respect to its simplicity by determining its length under the 'notational transformations' of chapter III. This reduced grammar will be a sequence of instructions of the form $\alpha \rightarrow \beta$. Given the grammar we can construct the relation $\mathcal{R}_2$ and one derivation from each equivalence class of derivations. The other derivations can be mechanically
recovered, \( x \) so that we \( x \) are able to determine the relation \( \gamma \) which holds between consecutive steps of restricted derivations. Th. 16 of \( \S \) 42*8 tells us that this determines \( \gamma \). Thus, given a grammar derived from a system of phrase structure by reduction, the underlying system of phrase structure can be uniquely recovered. A model for grammatical statement and the abstract structure associated with it must be related in this manner. I think that these comments are sufficient to indicate once again that the study of the relation between the general theory of linguistic structure and the particular form in which a grammar is presented is an important one, and that this relation must be given quite a careful analysis, at least if features of the grammar (such as simplicity) are to be taken into consideration in validating grammars.

In \( \S \) 50.5 we have found that the account given in the preceding paragraph somewhat overstates the case. Several important problems remain unresolved which question seriously the adequacy of the theory \( x \) in its present form. For one thing, we cannot tell by inspection of a grammar whether or not it is in fact a grammar of phrase structure. Secondly, it is not possible to carry out the reduction to a grammar in the case of any possible language. In fact, we will see below that the reduction to a proper linear grammar (Reduction 3), cannot be carried out in the case of English, and that although reduction to a finite (unordered) grammar may be possible, it will apparently be too complex to be of much interest, if we attempt to cover \( x \) sentences of other than the simplest construction. Finally we have asserted (and will see below, in greater detail) that if we \( x \) hope to find a simple enough system of phrase structure, we will apparently not be able to state the mapping \( \gamma \) in a grammar of the contemplated form, so that there is a gap between phrase structure and lower levels. We return to these questions in the appendix to this chapter( ), and the greater part of the following three chapters \( x \) will devoted to investigating these inadequacies in greater detail and exploring ways to remedy them.
In the light of the connection now revealed between the system $F$ and the form of grammars studied in chapter III, we can state a final condition on $F$, thus arriving at an evaluation procedure for constituent analysis. Suppose that we have a corpus of utterances which has been *analyzed* in terms of the system $G$ and projected to a set $G_1$ of first order grammatical sentences in the manner discussed in chapter IV.\(^{28}\) We now construct a system of phrase structure (i.e., a specific model of the system $F$) with the property that every string of $G_1$ becomes the image under $\mathcal{C}^F$ of some $F$-marker, in other words, with the property that each string of $G_1$ receives a constituent analysis. We further require that no string shorter than the longest string in $G_1$ receives a $F$-grammatical $F$-marker.\(^{29}\) In the manner which we have just outlined, \(\varepsilon\) we next reduce this system of phrase structure to a grammar, i.e., a sequence of conversion statements of the form "\(E(\gamma \rightarrow \delta)\)" (read "it is obligatory to rewrite $\gamma$ as $\delta$") or "\(P(\delta \rightarrow \gamma)\)" (read "it is permitted to rewrite $\delta$ as $\gamma$"). We then evaluate this grammar in the manner of chapter VII. That is, we investigate the simplicity of the grammar, understood as the amount of generalization it provides, and measured in terms of the possibility of compression with selected notational devices. The best
constituent analysis will be the highest valued system having the structure of $P_\frac{1}{2}$ and related in the proper way to $G_1$. Each interpretation of $P_\frac{1}{2}$ for $G_1$ projects $G_1$ to a new set, and if recursive production is permitted, this projection may be infinite.

This account requires two qualifications. First, the order of descriptive procedures outlined has no theoretical significance in our terms, as we have emphasized above. This is simply a statement of compatibility among levels. Second, the interpretation of $P_\frac{1}{2}$ is not evaluated by itself, but conjointly with the partial grammars derived from other levels. That is, complication of $P_\frac{1}{2}$ may be permitted if this leads to much simpler statements on other levels.

Although this account is schematic in parts and contains certain gaps, it is possible to proceed to investigate its empirical consequences in construction of actual grammars. In fact, the morphophonemic study of Appendix, chap. V is an instance of a grammar constructed along these lines, and containing a partial validation. I.e., it is shown that grammar that no interchange of the order of the statements of that grammar can give a simpler grammar. In chapter VII we will consider English phrase structure from this point of view.

To the question: what is ultimately involved in the proper choice of $P_\frac{1}{2}$, we have given the general answer: the total simplicity of the grammar for which $P_\frac{1}{2}$ provides the underlying phrase structure. More specifically, the more features of the language that can be stated in terms of
the elements of $P$, the simpler will be the grammar which
underlies, since these features will not have to be
independently stated. Thus each such feature can be taken
in a loose sense as a criterion for $P$. This is actually a
loose sense, because the only ultimate criterion for $P$ is
the total simplicity of the grammar. Simplicity is a
measure for a whole system. If these features appear on
other levels than $P$, we have independence of levels, with
$P$ being 'validated' in terms of these other levels. We
will see below that $P$ is not independent of the 'higher'
level of transformations.

The following features appear to be criteria for $P$ in
English, in this loose sense. (I.e., these features are
all stateable in terms of roughly the same set of elements,
and this set of elements in fact satisfies the axiom system
for $P$. Hence $P$ might as well be taken as having these
elements as its 'vocabulary'.)

1. The rule for conjunction
2. Intrusion of parenthetical expressions
3. Ability to enter into transformations
4. Certain intonational features.

We will investigate these and other 'criteria' for $P$ in
the next chapter. After having developed transformational
analysis and applied it to English, we will see that the first
two features mentioned, and many others, are special cases
of these, so that only three and four remain.

There are many other notions which must be developed
in $P$, and although I do not know how to do justice to them,
I think it might be profitable to at least suggest how they might fit into the system which has been presented, if only as a sketch of some of the remaining difficulties. Chief among these are the related set of notions 'grammatical relation' (e.g., actor-action or subject-predicate, verb-object, etc.), 'head', 'modifier', 'coordinate construction', 'subordinate construction', etc.

The theory of grammatical relations can be based on a set of functions that might be called 'selectors'. These are functions which, for each prime \( P_1 \), select a unique occurrence of some prime \( P_2 \) in each \( X \) directly represented by \( P_1 \).

**Def. 29.** \( f \) is a selector in \( P \) if and only if for all \( P_1, X \), there is a unique \( P_2, Z \), such that

\[
f(P_1, X) = (P_2, Z),
\]

where (i) \( Z \) is an occurrence of \( P_2 \) in \( X \)

(ii) \( P_1 \not\in U \leftrightarrow \exists_1(P_1, X) \leftrightarrow Z \not\in U\)

**Def. 30.** \( F \) is a complete set of selectors for \( P \) if and only if

(i) \( f \in F \Rightarrow f \) is a selector in \( P \)

(ii) for all \( P_1, P_2, Z, X \),

if \( f(P_1, X) \) and \( Z \) is an occurrence of \( P_2 \) in \( X \), then there is a unique \( f \in F \) such that \( f(P_1, X) = (P_2, Z) \).

Thus a complete set of selectors would enable us to assign a unique function \( f \) to each occurrence of \( P_1 \) in a string \( X \), if we know to what kind of
constituent this occurrence belongs. Given \( NP^0VP \), and \( f_1(Sentence, NP^0VP) \), there would be a certain \( f_s \) associated with \( NP \), and a certain \( f_o \) associated with \( VP \). In terms of \( F \) we can investigate the logical status of such notions as subject, object, etc., given a prior account of grammatical relation.

**Def. 31.** A **grammatical relation** (or **construction**) is an ordered pair \( (P_1, X) \) such that \( s^1(P_1, X) \).

Thus, \( subject \rightarrow predicate \) is the grammatical relation \( (Sentence, NP^0VP) \), and \( verb \rightarrow object \) is the grammatical relation \( (VP, VP^0NP) \). Note that under this formulation, a string can enter into various grammatical relations. Another approach would be to define a grammatical relation as an ordered \( n \)-tuple of the primes that form a constituent; e.g., to define \( subject \rightarrow predicate \) as the grammatical relation \( (NP, VP) \).

The many unanalyzed problems in this area of syntax make it difficult to find a crucial instance to decide between these formulations, but it seems likely that the extra generality given by **Def. 31** will be required for descriptive adequacy.

**Assuming grammatical relation** to be defined, we can explain each of the notions \( subject \), \( object \), etc., as an ordered pair of a selector \( f \) and a grammatical relation \( Q \). Given the grammatical relation \( (P_1, X) \), and the selector \( f \), \( f(P_1, X) \) will be a certain prime of \( X \), the subject of the grammatical relation, if this is the \( subject \rightarrow predicate \) relation and \( f = f_s \) (which might, for instance, be the selector that selects the first prime of a two-prime string, or the first head (cf. below), etc.).
To give further content to this area of syntax, it is necessary then to enter into the construction of a set $F$.

2. Given that $\{P_1, P_{i1} \ldots P_{in}\}$, we can distinguish three interesting cases:

(i) $P_1 = P_{i1}$, for some $i \leq n$

(ii) $n = 1$

(iii) neither (i) nor (ii)

Case (i) is, essentially, the case where in Harris’ Morpheme-to-Utterance characterization of constituents, the superscripts are not raised. Case (ii) amounts to giving the membership of a class. We might give names to these cases as follows:

Def. 32. Given a grammatical relation $Q = (P_1, P_{i1} \ldots P_{in})$, if $Q$ is:

an instance of case (i), (25), then $Q$ is a recursion of $P_1$;

(ii), (25), then $Q$ is a listing of $P_1$;

(iii), (25), then $Q$ is a partition of $P_1$.

Since $\{\}$ is irreflexive, (ii) and (i) are exclusive. Hence these are three distinct cases.

The notion of 'head of a construction' is somewhat ambiguous. In one sense, by a head of $P_1$ is meant a prime that appears in every development of $P_1$. In the case of a recursion, we might say that the head is $P_1$ itself.

Def. 33. $\text{Hd}(P_1, P_1)$ if and only if (i) for all $X$, if $(P_1, X)$ is a grammatical relation but not a recursion, then $P_{\text{prime}} \subseteq X$, where $P_{\text{prime}}$ is prime, or (ii) there is an $X$ such that $(P_1, X)$ is a recursion and $P_1 = P_1$. 
In terms of this notation, we might suggest an extension of the axiom system for $\mathcal{F}$.

**Ax.8.** Every partition has a head. I.e., if $(P_i, X)$ is a partition of $P_i$, then there is a $p^X$ such that $\text{Hd}(p, P_i)$.

This condition would rule out such obviously inadequate interpretations for $P_i$ as that offered below, where $P_i$ would have the $\beta$ elements and the primes of $P_i$.

**Ax.9.** The heads of $P_i$ (excluding $P_i$) stand alone in some fixed and unique order in a partition of $P_i$. I.e., the heads of $P_i$ (excluding $P_i$) can be ordered in a fixed and unique way as $P_i, \ldots, P_n$ such that

$$\mathcal{F}_1(P_i, P_i^{\wedge \ldots \wedge P_n})$$

**Def. 24.** In this case, the grammatical relation $(P_i, P_i^{\wedge \ldots \wedge P_n})$ is called a coordinate construction.

If there is any partition of $P_i$, then **Ax.9** guarantees the existence and uniqueness of a coordinate construction for $P_i$.

We can now define the $k$th head of a grammatical relation. There are several ways open, depending on how we interpret the $k$th head. We might mean simply the $k$th occurrence of a head in a given partition, or, alternatively, the occurrence of the element in the given partition which is in fact the $k$th prime in the coordinate construction guaranteed by **Axiom 9**. In this case, the $k$th head of one partition of $P_i$ will always be identical with the $k$th head of another partition of $P_i$, but this is not necessary in the former case. If we choose the latter formulation, **Ax.9** is crucial; with the former version, we can dispense with this axiom. I see no
way to choose between these alternatives. Continuing
arbitrarily with the former, we have

Def. 35. $H_k(P_1, X) = (P_q, Z)$ if and only if

either (I) $I(P_1, X)$ and there is a $Z_1, \ldots, Z_n, P_1, \ldots, P_q$

such that (i) $1 \leq k \leq n$

(ii) $Z_1 \prec Z_2 \prec \ldots \prec Z_n$

(iii) $P_k$ is an occurrence of $P_1$ in $X$

(iv) $P_1, \ldots, P_n$ are exactly the heads

of $P_{i_1}$

(v) $P_q = P_k$ and $Z = Z_k$

or (II) $P_q = Z = U$

Th. 35. $H_k$ is a selector, for $k \geq 1$.

Analogously, we can define $k^{th}$ subordinate (non-head) of a
grammatical relation.

Def. 36. $S_k(P_1, X) = (P_q, Z)$ if and only if

either (I) $I(P_1, X)$ and there is a $Z_1, \ldots, Z_n, P_1, \ldots, P_q$

such that (i) $1 \leq k \leq n$

(ii) $Z_1 \prec Z_2 \prec \ldots \prec Z_n$

(iii) $Z_1$ is an occurrence of $P_1$ in $X$

(iv) $P_1, \ldots, P_q$ are exactly the non-head

primes of $P_{i_1}$

(v) $P_q = P_k$ and $Z = Z_k$

or (II) $P_q = Z = U$

Th. 36. $S_k$ is a selector, for $k \geq 1$.

Given any ordered pair $(P_1, X)$, we can find the $k^{th}$ head and
The $k$-th subordinate of this pair, and these will be units just in case either $(F_k, X)$ is not a grammatical relation, or it is a grammatical relation but contains fewer than $k$ heads or $k$ non-heads, respectively. In terms of $S_b_k$ we should be able to define subordinate construction, and perhaps modifier. But it is not clear to me when a non-head is properly called a modifier, and whether the grounds for this are formal.

Finally, we have

$$Th. \mathcal{F} = \{H_d_k | k \geq 1\} \cup \{S_b_k | k \leq 1\}$$

is a complete set of selectors for $P_k$.

Returning to the end of §5.2.1, let $Q$ be the grammatical relation $\text{subject} \rightarrow \text{agent} \rightarrow \text{actor} \rightarrow \text{action}$. Then $Q$ is the ordered pair $(\text{Sentence, NP}' \text{VP})$. The notion of 'subject' can be understood as the ordered pair $(H_d_1, Q)$, and $H_d_1(Q) = \text{NP}$ is the subject of the grammatical relation $Q$.

This discussion only scratches the surface. There are many ways in which it could be generalized or altered completely. In particular, the definitions are fairly arbitrary in that the terms defined are often used in quite different ways. The additional axioms have only the justification that they give a somewhat more organized picture of constituent structure. But of course this is not a sufficient motivation. There are other ways in which the system could have been elaborated. The difficulty here is that not enough detailed work is available on constituent structure of actual languages for us to be able to determine what the effect would be on the grammar of these languages if one
or another conception is applied. Hence there must be quite a large element of speculation even in such skeletal theoretical work as this. It can do little more in this instance than suggest problems and gaps empirical study might direct itself.

To parse a sentence is essentially to recover its \( P \)-marker. We know that this may not always be possible, since a string may have several \( P \)-markers. It is interesting to see what further information is needed about a string to enable its \( P \)-marker to be recoverable. Or to pose a more specific problem, is there some set \( L \) of elements such that for any string \( X \), which, we are assured, is in \( \mathfrak{S}t(P) \), if an element of \( L \) is assigned to each occurrence of a prime in \( X \), then a \( P \)-marker of \( X \) is uniquely determined by this sequence of elements of \( L \). Furthermore, we can investigate the possibility of constructing a mechanical procedure for recovering the \( P \)-marker from the sequence of elements of \( L \).

Though this problem is still not a clearly defined one (since there are trivial solutions), the method of construction of \( P \)-markers has incidentally shed some light on what might be an interesting solution. We know from the remarks leading to that construction that \( c \)-traces will not serve as the set \( L \) (nor, of course, will \( p \)-traces). However the set of pairs \( (\Sigma, Q) \), where \( \Sigma \) is a \( c \)-trace and \( Q \) a \( p \)-trace, can serve as such a set \( L \). We might approach this problem in a somewhat different way with selectors and the notion of grammatical function derived from them as a starting point. If the problem can be more clearly defined, we might investigate the possibility of recovering \( P \)-markers from an
assignment of elements of $L$ only to certain terms in $X$. This question deserves more serious consideration. It is connected with the interesting problem of 'operational grammar' (cf. §2.1, and fn. 2, chap. I).

It is interesting to investigate the various kinds of constructional homonymity that can arise in $P$. We have a real case of constructional homonymity when several $P$-markers are mapped by $\mathcal{F}^P$ into the same grammatical string in $W$. We cannot quite discuss this case yet, since we have not yet considered the problem of how constituent structure is carried over under mappings (this will be an important part of the discussion in chap. VIII, cf.). But we can discuss a closely related problem, namely, the case in which several $P$-markers are assigned to a given string in $G(P)$, i.e., the case of several $P$-markers with a common product (cf. Def. 15, §4A.2). Two simple and interesting cases of homonymity are these:

(I) A string may be broken up into constituents in two different ways.

(II) A string may be broken up into parts in only a single way, but those parts may be constituents of different sorts, i.e., they may be represented by different primes.

Examples of both kinds can be found in English. The cases in 4.1.2 ("old men and women", etc.) are instances of (I). As an instance of (II) consider

Flying planes can be dangerous
Here "planes" can be the object of "fly", or "flying" can be an adjective modifying planes. (32) In both cases the constituent analysis is "flying planes -- can be dangerous". But in one case the noun phrase subject is an instance of Adjective Noun (hence in this case a plural noun phrase), and in the other, an instance of some form of Verb Noun (forming, in this case, a singular noun phrase).

In Two Models, Hockett utilizes cases of homonymy to demonstrate the independence of various syntactic notions, in particular, in §3.2, the independence of construction from form and order. Recapitulating his argument, if our system permitted only case(I) types (difference of constituent boundaries) or case (II) types (difference of construction), it would necessarily be inadequate, as these examples show. (33)

The distinction between (I) and (II) is related to the fact that P-markers are characterized by c-traces and by p-traces. A 'pure' instance of case (I) would be a case in which the P-markers of X assign the same c-traces to elements but different p-traces. 'Pure' instances of (II) arise when the c-traces differ, though the p-traces are identical. Both types are possible. The analysis we have suggested here (but cf. fn. (32)) for (27) is an instance of a pure case of (II). The cited instances in §3.2 of (I) may, however, be mixed cases in which both p-traces and c-traces differ, depending on just how the analysis is set up (but cf. §4.2, last paragraph).

57. In §3.2 we saw that considerations of descriptive adequacy
require that $P$ be non-restricted for English. In particular, non-restriction is necessary if "John", "the rearming of Germany", "a man", and "several men" are all to be NP's. Such considerations can be extended, leading to further requirements on $P$.

The notion of non-restriction is reminiscent of the notion of degree of grammaticalness of chapter IV. There it was observed that, e.g., "horse" and "justice" are both nouns, although they are rarely or never substitutable. In the case of "horse" and "justice" we have now repeated this observation, pointing out that they are both NP's, although perhaps never substitutable as NP's. We note then that in certain cases (when the phrases in question consist word by word of members of the same syntactic category), the notions of partial grammaticalness and non-restriction coincide. For such cases we can state the requirement of non-restriction axiomatically as a condition on $P$, in terms of the notion of *optimal analysis* introduced in §32. This axiom will state a condition of compatibility between $P$ and $Q$.

That is, we require that for each optimal string $\alpha$ of $Q$, there must be a corresponding prime $P_{\bar{\alpha}}$ of $P$ such that a word $w$ is in $\alpha$ (i.e., $g(\alpha, w)$), in terms of $\overline{\alpha}$, just in case on the level $P_{\bar{\alpha}}, f(P_{\bar{\alpha}}, w)$. But the elements of $Q$ do not literally occur in $P$. Nevertheless, we know that each $w$ in $\overline{\alpha}$ can be analyzed in terms of its morphological heads and affixes, i.e., it is associated with a certain string $\overline{\alpha}$ such that $\overline{\alpha}(m) = w$. And we have described a relation $\gamma$ in $\overline{\alpha}$ between $P$ and $Q$ that comes close
enough to embedding $\overline{M}$ in $\overline{P}$ for our present purposes. Thus we can give this condition of compatibility on $\overline{P}$ and $\overline{G}$ as follows:

**Ax. 10.** For each $q$ in the absolute analysis of $\overline{G}$, there is a prime $p$ in $\overline{P}$ such that $q(p, q)$ if and only if there is an $m$ and a $q$ such that $f(p, q)$, $\gamma(q, m)$, and $\overline{f}(\overline{m}) = \overline{w}$.

This requirement and the motivation for it can be rephrased in the terms of chapter IV (cf. § 32). Suppose $Y$ and $Z$ are instances of the same structural form, i.e., of a single sequence of classes. Then we would like to require that $X$ and $Y$ are phrases of the same sort. But the difficulty remains that 'same structural form' is a notion relative to degree of grammaticalness, and any two strings (of the same length) are of the same structural form at least on the lowest level of grammaticalness. Thus this requirement has no significance unless we fix an absolute sense to 'same structural form', as we have in fact done in § 32 by defining the absolute analysis. In particular, suppose that Adjective and Noun are classes in the absolute analysis, but that on the level of first order grammaticalness, we have subclasses $A_1$ (containing "green") and $A_2$ (containing "apparent") of adjectives, and subclasses $N_1$ (containing "grass") and $N_2$ (containing "failure") of nouns. Suppose further that in $P$ (without Ax. 10), the simplest grammar would have $f_1^P(NP^\ldots, A_1^\ldots N_1^\ldots)$ and $f_1^P(NP^\ldots, A_2^\ldots N_2^\ldots)$. 
Then we would not have the information in $P$ that "green grass" and "apparent failure" have the structural similarity that they are both $\text{Adjective} \text{Noun}$. It would certainly be a defect in the grammar if we did not know that "green grass" and "apparent failure" have the same phrase structure in a sense in which "green grass" and "the man" do not. This difficulty is avoided by $\text{Ax.10}$. Note that if the supposition of $\text{4.32}$ that the $\text{P} \text{N} \text{E}$ analysis cannot be the highest level analysis is correct, then $P$ must be non-restricted, and the example given below $\text{Th.17}$ is impossible. $\text{Ax.10}$, or some similar condition, will play an important role in the development of transformations. Cf.

We see that the requirement that $P$ must meet $\text{Ax.10}$ might lead to some complication of the grammar, as in the case just given. This might be avoided by reinterpreting $\text{Ax.10}$ as a further reduction in $P$ (cf. $\text{4.32}$). Then given the optimal classes, we can leave the grammar in the simplest form and require that $\text{Ax.10}$ be met in the reconstruction of $P$ from the grammar. There does not seem to be any particular problem in giving this formulation, and we omit it here.

We have given no formal requirement on $P$ from which it follows in a similar way that the more general case (e.g., of "John" and "the rearming of Germany", etc.) will be satisfactorily dealt with, nor do I see how one could be given. Our expectation is that the requirement of simplicity (cf. $\text{4.32}$) will compel the adoption of a phrase structure in which this more general case is also handled adequately. If detailed investigation of English syntax will show that this is not the case, then some such formal requirement
will have to be sought.

It remains to specify the relation between \( P \) and \( W \), that is, to axiomatize \( \Phi^P \). We have stated a very weak condition on a relation \( \gamma \), requiring that it be a concatenation-preserving relation between strings in \( P \) and strings in \( W \), relating primes to primes or units (cf. condition (1)). We must now relate \( \Phi^P \) to \( \gamma \) and complete the characterization of \( \Phi^P \). For expository convenience we will speak of \( \gamma \) too as a mapping (many valued) of \( P \) into \( W \), permitting ourselves the notation \( \gamma(p) = w \) for \( \gamma(p,w) \).

The general requirement (cf. \( \S 10,11 \)) is that the mappings of each level carry a certain set of markers of that level into the set \( \mu^W \) of grammatical strings of words, not necessarily exhausting this set (which is in turn mapped into grammatical utterances). We have already derived the product \( P \)-markers as the sets of strings of equivalent restricted \( s_1 \)-derivations. Recall that \( \text{Gr}(P) \) is exactly the set of products of restricted \( s_1 \)-derivations (Th. 2, \( \S 42,2 \)).

Thus we have

\[ \text{Ax. 16: If } x \in \text{Gr}(P), \text{ and } x \text{ is the product of the } P \text{-marker } \Pi, \text{ then } \Phi^P(\Pi) \in \mu^W. \]

Another general requirement (cf. \( \S 11,12,13 \)) is that the mappings of a level be single-valued. We have still not settled the domain of \( \Phi^P \). We know that it includes \( P \)-markers. A \( P \)-marker is a certain class of strings containing exactly one string in \( P \). We have seen that an interpretation of a string \( x \) in \( P \) is given by a class \( K \) containing \( x \) even when \( K \) is not a \( P \)-marker. It seems reasonable, then, to take as
the domain of \( \Phi^P \) sets \( K \) containing a single string in \( \mathcal{P} \), or more generally, ordered pairs of a set \( K \) and a string \( X \in \mathcal{P} \) contained in \( K \). Thus we add

Ax. \( \Phi_P \). The domain of \( \Phi^P \) is the set of pairs \( (X,K) \), where \( X \) is a string in \( \mathcal{P} \) and \( K \) is a set of strings containing \( X \). \( \Phi^P(X,K) \) is a unique string in \( \mathcal{M} \).

We have seen that \( \Upsilon \) carries strings in \( \mathcal{P} \) into strings in \( \mathcal{M} \). As a step towards relating \( \Phi^P \) and \( \Upsilon \), we note that \( \Phi^P \) can be subdivided into two components, a mapping \( \Phi_1^P \) into strings in \( \mathcal{M} \), and a mapping \( \Phi_2^P \) of strings in \( \mathcal{M} \) into strings in \( \mathcal{M} \). \( \Phi_2^P \) is then a subpart of the mapping \( \Phi^M \), which is defined on the level \( \mathcal{M} \). \( \Phi_2^P \), in other words, is an operation of placing word boundaries in strings of morphemes.

Ax. \( \Phi_1 \). \( \Phi^P(X,K) = \Phi_2^P(\Phi_1^P(X,K)) \), and \( \Phi_1^P(X,K) \) is a string in \( \mathcal{M} \).

We disregard for the time being the fact that the notion of compound mapping has no clear sense as yet.

The simplest way to relate \( \Upsilon \) and \( \Phi_1^P \) would be to require that \( \Phi_1^P(X,K) \) be one of the values of \( \Upsilon(X) \). But this we cannot do, since \( \Phi^P \) is not in general a concatenation preserving mapping. This complication of the mapping is necessitated, for one reason, by the possibility of discontinuous phrases. E.g., if we are to develop anything like a reasonable phrase structure for English, the sentence "I called him up" will contain the same verbal element (the compound verb phrase "called up") as does "I called up my friend". Both sentences will be represented by \( NP^P VP^P NP \), where \( VP \) represents "called up". So if we wish to retain a one-dimensional system of representation, we must pay for it by complicating the
mappings, or, alternatively, by a considerable complication in the characterization of $\xi$ (if this would indeed be possible). We will see in the analysis of English in chap. VII that there are many other such cases, particularly in the analysis of the auxiliary verb.

We can however, make this weaker requirement.

$$\text{Ax. 6d. } P^P(x, K) \text{ is a permutation of the primes of one of the values of } \gamma(x), \text{ where } \gamma \text{ meets condition (1).}$$

56.2 There is actually no interest in $\gamma$ as such, and we can consider it eliminated in Ax. 1d. by condition (1). It was introduced above mainly to bring out more clearly the nature of $P^P_1$. The excess specificity of $P^P_1$ over $\gamma$ comes from the fact that $P^P_1$ takes into account the interpretation of the string $X$, not just the string itself. In the case of $X \in \text{Gr}(P)$, this means that $P^P_1$ can be phrased in terms of the full constituent structure of $X$, and the whole 'history' of its development. The function of this added information can be seen by comparing $\gamma$ and $P^P_1$.

(i) $\gamma$ cannot be single-valued. Thus we may have to represent a long component of agreement in number, for instance, by assigning to the verb an element $\text{comp}$, which is mapped by $\gamma$ into either the morpheme singular or the morpheme plural, depending on which of these morphemes occurs with the preceding Noun Phrase. In the grammar of English presented below (cf. , chap. VII) just this situation occurs. While in this case it would have been possible, with little extra complexity, to handle agreement directly in the grammar, in the case of a language
with a more complex componential structure (e.g., Modern Hebrew, as presented in the appendix to chap. VII) this would lead to great additional complexity. \( \Phi^P_1 \) can be single-valued, however, since knowing the constituent structure, we can determine the morphemic form that must be assumed by the long component in other parts of the sentence without ambiguity.

(ii) \( \gamma \) must preserve concatenation, since the exact manner of the splitting of a discontinuous constituent is determined in general by the constituent structure of the remainder of the sentence. \( \Phi^P_1 \), of course, is not limited in this way.

(iii) We cannot require that the image under \( \gamma \) of a string from \( \text{Gr}(P) \) be a unique member of \( \text{M}^W \) (i.e., a unique \( \text{M} \)-marker). Thus not only is \( \gamma \) many-valued, but it may even associate a string \( x \in \text{Gr}(P) \) with several strings in \( \text{M}^W \). How the mapping takes place may depend on the constituent structure of \( X \). I have not found any instances of \( \text{M}^W \), and it may be that it should be excluded axiomatically, by adding another clause to condition (1).

56.3. We can further subdivide \( \Phi^P_1 \) into a series of steps. This is necessary in practice because of the immense complexity of this mapping when it is stated in detail. Thus we regard \( \Phi^P_1 \) as a compound transformation \( \Phi^P_1 \Phi^P_1 \ldots (\Phi^P_{-1}) \ldots \).

There are a variety of problems left unresolved in this discussion. We have not given an explicit statement of how a transformation takes into account the constituent structure of a string. And we have not shown how the actual
statement of the mapping can be given in the grammar. We have not explained what a compound transformation might be. In fact, this is an impossible notion in our present formulation, since the argument of a mapping is a pair \((X, K)\), and its value is a string \(X^i\). Finally, we have not explained in what sense the result of a mapping, the string of words which is its value, can be said to have a constituent structure. This elaboration is certainly necessary. We must be able to refer to the noun phrases, verb phrases, etc., of sentences given in words.

We will leave these problems unanswered now. We will be able to characterize \(P^\text{DP}\) more explicitly below, when the requisite notions will have been developed for transformations.
Footnotes - Chap. VI.

(1) (p. 190) Methods, chap. 16.

(2) (p. 190) Immediate Constituents.

(3) (p. 191) In Harris' system, these include the rules in which the same superscript appears on both sides of the identity sign, but not only these.

(4) (p. 191) Meaning in Linguistics, From a Logical Point of View, p. 54.

(5) (p. 192) We must be careful about permitting generation of sentences of length $\leq n$ by this method. Otherwise, the simplest grammar will simply state that all word sequences are grammatical sentences. This absurdity is avoided if we insist that none of the new sentences generated on the level $P^n$ be of length $\leq n$, i.e., that these grammatical sentences already be exhausted by the methods of chapter IV. No doubt some weaker condition can be found which will also exclude this absurd result, but it must be remembered that some condition is necessary.

(6) (p. 198) Cf. § 13.1 for definition of this notation.


(8) (p. 200) But Cf. § 46.

(9) (p. 200) As shown by pair test with two Philadelphia speakers. Though it is possible to say these so that stress and juncture differ, the pair test showed no identification in normal speech.

(10) (p. 201) This follows from Harris' treatments of constituent analysis, though I do not state it explicitly.

(11) (p. 201) Cf. § 10, § 6.3

(11') (p. 202) As an aside, it should be reemphasized that the consideration of intuitive adequacy is perfectly in place here, since we are now trying to characterize pre-systematically the structure of an adequate constituent system, but it will have to be avoided, of course, when we consider the method of finding or evaluating an interpretation for the formal structure which we are now trying to develop for a given language. This method of discovery or evaluation will have to be such that the cases of constructional homonymity given above will turn out to have
two analyses in the highest valued grammar.


(13) (p. 209) Where matched parentheses might, e.g., be marked with the same subscript.

(14) (p. 211) I.e., $P_{n}^{n} \ldots P_{n}^{n}$ is an occurrence of $P_{n}^{n}.$

An occurrence of $X$ has been defined as a string ending in $X,$ and
our formal work must always use 'occurrence' in this sense.
But in informal exposition the word 'occurrence' can be
understood in its normal sense, as if an occurrence of $X$ in
$Y$ were one 'token' of the type of $X$ in $Y.$


(16) (p. 222) Cf. §32 and §57. The fact that $P$ is non-restricted
for English will play a certain role in establishing transformations
in English, see below.

(17) (p. 226) I.e., it limits the possibility of having the
same superscript on both sides of the identity sign in Harris'
treatment of phrase structure.

(18) (p. 230) For a specific and fairly complex instance of such
a system, cf. Appendix to chap. VII.

(19) (p. 230) Cf. references in fn. 13, chapter II.

(20) (p. 231) It is not necessary in general to distinguish
between derivations in the sense of chapter III and $\delta$-derivations.

(21) (p. 233) We will often drop the concatenation sign where
there can be no ambiguity.

(22) (p. 235) We have seen general reasons why this is the case
in §21.2. The appendix to chap. V shows clearly that in practice,
manipulation of order can effect great reduction on the
morphophonemic level. Now we investigate the function of
order in simplifying our characterization of $P,$ citing later
just one of the many ways in which such simplification can
come about.

(23) (p. 236) These problems were introduced in the general
discussion in chapter III in §21.2-3.
(24) (p. 239) We must assume this now, although below we suggest that this is not the case, and that the burden of infinite projection falls on the next higher level.

(25) (p. 242) We will see later that a constituent analysis is provided indirectly for every grammatical utterance.

(26) (p. 247) More properly, $E^M$, because this is what was really analyzed in the morphophonemic study. Cf. §5.2 for the problem involved in analyzing $E^P$.

(27) (p. 247) Put of $E^P$. §5.2.

(28) (p. 250) We might suppose further that we have analyzed $S_1$ in terms of $M$, and have projected to a set $S_1$. Cf. last paragraph of §5.2.

(29) (p. 250) Cf. fn. 5, above. Here too, as in the case of $M$, we omit the discussion of the analogue to §29. That is, actually, just as in applying $G$, we can try to find a method for redrawing the borders of $S_1$ so as to exclude strings that are exceptional from the point of view of phrase structure.

(31) (p. 255) Chap. 16, Methods.

(32) (p. 250) Cf. chap. VII, for a further discussion of this case. The analysis given there makes (27) a case of (I) not (II). In chapter IX, we return essentially to the conception given here.

(33) (p. 260) The fact that our theory permits both types shows that it may be adequate, not that it is adequate. To show this it is necessary to establish that the highest-valued interpretation of the system automatically yields this solution. We will investigate this in subsequent chapters.
Appendix to Chapter VI. On the Range of Adequacy of Various Types of Grammars.

In § 44 we pointed out that one of the motivations for constructing higher levels such as phrase structure was to permit the generation of infinitely many sentences, since the method of projection outlined in chapter IV produced no sentences longer than the longest sentence actually present in the (necessarily finite) corpus. Actually it is not necessary to go to higher levels to achieve this goal. Suppose that the methods of chapter IV have projected the corpus to a set $G_1$ of first-order grammatical sentences, where the sentences of $G_1$ are all less than $n$ words in length, for some fixed $n$. Then we can construct rules of a recursive character framed solely in terms of words which enable us to generate the strings of $G_1$, and no strings less than $n$ words in length which are not in $G_1$ (cf. fn. 5), and we can define "first-order grammatical sentences" more generally as sentences which are generated when we select the simplest grammar of this sort, and let it run on freely. A natural way to construct such a grammar is by considering language to be a Markov process with a finite number of states, where the transition from state to state is marked by the production of a word (for our purposes we disregard transitional probability). We can present a finite grammar for a language produced in this way by stating what transitions from state to state occur, and what word accompanies the transition. A language which can be described in this way we call a "finite state language."

By a "language" we will mean, as before, a set (finite or infinite) of strings, each finite in length, and constructed out of a finite number of symbols (primes). For uniformity of terminology we will describe an arbitrary finite state language as follows. There is a set of states $S_0, \ldots, S_n$. Transition from state $S_i$ to state $S_j$ is marked by the production of the prime $a_{ij}$. It is not necessarily the case that $a_{ij} = a_{kl}$ for $i \neq k$ or $j \neq l$. If $a_{ij} = U$ (the identity element), we say there is no transition between state $S_i$ and state $S_j$. A string $\Sigma$ belongs to the language just in case it is produced by a finite sequence of states $S_{a_0}, \ldots, S_{a_m}$, where $a_0 = a_m = 0$, \ldots.
and where $\sum$ is $n$ primes in length (recall that $U$ is not a prime). As the 'grammar' of such a language we can present the finite list

\[(38) \quad a_0, a_1, \ldots, a_m, a_{n0}, \ldots, a_{nn},\]

which completely characterizes the language.

In this chapter we have discussed a different way of describing languages, through recourse to constituent analysis into strings which are longer than the word, (i.e., phrases), and whose distribution can be discussed as a unit. In §50 we showed how every description in terms of phrase structure can be reduced to a grammar consisting of an initial string Sentence (more correctly, $\# \text{Sentence} \#$, cf. §50.2) and a set of instructions "rewrite $\alpha$ as $\beta$", and furthermore, we saw that the underlying system of phrase structure can be uniquely reconstructed from this grammar.

We have seen that not every such grammar uniquely describes some system of phrase structure. However, in studying the range of adequacy of description in terms of phrase structure it is convenient to make the assumption that every such grammar does correspond to a unique system of phrase structure, and to study the range of such grammars. The discrepancy will not be relevant to our immediate discussion.

We therefore consider grammars having the following form. We have a finite set $\sum$ of initial strings $\sum_1, \ldots, \sum_n$ (in fact, a single initial string will obviously suffice). We have in addition a finite set $F$ of instruction formulas $X \rightarrow Y$, where $X$ and $Y$ are strings, $Y$ differing from $X$ only in that a single prime of $X$ is replaced by some string (possibly a prime) to form $Y$. A grammar will thus have the form

\[(39) \quad (i) \sum : \sum_1, \ldots, \sum_n \]

\[(ii) F: \quad X_1 \rightarrow Y_1 \]

\[X_2 \rightarrow Y_2 \]

\[\vdots \]

\[X_m \rightarrow Y_m \]
where $\Sigma_1, X_1, \ldots, X_n$ are strings of finite length, and for each $j$ there is an $a, b, c, d$ such that $X_j = abcd$, $b$ is a prime, and $Y_j = aab$.

We define the notion of "derivability" as follows:

**Def. 37.** $a$ is derivable from the set $K$ in terms of $F$ if and only if $a$ is a string, $K$ is a set of strings, and $F$ is a set of instruction formulas meeting the condition given above for (3911), and either (i) or (ii) is the case:

(i) for some $b, c, X_1, Y_1, a = b^cX_1^c$, $X_1 \rightarrow Y_1$ is an instruction formula of $F$, and $b^cX_1^c$ is in $K$

(ii) there is a string $b$ such that $a$ is derivable from $\{ b \}$ in terms of $F$, and $b$ is derivable from $K$ in terms of $F$.

The grammar (39) determines two interesting sets of strings. First of all we have the set of strings which are derivable from $\Sigma$ in terms of $F$. Conceiving of $\Sigma$ as a set of axioms and $F$ as a set of rules of inference, the set of derivable strings would be just the set of theorems of the system $[\Sigma, F]$. Any set of strings which is derivable from some system $[\Sigma, F]$ of the form (39) we will call a "derivable language."

Considering systems $[\Sigma, F]$ of the form (39), we note that there may be certain derivable strings from which no further strings are derivable. Any such string we will call a "terminal string" with respect to $[\Sigma, F]$. Given (39), a string $X$ will be a terminal string with respect to $[\Sigma, F]$ if and only if it does not contain any of the strings $X_1, \ldots, X_n$ as a substring. In the interesting cases there will be a set $F$ of primes with the property that none of the primes which are developed in converting $X_1$ to $X_n$ are primes of $F$, and furthermore, every terminal string is a string in $F$. However, this is not necessarily the case. Cf. final remarks of §50.3. The set of terminal strings is the second important set which is determined by the grammar (39). Any set of strings which is a terminal set for some system $[\Sigma, F]$ of the form (39) we will call a "terminal language."

In discussing the theory of phrase structure in chapter VI we have seen that
in the interesting cases, terminal languages are simply languages, i.e., sets of strings of words. And for any terminal language L, the derivable language \( L_d \) of which \( L \) is a subset is essentially the system of phrase structure of \( L \). More precisely, given a string of \( L \) and its derivation, we can (assuming that certain further conditions are met—cf. 50.1-3) determine the constituent structure and the grammatical relations in this string.

We have now defined three types of language, finite state languages, derivable languages, and terminal languages. Since any language of any of these types is determined by a finite grammar, there are only a denumerable number of languages of any of these three types. Hence many languages are not of these types, since the set of languages (a language being a finite or infinite set of strings) is non-denumerable. It therefore makes sense to ask how these three types of language are related, and what sorts of language lie beyond their bounds. We can see quite easily that these types are not equivalent, and that rather simple types of language fall beyond these bounds.

**Th.28.** (i) Every derivable language is a terminal language, but not conversely.

(ii) Every finite state language is a terminal language, but not conversely.

(iii) There are derivable, non-finite state languages, and finite state, non-derivable languages. (31)

Suppose that \( L \) is a derivable language. Clearly there are only a finite number of primes from which all strings of \( L \) are composed. Suppose that these are the primes \( a_1, \ldots, a_q \). Suppose that \( b_1, \ldots, b_q \) are primes which do not occur in \( L \). Then we can add to (3911) the finite set of instruction formulas

\[
\begin{align*}
(40) \quad & a_1 \rightarrow b_1 \\
& a_2 \rightarrow b_2 \\
& \quad \vdots \\
& a_q \rightarrow b_q
\end{align*}
\]
giving us the set $F'$ of instruction formulas. Let $L'$ be the language determined by $[\Sigma, F']$. The terminal strings of $L'$ are simply alphabetic variants of the derivable strings of $L$, and these two sets have no primes in common. But if one set of strings is a terminal language, then clearly any set which differs from this set only in that each prime is replaced by a new prime (distinct primes being kept distinct under the replacement) is also a terminal language. Hence $L$ must itself be a terminal language, giving us the first part of Th.281.

Suppose that $L$ is a derivable language with the grammar (39), and that $K$ is an infinite set of strings of $L$. Clearly at least one of the instruction formulas of $F$ must have been used an infinite number of times in deriving the strings of $K$. In other words, it must be the case that for every infinite set $K$ of strings of $L$ there is an infinite sequence $Q_1$ of distinct strings of $K$, an infinite sequence $Q_2$ of strings of $L$, a prime $p$ and a string $a$ such that when $Q_1$ and $Q_2$ are paired off term by term, each term of $Q_1$ is formed from the corresponding term of $Q_2$ by the replacement of $p$ by $a$.

With this property in mind, we can easily construct non-derivable languages. For example, let $L$ be the language containing just the strings

\[(41) \quad a, aa, aaaa, aaaaaaa, \ldots\]

and in general, all (and only those) strings containing $2^{2^n}$ occurrences of $a$. Then clearly we will not be able to find a sequence $Q_1$ which has the above property. Hence the language (41) is not derivable. However, (41) is not a terminal language either, and hence it does not establish the second part of Th.281.

To establish this, we note the following much weaker property of derivable languages. If $L$ is derivable, all but a finite number of strings of $L$ must be constructible from some other string of $L$ by the replacement of a prime by a string. Suppose that $L$ is the language containing just the strings

\[(42) \quad ab, cabd, ccsabdd, cccabddd, \ldots\]
and in general all (and only those) strings containing exactly \( n \) occurrences of \( a \) followed by \( ab \) followed by exactly \( n \) occurrences of \( d \). None of the strings of \( L \), in this case, is constructible from some other string of \( L \) by replacement of a prime by a string. Hence \( L \) is not derivable. \( L \) is, however, a terminal language, with a grammar containing \( Z \) as initial string and

\[
\begin{align*}
Z & \rightarrow ab \\
Z & \rightarrow cZd
\end{align*}
\]

as instruction formulas. This proves the second part of Th.281.

Note that (42) is clearly not a finite state language. Having produced the symbol \( b \), we must move into one of an infinite number of states, depending on how many \( c \)'s have already been produced. (38) Hence there are terminal languages which are not finite state languages, giving us a proof of the second part of Th.281i.

Suppose that \( L \) is an arbitrary finite state language with states \( S_0, \ldots, S_n \) and the grammar (38). Let \( L' \) be the language with the primes of \( L \), and in addition, the primes \( Z_0, \ldots, Z_n \). The grammar of \( L' \) contains the initial string \( Z_0 \) and the instruction formulas of the form

\[
\begin{align*}
Z_i & \rightarrow a_{i1} Z_j \quad \text{(for all } i, j \text{ such that } a_{i1} \neq U \text{ and } i \neq 0) \\
Z_i & \rightarrow a_{i0} \quad \text{(for all } i \text{ such that } a_{i0} \neq U) \\
\end{align*}
\]

Clearly the terminal strings with respect to the grammar \( \{ Z_0, (43) \} \) are exactly the strings of \( L \), so that the first part of Th.281i is established.

A slight modification of (42) gives us a derivable language which is not a finite state language. Let \( L \) be the set of strings

\[
\begin{align*}
\text{(45)} & \quad a, cab, ccabb, cccabbb, \ldots
\end{align*}
\]

Then \( L \) is not finite state language, for just the reasons that prevent (42) from being a finite state language. It is, however, derivable from the initial string \( a \) with the
single instruction formula

\[(46) \quad a \rightarrow cab\]

An example of a finite state, non-derivable language is also easily found. Let \(L\) be the language containing just the strings

\[(47) \quad aa, aaaa, aaaaaa, \ldots aaaa, aaaaaa, aaaaaaaa, \ldots\]

In other words, \(L\) contains every string of \(a\)'s of length \(2m\) or of length \(3n\), for some \(m\) and \(n\). It is clear that \(L\) cannot be derivable. For suppose \(X = aa \ldots aa\) has been derived from some grammar for \(L\). In order to derive a longer string from \(X\) it is necessary to develop one of the \(a\)'s into \(aaa\) or into \(aaaa\), depending on whether \(X\) is of the \(2m\) or the \(3n\) type. But in order to determine this, we must count the \(a\)'s in \(X\) -- i.e., the corresponding instruction formula must be \(a_1t\) least as long as the string of \(a\)'s in \(X\). Hence the number of instruction formulas must be infinite, since \(X\) can exceed any fixed finite length, and \(L\) is consequently not derivable. But \(L\) is a finite state language. It is generated by the six state grammar with \(a_{01} = a_{10} = a_{12} = a_{21} = a_{03} = a_{34} = a_{40} = a_{45} = a_{56} = a_{64} = a\), and \(a_{11} = U\) for every other pair \(i,j\) \((0 \leq i,j \leq 6)\). Thence Th.28111 is established.

The import of Th.28111 is that description in terms of phrase structure is essentially more powerful than description without higher levels in terms of a finite state grammar. That is, for the description of certain languages it is actually necessary to introduce higher levels such as phrase structure (as long as we restrict ourselves to these modes of description). In fact, it is no doubt true that every natural language is actually a finite state language. This is connected with the fact that there is presumably only a finite number of selectional relations (in the sense discussed in §49.1 and 49.3, and again below, in chapter VII) in actual languages. Hence, by Th.28, every actual language is a terminal language -- it can be described by some system of phrase structure. In investigating the adequacy of grammars which do
not go beyond the level of words or beyond the level of phrases, then, we do not turn our attention to the problem of whether it is literally impossible to describe some actual language in terms of such grammars, but rather to the question of whether it is convenient, economical, or useful to do so, whether description in such terms is revealing and gives insight into systematic properties of the language, intuition of native speakers, etc. Such considerations are of course much less clear-cut and any position taken is more tenuous and difficult to establish. Nevertheless, it is quite obvious to anyone who has taken the trouble to attempt to construct a finite state grammar for an actual language, that such a description, though perhaps possible, is so complex as to be practically useless and is obviously failing to reveal or make use of fundamental structural features of the language (the same might be said for a description that does not go beyond the level of phonemes). We may be able to gain some insight into the question of why a certain type of grammatical description is deficient in these respects by investigating the relatively simple languages which literally cannot be described in such terms. We often find that these languages which are beyond the scope of ordinary grammar are not based on fundamentally new means for constructing sentences, but rather extend the ordinary means indefinitely far beyond their finite limits in actual languages. For example, it is easily demonstrated that the language containing all strings of the form

\[(48) \; X^X, \text{ where } X \text{ is any string of the symbols } a_1, \ldots, a_n\]

and no others is not a terminal language (hence is neither a finite state nor derivable language). Each sentence of the language \((48)\) consists of a string \(X\) followed by the identical string \(X\). In other words, \((48)\) is constructed on the basis of a strict parallelism which is extended infinitely. But parallelism of this type is also found in ordinary languages. For example, in coordinate constructions of the type \(X^X, X, Y^Y, Y\), \(X\) and \(Y\) must be parallel constructions (cf. \(\S 59.1\) below). If \(X\) is a noun phrase, \(Y\) must be a noun phrase, etc. In ordinary languages however, the exact nature of this
parallelism can apparently be prescribed by a finite number of conditions, so that these languages do not transcend the limits of terminal languages. Almost invariably we find, however, that these cases of parallelism lead to considerable complication of the grammar. This is true in the case of and-constructions, as well as in the case of such more complex instances of parallel construction as sequences of relative clauses (37), etc.

Aside from its intrinsic interest in clarifying the coverage of various types of grammars, then, such investigations as those of this section may prove useful in that they serve as a guide to inadequacies of such grammars (and the conceptions of linguistic theory on which these grammars are based) in the case of actual languages. The following principle suggests itself as a methodological aid: wherever we find a certain process of sentence construction that appears with finite limits in actual languages, but that, if extended indefinitely, produces languages that cannot be described by a given type of grammar, then we may suspect that such types of grammar are inadequate for actual languages, for reasons of complexity, etc. Thus if a grammar is saved from literal inapplicability by virtue of the finiteness of some process (by virtue of the fact that the variety of actual forms can be exhausted by listing) we should seek to replace it by something more general that would succeed even if listing were impossible. Such a principle can of course serve as nothing more than a hint in the investigation of the adequacy of particular grammars and particular formulations of linguistic theory.

Looking back at the difficulties we have come across, from the point of view just suggested, it becomes evident that certain types of grammatical statement which have no place in the formulation of linguistic theory which we have so far provided might be quite effective in reducing the complexity of grammar. Returning to the case of parallel constructions, suppose that our grammar could include statements of the form (46). Then languages of that type could be accommodated. In particular, in the case of and-constructions, we could vastly simplify the grammar by statements such as this:

(49) if \( \ldots X \ldots \) is a sentence, and \( X \) is a constituent of this sentence, then \( \ldots X \text{and} X \ldots \) is a sentence.
Statements of the same type would enable us to eliminate half of the $2n$ instruction formulas in the example in the second paragraph of fn. 37. Suppose that a given language $L$ contains a certain set of strings which can be adequately described by a simple system of phrase structure, and in addition, contains the 'mirror image' of each of these strings (i.e., for each string $a_1^\ldots a_n$ which it contains, it also contains $a_n^\ldots a_1$). Then we might have to double the complexity of the grammar of $L$ to account for this set of 'inverse' strings, but if we had provided for the possibility of such grammatical statements as

(50) if $X$ is a sentence, then $X^{-1}$ ($X$ read from back to front) is a sentence.

Then this statement alone would account for all of the inverse strings, with no further complication of the grammar. More general statements of this type would also allow us to account in a natural way for certain cases of discontinuity of constituents (this problem cannot be handled within the system $F$, as formulated above), and to avoid many other troublesome difficulties. The investigation of such more elaborate types of grammatical statement (and of the type of linguistic level that underlies them and gives them significance) will occupy us below in chapters VIII and IX.

Notice that derivable languages can be constructed by a finite state process similar to the one described above, but with a different interpretation. Suppose that we have the grammar (39). Consider a process with the states $S_0, \ldots, S_n$ with transition from $S_i$ to $S_j$ marked by the production of the string $A_{i,j}$. For $1 \leq n$, let $A_{0i} = \sum_i$ (where $\Sigma_1, \ldots, \Sigma_n$ are the initial strings). That is, we move from state $S_0$ into the states $S_1, \ldots, S_n$ by producing one of the $n$ initial strings. From that point on the state of the process when the string $A$ has just been produced is determined by that subset of the set $X_1, \ldots, X_n$ (cf. (39)) containing just those strings which are substrings of $A$, and we move to the next state by replacing $X_i$ by $Y_i$ in $A$, for some $X_i$ in this subset. When we reach a string $B$ containing none of the strings $X_1, \ldots, X_n$ as substrings, we say that the process has returned to $S_0$. In less technical language, this means simply that
in deriving a sentence of the language we consider at each point in the derivation just the string which appears at that stage, and not the 'history of derivation' (i.e., the constituent structure) of this string. Cf. $\phi_{50.5.2}$, $50.5.4$. This requirement suggests new methods for constructing non-derivable languages, by considering processes of derivation which are non-markovian—i.e., where the development of a given string depends not just on the form of this string, but also on the forms of the strings from which it was derived. For example, suppose we again consider grammars with a finite number of initial strings and instruction formulas. And suppose we revise the notion of derivability so that given a derivation containing $m$ steps, we form the $m+1$st line by concatenating the first $m$ steps in order and then applying some instruction formula to the result. Thus given a derivation $A_1 \, A_2 \ldots \, A_m$, we form $A_{m+1}$ by applying an instruction formula to $A_1 \wedge A_2 \ldots \wedge A_m$. This process of derivation is non-markovian. Suppose we choose the initial string $a$ and the single instruction formula: $a \rightarrow aa$. Then we derive in this way exactly the language (41) containing just those strings that have $2^n$ occurrences of $a$, a language which, as we noted above, is neither derivable or terminal. This new process of derivation is more powerful because in developing a string further it takes into consideration the 'history of derivation' of the string; this, as we have seen, amounts to taking into consideration its constituent structure. We will find that such stronger forms of derivability are extremely useful in the case of actual languages. In fact, even the suggested formulation (49), with its reference to constituent structure, presupposes some elaboration of this sort.

For the reasons just suggested, I believe that the further study of the notions briefly discussed above may be useful. Many further questions immediately suggest themselves. Thus it is natural to ask such questions as the following: Are the sets of derivable (or terminal) languages closed under the operations of forming the logical sum or product; how is the notion of derivability extended if we drop the requirement that only a single prime can be developed in an instruction formula (note that it was just this requirement that makes it possible to reconstruct the system $P$ from the grammar), etc.
In developing the relation between the system of phrase structure and the form of grammar in §50, we went beyond grammars of the form (39) (this form is essentially the result of Reduction 2, §50.2), and considered also the order of statement of instruction formulas. Suppose that we have a set of instruction formulas:

\[ \lambda \rightarrow \lambda \quad (1 \leq \lambda \leq n) \], as in (39). Suppose that we now order the instruction formulas, and require that in constructing a derivation we proceed arbitrarily far down the list, beginning over again at the beginning when we reach the end of the list, until we choose an instruction formula to apply. This of course is no additional restriction at all. Suppose now that we require that certain of the instruction formulas be obligatory—that is, whenever we reach them in running through the list, they must be applied. This still adds no restriction, since we have not given any requirement as to how many instruction formulas are to be obligatory. But this refinement does offer a possibility of simplifying the grammar, as we have seen above. The most natural requirement to put on such a grammar seems to be one that guarantees that every time we run through the grammar (the sequence of instruction formulas) each non-terminated derivation be advanced at least one step—i.e., we cannot indefinitely run through the grammar vacuously. We thus defined a "proper linear grammar" as a sequence of instruction formulas \( \lambda \rightarrow \lambda \quad (1 \leq \lambda \leq n) \), where for each \( \lambda \) there is some \( \lambda \) such that \( \lambda \rightarrow \lambda \) is an obligatory instruction formula. Thus for example, if the string \( a \) can be converted into one of the forms \( b_1, \ldots, b_k \), we will present in the proper linear grammar the sequence of instruction formulas

\[
(51) \quad a \rightarrow b_1 \\
\quad a \rightarrow b_2 \\
\quad \vdots \\
\quad a \rightarrow b_k
\]

where only the last is obligatory.
To increase the generative power of such grammars, we permit each rule to be applied an indefinite number of times to a given string whenever the rule in question is selected for application. (Cf. Def. 25, § 50. *2, for a more careful statement). We define a "proper derivable language" (a "proper terminal language") as one which is derivable (terminal) in this way from some proper linear grammar.

Instead of investigating the relation between derivability and proper derivability directly, we turn to the more interesting question of whether every set of derivations which is produced by a grammar of the form (39) can also be produced by some proper linear grammar, and conversely. A set of derivations which can be produced by some grammar of the form (39) we will call producible; a set of derivations which can be produced by some proper linear grammar we will call properly producible. It is more interesting to consider the set of derivations produced by a certain grammar than the language derivable from it, since as we have seen, the underlying phrase structure for the terminal language determined by the given grammar can be reconstructed from the set of derivations produced by the grammar, though not in general from the set of derivable strings. In other words, the most interesting notions introduced above are "finite state," "terminal," "proper terminal," "producible," and "properly producible," with "derivable" and "properly derivable" being merely auxiliary notions.

We find that the two types of producibility are incomparable. I.e.,

Th. 29 There are producible sets of derivations that are not properly producible, and properly producible sets which are not producible.

The existence of sets of derivations which are properly producible but not producible has already been pointed out in § 50. 5.2. One simple set of this type is the following:

\[ \begin{align*}
D_1 & : a, b, a \\
D_2 & : c, b, c \\
D_3 & : c, b, a
\end{align*} \]
This set of derivations is produced by the proper linear grammar with initial strings $a$, $c$, and the following sequence of instruction formulas, where obligatory instructions are marked with "B!"

(53) $B: \ a \rightarrow b$
    $B: \ b \rightarrow a$
    $B: \ c \rightarrow b$
    $b \rightarrow c$

But obviously there is no set of instruction formulas which will produce this set of derivations, without producing also

(54) $D_4: \ a, b, c$

The point here is that as we noted above, the process of derivation from grammars of the form (39) is markovian. But a proper linear grammar can impose certain further constraints on derivability. We pointed out in $\gamma 50.5.2$ that this is not an advantage, but rather is a disadvantage of proper linear grammars, since it makes it more difficult to determine whether a given grammar actually does determine a system of phrase structure. On other levels where constituent structure does not play the same role, this extra power is a distinct advantage.

A grammar similar to (46) provides an example of a producible set of derivations which is not properly producible. Consider the grammar with the initial string $a$ and the following set of instruction formulas:

(55) (i) $a \rightarrow bab$
    (ii) $a \rightarrow cac$

The language derivable from this grammar consists of the strings

(56) $a, bab, cac, bbabb, bcaeb, cbabc, ccacc, \ldots$,

and in general, of all strings $zaZ^{-1}$, where $Z$ is a string of zero or more $b$'s and $c$'s,
and \( Z^{-1} \) is the mirror image of \( Z \). There are no terminal strings in this case, but there would be if, for example, we added the instruction formula

\[(57) \quad a \rightarrow d\]

Among the derivations produced by (55) we find every initial subsequence of (58i) and (58ii).

\[(58) \quad (i) \quad a, bab, bbbab, bbbbab, bbbbbb,...\]

\[(ii) \quad a, cac, ccacc, cccacc, cccccacc,...\]

If (57) is added to (55), we have as terminated derivations, in particular, every initial subsequence \( S \) of (58i) and (58ii) followed by the final term of \( S \) with \( a \) replaced by \( d \).

In forming a proper linear grammar to properly produce exactly the set of derivations produced by (55), we must make either (55i) or (55ii) obligatory. In case (57) is added, we must still make either (55i) or (55ii) obligatory, since if (57) is obligatory it will be impossible to run through the grammar more than twice. But clearly if (55i) is made obligatory, then it is impossible to produce the derivations which are initial subsequences of (58ii) and if (55ii) is made obligatory, then it will be impossible to produce those which are initial subsequences of (58i). (39) Thus Th.29 is established.

Th.29 tells us that certain proper linear grammars may not lead to systems of phrase structure, and that certain systems of phrase structure which can be represented by finite grammars are not representable by proper linear grammars. In other words, we can strengthen the statement made in §50.3.3. Even if Reduction 1 and Reduction 2 lead to a finite grammar, it still may not be possible to carry out Reduction 3 to a proper linear grammar.

This opens up a new possibility for testing the adequacy of the syntactic theory constructed in outline above. We know that ordering of the statements of the grammar can lead to considerable simplifications; i.e., that it is very useful to assume a
hierarchy of syntactic and morphological processes. The condition on a sequence of statements that gives a proper linear grammar (i.e., the requirement that every non-terminated derivation be advanced with each application of the grammar) seems a very natural one, and one that increases the value and utility of the grammar. Since we know that Reduction 3 is not always applicable even to a set of derivations producible from a finite grammar, we may ask whether the set of derivations which we require for some actual languages is in fact reducible to a proper linear grammar. A negative answer to this question will be strong evidence that our conceptions of syntactic structure are still inadequate.

In the next three chapters we will bring evidence in favor of the following thesis: the phrase structure of English cannot be reduced to a proper linear grammar; there is a subset of English sentences, interesting on quite independent grounds, which can be provided with a very simple system of phrase structure which is reducible to a proper linear grammar; each grammatical English sentence can be provided with a constituent analysis derivatively by considering its relation to this subsystem of phrase structure; this approach to syntax provides a much simpler grammar, reveals many underlying regularities of the language and offers formal explanation for much of the 'linguistic intuition' of the native speaker, the traditional practice of grammarians, etc.
Footnotes

(34) Such processes can be represented graphically by a 'state diagram'. Cf. Shannon and Weaver.

(35) The number of states and the length of the grammar \(38\) can sometimes be reduced by permitting a passage from \(S_i\) to \(S_j\) in several different ways, but this elaboration is of no importance to our discussion.

(36) This is not to say that finite state languages must have only dependencies of less than some finite length. That, of course, is false. For example the language containing just the strings acc. cca and bcc. ccb, with any number of c's, has dependencies of indefinitely great length, but is a finite state language.

(37) To avoid certain inconsequential considerations, we will consider that a language \(L\) is finite state, derivable, or terminal in case the language \(L^1\) is finite state, derivable, or terminal, respectively, where \(L^1\) is formed from \(L\) by replacing each string \(X\) of \(L\) by \(\#^X\#\), where \(\#\) is a prime of \(P\) (i.e., it is never developed by an instruction formula). Cf. \(\S\) 50.2.

(38) E.g., in sentences of the type \(NP \wedge rel \wedge VP_1 \wedge VP_2\) (e.g., "the cars that were damaged were towed away") the segments \(NP \wedge VP_2\) and \(rel \wedge VP_1\) must be parallel in certain respects—both \(VP_1\) and \(VP_2\) must contain the kinds of verbs that go with \(NP\), \(rel\) can be \(who\) only if \(NP\) is animate, there are parallel positions in \(VP_1\) and \(VP_2\) where, e.g., \(himself\) can occur only if \(NP\) is singular masculine, etc. The situation is still more complicated when we have a sequence \(NP \wedge rel \wedge VP_1, rel \wedge VP_2, \ldots, rel \wedge VP_n, VP_n + 1\).

Some of the advantages of a description in terms of phrase structure over a finite state grammar are evident from this example. Suppose that sentences of the form \(NP \wedge rel \wedge VP_1 \wedge VP_2\) are to be produced by a finite state grammar. Suppose that by choosing a given string for \(NP\) (e.g., the care) we impose \(n\) conditions on the choice of \(VP_2\). Then that part of the state diagram that accounts for the strings of the form \(rel \wedge VP_1\) must be multiplied \(n\)-fold into \(n\) subdiagrams which differ only in the names of the states that appear (neglecting now the effect of the choice of \(NP\) on the choice of \(rel \wedge VP_1\) itself). Since there are an immense number of possibilities for \(rel \wedge VP_1\), the grammar becomes enormously complex at this point. This particular difficulty can be avoided in terms of phrase structure if we set up the primes \(NP_1, \ldots, NP_n, VP_1, \ldots, VP_n\), and choose as instruction formulas the \(2n\) formulas: \(Sentence \rightarrow NP_1 \wedge VP_1; \ NP_1 \rightarrow NP_2 \wedge rel \wedge VP_1\) \((1 \leq 1 \leq n)\).

More generally, the advantage of description in terms of phrase structure shows up whenever we have a set of strings which can occur in various positions, where the choice of strings that are permitted to follow them depend on the choice of strings that precede them. Thus in the above example \(rel \wedge VP_1\) can occur only in the contexts \(NP_1 \rightarrow VP_1, \ldots, NP_n \rightarrow VP_n\). Similarly, many strings can appear as noun phrases, but what strings may follow them depends on whether the noun phrase is subject, object, etc. In any such case, description in terms of a finite state grammar will be pointless, even if possible.
To give a more careful proof that this set of derivations is not properly derivable, we may show that for any proper linear grammar which produces a derivation a subsequence $S$ of (561) and which simultaneously produces every initial subsequence of (5811), the number of times that (551) must occur as an instruction formula in this grammar is an increasing function of the number of lines in $S$; hence the grammar must contain an infinite number of lines, if it is to produce all of the initial subsequences of (561). Note that although the set of derivations given by (55) (with or without (57)) is producible but not properly producible, nevertheless, the language derivable from (55) (or (55) and (57)) is also properly derivable, unless further restrictions are placed on the notion "proper linear grammar." In fact, every derivable language is properly derivable, e.g., by a sequence of instruction formulas which contains every instruction formula for the given derivable language in any order followed by a sequence of obligatory instruction formulas $x \rightarrow x$ for each $x$ which is a left-hand term in one of the instruction formulas for the derivable language. Naturally, the set of derivations from such a grammar will not lead to a system of phrase structure, since the irreflexivity of $\vdash$ will be violated.
Chapter VII - Description in Terms of Phrase Structure

58. Having developed the level \( P \) abstractly, we can now attempt to determine its effectiveness by applying it to the description of actual language material. The sketch of English phrase structure that follows is presented both for illustration and for later reference. It is obvious that it could not be complete or balanced without far exceeding the scope of this study. Some parts of English phrase structure will be omitted completely, others will be very broadly sketched, and others given in fair detail, the place and degree of detail being determined partly by the desire to determine how satisfactory an analysis of English can result from a rigorous reliance on the conceptions of chapter VI, and partly by the needs of the theoretical exposition to follow in the next chapter where we will attempt to develop the means for a more adequate grammatical description.

If more detail were given, I think that the points to follow in the next chapter could in fact be made even more strongly, though the detail might obscure the demonstration of their validity.

The notations used here will be those of chapter III, with occasional minor and self-explanatory deviations. The primes of \( P \) will be labeled so as to suggest their customary names (e.g., \( IP \), \( Y \), etc). A more rational labeling system would permit certain statements to be made more elegantly (if the notational devices were extended in simple and obvious ways). In general, there has been no attempt here (as there was in the appendix to chapter V) to present the grammar in the
completely reduced form, but I have attempted to set up the elements just the way they would appear in the most elegant form. In other words, the grammatical statement which we will gradually develop in this chapter, and finally present in \( \frac{457}{2} \), is, subject to the qualifications given above, an expanded version of the simplest statement I have been able to develop.

\[ \text{Eq. 1} \]

While we cannot give a thorough validation of the particular analysis of phrase structure which will be proposed, nevertheless at least some of the reasoning involved can be sketched. In \( \frac{453}{2} \), four grammatical features were mentioned as criteria, in a loose sense there outlined, for the analysis of English. The first and most useful of these is the rule for conjunction.

The rule giving the distribution of "and" can be used as a criterion for constituent analysis in the following way. If we have a context \( Z_1 \) \( \text{and} \) \( Z_2 \), and if both \( X \) and \( Y \) occur in this context, we can determine whether they occur as constituents by seeing whether \( X \text{and} Y \) can occur in this context. This is a fairly sensitive criterion for the determination of phrase structure, though considerable qualification is necessary. It is clear that it works effectively in the extreme cases. Thus given

(1) the men walked down the road
(2) the boys walked down the road
(3) the men disappeared into the distance

we have also
(4) the men and the boys walked down the road
(5) the men walked down the road and disappeared into the distance indicating that "the men", "the boys", "walked down the road", and "disappeared into the distance" are constituents here. But given
(6) the little tug chugged up the river
(7) the great liner sailed down the river
we do not have
(8) the little tug chugged up the and great liner sailed down the river
indicating that "little tug chugged up" and "great liner sailed down" are not constituents.

Before applying this criterion, it is important to emphasize again its real status. The rule for conjunction is a criterion for constituent analysis in the sense that if we choose constituents so that only constituents of the same type can be so repeated, we have a very simple and concise rule for conjunction (namely, we can state that if $X$ and $Y$ are constituents of the same type and the same internal structure, in a sense to be specified, in the context $Z_1 \wedge - \wedge Z_2$, then $Z_1 \wedge X \wedge \wedge Y \wedge Z_2$ is also a sentence), whereas if we choose them so that this is not the case, the rule for the occurrence of and will be exceedingly complex. The examples cited indicate that this criterion coincides with intuition in certain clear cases. But the correspondence with intuition supports this criterion only in a derivative sense. It really
gives direct support only to the general policy of applying considerations of simplicity to the determination of constituent structure. Similarly, if we cannot (or, for other reasons, do not) set up as constituents exactly the elements with this property of entering into conjunction, this is not to be understood as a repudiation of the criterion of conjunction. The particular criteria of analysis that we set up have only the function of collecting under one heading certain related ways of simplifying the grammar. The ultimate criterion is total systematic simplicity, and there are no exceptions.

We can apply the conjunction criterion directly to the determination of the place of the major constituent break. Given the sentence

(9) My friend enjoyed the book

we might consider analyzing it in any of the following ways, as a first step:

(10) My friend enjoyment the book
(11) My friend enjoyed the book
(12) My friend enjoyed the book

But we have

(13) My friend liked the play and enjoyed the book

This rules out the direct analysis (12) into three immediate constituents (since "enjoyed the book" is repeatable as a unit), and gives (10) as one permissible analysis. That this is the only permissible analysis follows from the non-grammaticalness of
(14) My friend enjoyed and my family liked the book which rules out (11). Application of this criterion thus supports the intuitive analysis

(15) \( f(Sentence, Noun \ PhD; Verb \ Phrase) \)

as the basic analysis of the sentence. Thus the basic grammatical relation (cf. 454) is actor-action(subject-predicate).

Further investigation shows the possibility of

(16) My friend enjoyed the book and the play
(17) My friend read and enjoyed the book
so that there is a secondary analysis: \( f(Verb \ Phrase, Verb \ Noun \ Phrase) \), and a secondary grammatical relation verb-object.

It is worth noting that the decision between (10), (11), and (12) cannot be made in terms of substitutability, at least in any obvious way. There is little to choose between the alternative sets of substitution classes arising from these analyses in terms of size, freedom, etc.

Now consider the sentence

(18) My friend has been reading the book

We might consider analyzing the verb phrase in (18) into either (19) or (20):

(19) has been - reading the book
(20) has been reading - the book

The conjunction criterion in this case favors (19), since we have (21) as a natural English sentence, but not (22).

(21) My friend has been reading the book and smoking a cigar
(22) My friend has been reading and has been enjoying the book
This suggests that the primary analysis of the verb phrase can be into auxiliary phrase and a second type of verb phrase, the latter being analyzable into verb and object. In the reduced form, we have as the beginning of the grammar:

(23) Sentence → NP^0 VP

VP → VP_A^1 VP_1

VP_1 → V^1 NP

The conjunction criterion will play an important role in this analysis, and a much more careful investigation of the distribution of "and" is certainly called for. In particular, it is necessary to study the parallel internal structure required for elements to be joined by "and". Here we have discussed only the external parallel, the fact that they must both be represented by the same element and must play the same role in the sentence. We will see below (cf. ) that this study requires a prior study of transformations.

In carrying out the investigation of conjunction, it will be important to bear in mind that we are only interested in the systematic behavior of "and", where 'systematic' is defined as in chapter IV. (1) Given a set of first order grammatical categories, and a linguistic corpus, we have a set of sentences generated. In this set we will find instances that are true, false, humorous, logically contradictory, enlightening, meaningless, etc. Exactly what are the boundaries of this set (which defines systematic behavior in the first approximation to grammaticialness given in chapter IV) depends on the subtlety of our category analysis. We are interested in describing just this set, which will include many sentences.
that no one would ever say, under normal circumstances. If
this set is not inclusive enough to contain sentences of
these various types, it will fail to meet presystematic
requirements, as we have seen. When we say that (13), (16),
(17), and (21) are grammatical, but not (14) and (22), what
is really asserted is that in an adequate corpus of
English, the sentence forms of which the former are instances
will be grammatical in the sense of chapter IV, but the
sentence forms of which the latter are instances will not.
The difference between unnatural (or meaningless) instances
of grammatical sentence forms, and unnatural (or meaningless)
instances of ungrammatical sentence forms is crucial for us.

A second criterion is that of intrusion of such
careful elements as "in my opinion", "as it turned out", "however", etc., with a special comma intonation. This is
a less sensitive criterion than conjunction, since though it
selects a set of major constituent breaks where intrusion can
occur, it does not differentiate among these as to relative
order. But the distinction between those places where
parenthetical intrusion can occur and where it cannot, can
be used as support for an analysis, regarding the points of
intrusion as more 'major' than the points of no intrusion.
The choice of (10) over (11) is thus supported by the
existence of (24) but not (25).

(24) My friend, as you can see, enjoyed the book
(25) My friend enjoyed, as you can see, the book

However, this criterion conflicts with conjunction in
certain instances. Thus we have
(26) His performance was, in my opinion, very undistinguished.

(27) He should, in my opinion, have done much better.

These are subsidiary breaks by the conjunction criterion. The conflict can be resolved by giving two sources for parenthetical intrusion, first, intrusion at major constituent breaks, and second, intrusion at the points where a certain class of adverbs (e.g., "certainly") occur. Thus we have

(28) His performance was certainly very undistinguished.

(29) He should certainly have done much better.

These sources are both necessary. The second is needed to explain (26) and (27), the first, to explain

(30) He was killed, so I heard, in an automobile accident.

The third criterion mentioned in §5.3 concerned the simplification of transformations. We will return to this below, when the notions involved have been developed. We will see then that a more integrated conception of criteria of analysis, including those of this chapter and a large number of others, will be a by-product of the development of transformational analysis.

The final criterion mentioned in §5.3 (and one which we can barely touch on here, without going far afield) is the position of junctures, more generally, the rules for stress and pitch. There is a striking correspondence between the degree of stress and constituent boundary. To take a simple case, consider

(31) The old man in the corner has been reading the newspaper.

The heaviest stress is normally on "newspaper". The stress on "corner" is heavier than that on "man" or "reading". These in
turn have a heavier stress than "old", which is more heavily stressed than "the", "has", "been", "in".

Suppose we mark constituent boundaries (except word boundaries) by a vertical line at the end of each constituent. Thus a word which is at once the final word of n constituents (e.g., "corner" concludes the Noun Phrase and the Prepositional Phrase) will be followed by n vertical lines. Then, considering the sentence as a whole as its own largest constituent, we have

(32) The old man | in the corner || has been reading | the newspaper ||
     
               NP    PP       VP    NP

Sentence

The rule that the degree of the major stress of a word is proportional to the number of vertical bars following the word in question determines the relative stress of words in the proper way in this instance, when we add the subsidiary rule that certain words ("the", "in", "has", "been", etc.) always have zero stress. Thus the rules for stress can be considerably simplified with a proper choice of constituents, or in other words, stress is a criterion for constituent analysis.

The analysis proposed for (32) conflicts with the analysis (23), and with the choice of (19) over (20). This conflict of criteria can be resolved by noting that, because of the subsidiary rule concerning "has", "the", etc. "been", "in", etc. (3) we would have exactly the same statement of relative stress if we had put the vertical bars in accordance with (23), since the bars after "been" would have no effect. Thus this
criterion is neutral with respect to the choice between (19) and (20), and we could just as well have written (32) in accordance with (23).

The rule for predicting stress as sketched above is intended to be suggestive, not definitive. It will not account successfully for other cases without elaboration. Recent linguistic work has shown that the study of suprasegmental features such as stress and juncture is a complex one, and the relation between the syntactic structure and suprasegmental patterning requires a careful study. Although it has often been suggested that constituent structure be determined by considerations involving suprasegmentals, I am not acquainted with any attempt at a general statement as to how this might be accomplished. This is an important question, but once again further discussion and investigation would carry us into areas that have arbitrarily been placed beyond the scope of this study.

We might consider, as a criterion for syntactic analysis, such a phenomenon as intermittent pause. E.g., it might be argued that (9) can be said with a pause at the break suggested in (10), but not at the break suggested in (12). There are other cases where a sentence can be said so that its interpretation is unmistakable, but is ordinarily said so that its interpretation is ambiguous. E.g., cf. §48.2 and fn. 9, chap. VI. But if the datum we are analyzing is normal speech (with the pair test as the basic operational test), then it seems preferable to handle such possible readings as derivative from the grammar (rather than as criteria for the grammar), i.e., to explain them as being a reflection of the fact that the
speaker knows (on other grounds) the place of the constituent breaks. The task of the linguist, then, becomes the isolation of these other grounds. This is the same problem that we have discussed above in §26.2.

Other criteria for constituent analysis will appear below, as we investigate the actual formulation of the syntactic description.

§60.1. Since the auxiliary verb phrase $V^A_P$ plays a central role in English syntax (particularly, as we will see below, in transformational analysis), we must analyze it in some detail. The forms to be accounted for under the auxiliary verb are given in Table 1, with "take" as the verb.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
<th>Column III</th>
<th>Column IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. takes</td>
<td>took</td>
<td>take</td>
<td>-</td>
</tr>
<tr>
<td>2. is taken</td>
<td>was taken</td>
<td>are taken</td>
<td>were taken</td>
</tr>
<tr>
<td>3. is taking</td>
<td>was taking</td>
<td>are taking</td>
<td>were being taken</td>
</tr>
<tr>
<td>4. has taken</td>
<td>had taken</td>
<td>have been taken</td>
<td></td>
</tr>
<tr>
<td>5. will take</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>6. is being taken</td>
<td>was being taken</td>
<td>are being taken</td>
<td>were being taken</td>
</tr>
<tr>
<td>7. has been taken</td>
<td>had been taken</td>
<td>have been taken</td>
<td></td>
</tr>
<tr>
<td>8. will be taken</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>9. has been taking</td>
<td>had been taking</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10. will be taking</td>
<td>-</td>
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<tr>
<td>11. will have taken</td>
<td>-</td>
<td>-</td>
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<tr>
<td>12. will have been taken</td>
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<td>-</td>
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<tr>
<td>13. will have been taking</td>
<td>-</td>
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<tr>
<td>14. has been being taken</td>
<td>-</td>
<td>have been being taken</td>
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<tr>
<td>15. will be being taken</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>16. will have been being taken</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.

"will" has been used as the representative of a class $M$ containing "will", "can", "may", "shall", "must", "would", "could", "might", "should". We can fill the gaps in column II by taking "would", "could", "might", "should", "must" as
compounds, these being, respectively, the 'past tense' forms of "will", "can", "may", "shall", "must". It turns out that this analysis does in fact simplify the direct description of the auxiliary phrase, even though the gaps in column III remain. But its ultimate acceptability also depends on its effect on the formulation of other statements of the grammar, such as 'sequence of tense' statements, which I have not investigated.

Column IV can be disregarded for the time being, as involving a special fact about the morpheme "be", not a feature specific to the auxiliary phrase. This is evident from the fact that "be" in its use as the main verb also has the peculiarity that it has a separate form for past singular and plural.

Rows 14, 15, and 16 contain forms whose status may be in some doubt. Whether they are to be included or not depends on a grammaticalness decision. The analysis of the auxiliary phrase is simpler if they are included, or in other words, they conform to the 'structure' of the verb phrase. They certainly have a clear meaning, which can be expressed in no other way, and I have noted several instances of these forms in normal conversational speech. If they are excluded on some grounds, then no doubt such forms as "he has been being very cooperative today" will be excluded on the same grounds. If so, this exclusion will require a special statement, outside the analysis of the auxiliary phrase, since in this latter case the second "be" is the main verb, not the auxiliary. (6)

In any event, the decision as to their grammaticalness will
apparently not affect the analysis to be given of the auxiliary phrase. If this analysis is the simplest when they are admitted, then this analysis plus the rule excluding them will, apparently, be the simplest analysis if they are excluded.

The words in the auxiliary phrase have the property of containing no major stress (other than contrastive stress, which any word can assume). There are certain other elements which might be considered as belonging to the auxiliary phrase, including "ought to", "used to", "have to", the passivizing "get", "want to", etc. All of these will be discussed below (several, repeatedly) in this chapter and in chapter IX.

60.2. We have in the grammar the statement $\text{VP} \rightarrow \text{VP}_A \text{VP}_B$ (cf. (23)), and we now face the problem of giving the statement $\text{VP}_A \rightarrow \ldots$ in such a way as to include in $\ldots$ all the forms of table.1 and no others.

As the components of $\text{VP}_A$, we may take the elements:

(i) ed, C

(ii) M = will, shall, can, may, must

(33)(iii) have\text{en}

(iv) be\text{ing}

(v) be\text{en}

ed is the morpheme of the past tense. C is an element of $\overline{P}$ (i.e., a lowest level element of the level $\overline{P}$) which carries the long component of number. The analysis of C into morphemes is thus effected by a mapping (cf. §58.2). Given these elements we can analyze the auxiliary phrase as
(34) $VP_A \rightarrow \left\{ \begin{array}{l}
ed \hspace{1cm} M \hspace{1cm} \text{have}^\text{en} \hspace{1cm} \text{be}^\text{ing} \hspace{1cm} \text{be}^\text{en} \\
C \end{array} \right.$

Thus an auxiliary phrase must contain as its initial element either \text{ed} or \text{C}, and this initial element can be followed by zero or more of \text{M}, \text{have}^\text{en}, \text{be}^\text{ing}, \text{be}^\text{en} in that order. \text{M} can then take any of the forms in (33ii).

But the sequence of morphemes which results from an application of (34) is not in the correct order. Thus to complete the statement we give the following rule:

(35) Let $Y$ be any one of \text{ed}, \text{C}, \text{en}, \text{ing}, and let $Z$ be a prime.

Then $X^\text{en}Y^\text{en}W \rightarrow X^\text{en}Z^\text{en}Y^\text{en}W$,

where $+$ is the marker of word boundary, \text{i.e.}, $\uparrow$ is the concatenation operation of the level $W$.

When (35) is properly formulated, it will thus appear as a part of the mapping $\Phi^P$ which carries the products of derivations in $P$ into strings in $W$. The result of the application of (34) (when M is analyzed by (33ii)) is thus part of a string of $\Theta P$, and we have here the first instance in which it is required that $\Phi^P$ not be order-preserving (cf. § 58).

These two statements, one rule of the grammar, one statement of a mapping, serve to generate exactly the forms in Table 1 (if rows 14, 15, and 16 are to be excluded, then a statement must be added to this effect). As an example of their operation, suppose we have $VP_A^\text{take}$, and $VP_A$ is analyzed by (34) as

(36) $\text{ed}^\text{en} \text{have}^\text{en} \text{be}^\text{ing} \text{take} = \text{ed} \text{have}^\text{en} \text{be}^\text{ing} \text{take}$
Applying rule (35) to a string of \( \text{Gr}(P) \) three times (with \( Y=\text{ing}, \ Y=\text{en}, \ Y=\text{ed} \)), we derive

\[(37) \text{ have}^{\text{ed}} + \text{be}^{\text{en}} + \text{take}^{\text{ing}}\]

which by the morphological rules (i.e., the mappings of \( \mathbb{M} \) into \( \mathbb{W} \)) is carried into

\[(38) \text{ had}^{\text{en}} + \text{been} + \text{taking}\]

Similarly, when (35) is correctly formulated as a mapping, every other element of Table 1 and only these elements, can be generated. Column II results when the initial element of the \( \text{VP}_{A} \) is \( \text{ed} \), columns I and III, when the initial element is \( \text{C} \).

\( \text{C} \) will in turn be analyzed by a mapping \( \Phi_{11}^{P} \) (cf. \( \Phi_{11}^{58.3} \)) whose content will be roughly as follows, where for noun phrases, \( \text{S} \) is taken as the plural morpheme, and \( \Phi \) as the singular morpheme (thus each \( \text{NP}^{\phi} \) becomes either \( \text{NP}^{\phi}\text{S} \) or \( \text{NP}^{\phi}\Phi \)):

\[(39) \text{C goes into } \text{S in the environment } \text{NP}^{\Phi} \text{ ... , and into } \Phi \text{ into the environment } \text{NP}^{\phi}\text{S} \text{ ... .}\]

Thus the long component of number has the peculiar feature that the verb is 'singular' when the subject is 'plural', and vice versa (e.g., "the boy takes", "the boys take").\(^7\) That the same morpheme \( \text{S} \) is affixed to both the noun and the verb is evident both from the fact that this is a long component, and from the fact that the same morphophonemic statements must be made about both elements (e.g., both are voiced after voiced consonants, etc.).\(^8\) When \( \text{C} \) becomes \( \text{S} \), we have column I, when it becomes \( \Phi \), we have column III. Note that several rules are required in the morphology to
account for the special behavior of be and have with S and Ø (just as rules have to be given in the morphology to state the phonemic shapes of particular verbs with en, ed, and ing, as well as S) and for the fact that the elements of M (cf. (33ii)) do not change their shape with S, i.e., that for mcm, m^S-m^Ø (e.g., "will" can be either singular or plural).

At this point it appears that a possible alternative analysis would be to have only one morpheme of number, S, and to say that it may or may not occur, instead of having two morphemes S and Ø with the requirement that one of the other must occur both with nouns and verbs. But it will appear below ( ) that the latter analysis, with the zero morpheme, must be accepted.

The desirability of retaining this extremely simple account of the auxiliary verb phrase seems to be the major reason for the analysis on the morphological level of took into the morphemes take^ed (i.e., "take" plus past tense), similar to hiked = hike^ed, or of sung into sing^en, like eaten = eat^en. Outside of this consideration, there would seem to be little reason to relate take and took, sing and sung at all. But this consideration is a compelling reason. (9) Since this is obviously the correct analysis, it appears that the level of morphology is not independent of the level of phrase structure. i.e., that higher level syntactic considerations furnish criteria for morphological analysis.

In this analysis we have considered the elements ed, S, Ø, en, ing as belonging to the auxiliary phrase, although we saw earlier that the constituent break is, e.g., John-was-eating,
thus $NP_{i} \cdot VP_{A} \cdot VP_{i}$. These two analyses are not strictly compatible, unless the conjunction criterion can be restricted to apply, somehow, after the mapping whose content is (35) has been carried out. Such a condition has no clear meaning in terms of our present theoretical development. Cf. also §58.3. We will proceed as if there were no problem here, and will return to this later with more adequate means.

60.3. Further light is shed on the internal structure of the auxiliary verb phrase by the investigation of infinitive phrases. Consider the constituent analysis of

(40) To prove that theorem was difficult

Clearly, the major constituent break is after "theorem", making "to prove that theorem" the noun phrase subject. The constituents of this noun phrase are customarily taken to be the infinitive "to prove" and the noun phrase "that theorem". But this analysis is not satisfactory. It can be preserved only at the cost of duplicating the description of the verb phrase. To show this properly would require a detailed comparison of the alternatives, but note that part of the description of the verb phrase will be a statement of the objects, prepositional phrases, etc., with which each verb can occur, i.e., a statement, for each verb $V_{i}$, of the set $K_{i}$ of conditioning contexts in which verb can become $V_{i}$. But if our constituent analysis of the noun phrases in question is $\text{Infinitive}^{\alpha} \cdot X$, then this information must be repeated in stating the development of $\text{Infinitive} \rightarrow V_{i}$. That is, we will have to give the general analysis of noun phrases of this type by such a statement as (41)
(41) \[ NP \rightarrow \text{Inf}^X \]
\[ \text{Inf} \rightarrow \text{to} \left\{ \begin{array}{c} V_1 \\ \vdots \\ V_n \end{array} \right\} \text{ in env.} \left\{ \begin{array}{c} K_1 \\ \vdots \\ K_n \end{array} \right\} \]

where $X$ is a statement of everything that can occur after a verb to complete the verb phrase, and $K_i$ is a detailed specification of the conditioning context for $V_i$.

But any complete verb phrase can be turned into a noun phrase by prefixation of "to". Thus the most economical description of such noun phrases is simply

(42) \[ NP \rightarrow \text{to}^V VP \]

so that in (40) the constituents are "to" and "prove that theorem". This avoids the superfluous specification of $X$ and the $K_i$'s given in (41). The internal construction of the noun phrase is now given completely by the analysis of the verb phrase, an analysis which will in any case have to be given separately. Various alternative ways of preserving the analysis of $NP$ as $\text{Inf}^X$, where $\text{Inf} \rightarrow \text{to}^V$, also appear to be redundant and complex as compared to (42). The unreasonableness of taking the infinitive as $\text{to}^V\text{verb}$, rather than $\text{to}^V\text{Verb Phrase}$, becomes particularly clear in cases like

(43) \[ \begin{cases} \text{to keep the soldiers under control wasn't easy} \\ \text{to call these people up is silly} \end{cases} \]

This analysis requires some elaboration, however. Not every verb phrase can occur after to, but only those containing neither $\text{C}$ nor $\text{ed}$, nor any member of $M$(cf. (33)). That is, we
can have "to take", "to have taken", etc., but not "to took", "to takes", "to can take", etc. The simplest way to handle this situation is to revise (34), replacing it by

\[(44) \quad VP_A \rightarrow VP_{A1} < VP_{A2}>\]

\[VP_{A1} \rightarrow \{ed\} <M>\]

\[VP_{A2} \rightarrow <have\text{'en}> <be\text{'ing}> <be\text{'en}> (10)\]

Then as a complete specification of infinitive phrases, we have simply

\[(45) \quad NP \rightarrow to <VP_{A2}> VP_1\]

\[\textbf{Q.4.} \quad \text{This completes the internal analysis of } VP_A, \text{ but it is necessary to add statements of certain contextual restrictions on the form of the auxiliary phrase. This is not surprising, since we know that } P \text{ is a non-restricted system for English.}\]

The principle restrictions are these:

\[(46) \quad \text{(i) } be\text{'en} \text{ can occur only when the following verb is transitive,}\]

\[(\text{ii) } be\text{'ing} \text{ cannot occur in the environment}\]

\[\begin{cases} V_m \text{‘that’} \\ V_n \end{cases}\]

where \(V_m = \text{know, believe, think,}\ldots\)

\[V_n = \text{own, like, belong to,}\ldots\]

Thus we cannot have "John is barked", "John is knowing that he will come", "John is owning his house", etc. The first restriction ((46i)) must certainly be recognized if the conception of "systematic behavior" (i.e., "grammaticalness") developed in linguistic theory is to be judged adequate. (11)

About the second ((46ii)) there may be more question.
This general account of the auxiliary phrase will underly much of the discussion and analysis given below. Later considerations will require that it be slightly recast, but the basic feature of the analysis, namely, rule (35) will be retained throughout.

60.5. Independent support for rule (35) comes from investigation of the noun phrase (and we will see presently, from further investigation of the verb phrase). Alongside of (40) and (43) we have

\[
\begin{align*}
\text{(47) } & \begin{cases}
\text{proving that theorem was difficult} \\
\text{keeping the soldiers under control wasn't easy} \\
\text{calling these people up is silly}
\end{cases}
\end{align*}
\]

It is clear that these are perfectly parallel in their construction to (40) and (43), and that notions of 'verbal noun', etc., are unsatisfactory for the same reasons as is 'infinitive'. By virtue of (35), however, we can analyze these as \textit{ing\textsuperscript{Verb Phrase}}, so that "to" and "ing" are elements of exactly the same kind. Thus, where \( \text{NP}_{I} \) is a special kind of noun phrase (in fact, a subclass of abstract nouns), we have

\[
\begin{align*}
\text{(48) } & \text{NP}_{I} \rightarrow \begin{cases}
\text{to} \\
\text{ing} \\
\text{VP}_{A2} \\
\text{VP}_{2}
\end{cases}
\end{align*}
\]

replacing (45). The fact that \textit{ing} occurs as a suffix (not as a preposed word, like to) is an automatic consequence of (35), which, as we have seen, was independently motivated.

One restriction seems necessary here. \textit{being} cannot occur after \textit{ing} (e.g., there is no "being eating dinner when they arrive would be quite impolite", though there is ""to be eating dinner when they arrive would be quite impolite").
It will appear below that (48) plays a crucial role in the analysis of the verb phrase. We note already that (48) might be said to furnish a criterion (in much the same sense as does the rule for conjunction) for the establishment of the major constituent break in the sentence, i.e., for the choice between (10) and (11) as analyses for (9). Given \( N^aV^bN \), \( N^aV^bPP \) (PP = Prepositional Phrase), etc., if we set up a prime \( \Omega \) representing \( N^aV \), and analyze these sentences into \( \Omega^aN \), \( \Omega^aPP \), etc., in accordance with (11), then this prime \( \Omega \) will be of no further utility in stating grammatical rules -- it will appear nowhere else, and in (48), we will have to list all the forms \( V^aN \), \( V^aPP \), etc. as new elements, which have played no previous role in the grammar. But if we analyze these sentences into \( N^aVP \), with \( VP \) representing \( V^aN \), \( V^aPP \), etc., in accordance with (10), then this prime \( VP \) can appear again, with no further discussion, in (48), to be analyzed in the grammar by the same statement as develops \( V^aN \), \( V^aPP \), etc., from \( N^aVP \) when \( VP \) occurs in \( N^aVP \). The various forms of \( VP \) need not be listed in (48). In other words, we might say, in a loose sense, that a corollary of the principle of simplicity is that \( P \) should be constructed in such a way that each prime will appear in many statements of the grammar.

One way to test the adequacy of a formal construction is to compare the analysis to which it leads with our intuitive conception of linguistic form (cf. §7). Correspondence can be taken as a partial corroboration for the abstract underlying conceptions from one viewpoint, or, from another viewpoint, as providing a formal linguistic basis for these intuitions.
We have not yet analyzed the verb phrase, but clearly one possible analysis will give the sentence type

\((49) \text{NP} \overset{\text{i.e.}}{=} \text{NP}\)

e.g., "John is my friend". But we have seen that one form of the NP is \(\text{NP}_1\) as in (48). Thus we should be able to have

\((50) \text{NP} \overset{\text{i.e.}}{=} \text{NP}_1\) —e.g., "the important thing is to be happy"

Thus it follows from the analysis to which we have been led that there should be two ways of interpreting a sentence of the form

\((51) \text{NP} \overset{\text{is} \overset{\text{ing}}{\text{ing}}}{} \text{VP}_1\)

either as a case of (50), with \(\text{is}\) as the main verb and \(\text{ing} \overset{\text{VP}_1}{}\) as the \(\text{NP}_1\), or, with \(\text{is} \overset{\text{ing}}{\text{ing}}\) as the auxiliary, as a case of

\((52) \text{NP} \overset{\text{VP}_1 \overset{\text{VP}_1}{}\text{VP}_1}{}\)

But this is in fact quite in accord with intuition.

Clearly we understand (53) and (54) in quite different ways,

\((53) \text{The important thing is winning}\)

\((54) \text{Our team is winning}\)

with quite different constituent structure. (53) is an instance of (50), and (54) of (52). Despite some clumsiness, we might consider

\((55) \text{his suggestion was becoming famous}\)

to be a case of constructional homonymy.

\(60.6\). To sum up our major conclusions at this point, the basic structure of the English interpretation of \(\text{P}\) can be stated as follows:

6.1. We have not yet discussed VP₁, except to point out that one analysis is as VₐNP. The investigation of the distribution of the infinitive elements to end ing in the verb phrase provides a further analysis. Consider first the case of to-phrases. There are three fundamental patterns for the occurrence of these in the Verb Phrase. Where "to-phrase" denotes any instance of NP beginning with to (cf. (48)), we have three classes of verbs, Vₐ, Vₐ, Vₐ such that

\[(57) \ (i) \ VP₁ \rightarrow Vₐ \ NP \ \text{to-phrase} \]
\[(ii) \ VP₁ \rightarrow Vₐ \ NP \ \text{to-phrase} \]
\[(iii) \ VP₁ \rightarrow Vₐ \ NP \ \text{to-phrase} \]

Vₐ = want, like, ask, beg, ...
Vₐ = advise, compel, order, persuade, ...
Vₐ = decide, demand, begin, try, fail, ...

Conforming to (57i), we have "they want him to come", "they want to come"; conforming to (57ii), "they advise him to come"; conforming to (57iii), "they decided to come".
For *ing*-phrases (where "*ing*-phrase" denotes any \( NP_1 \) beginning with *ing*) we have a parallel situation, with classes \( V_\alpha, V_\beta, V_\gamma \) such that

\[(58) \quad (i) \quad VP_1 \rightarrow V_\alpha < NP > \text{ ing-phrase} \]
\[(ii) \quad VP_2 \rightarrow V_\beta NP \text{ ing-phrase} \]
\[(iii) \quad VP_1 \rightarrow V_\gamma \text{ ing-phrase} \]

\( V_\alpha = \) imagine, prefer, visualize, ...
\( V_\beta = \) find, catch, ...
\( V_\gamma = \) urge, postpone, regret, forget, stop, ...
\( V_\gamma = \) avoid, begin, try, ...

Conforming to (58i) we have "I can't imagine him riding a horse" or "I can't imagine riding a horse"; to (58ii), "I found him smoking"; to (58iii), "I urged putting it off for a few days".

The distinction between subclasses of \( V_\gamma \), though not of \( V_\alpha \), is necessitated by the somewhat broader distribution of *ing*-phrases. An \( NP_1 \) can occur after a possessive adjective ("the man's", "my", etc.), but only if it is an *ing*-phrase. It is thus necessary to distinguish between *ing*-phrases which follow possessive adjectives, and those that can not. Thus we have "I regretted their refusing to come", but not "I avoided their refusing", etc. An analogous distinction might be necessary in \( V_\alpha \), but it appears that all members of \( V_\alpha \) can occur with these adjectives. Thus we have "I imagined their being lost in the storm", etc. This distinction seems to vary somewhat from speaker to speaker, as does the distinction between \( V_\alpha \) and \( V_\gamma \).
It may be that $V_{\gamma_1}$ should be treated together with $V_{\alpha}$, with the possessive adjective as a variant of the 'objective' subject. We return to this later on.

Since to-phrases and ing-phrases are noun phrases (by (48)), we might regard (57) and (58) as being special cases of transitive verb phrases, with either one or two objects. Thus "they want to come" would be a case of NP-V-NP, like "they want a drink", and "they want him to come" would be a case of NP-V-NP-NP like "they asked him a question". But this counter-intuitive analysis is excluded by a closer consideration of such sentences. We will return to this below, after analyzing the transitive verb.

64.2. There is also a class of verbs $V_j$ that occurs with ing-phrases but not with to-phrases.

\[(59)\] $VP_1 \rightarrow V_j^{NP} \langle ing \rangle \ VP_1 \ (12)$

\[V_j = \text{see, feel, hear, watch,..}\]

Thus we have "I saw him come", "I saw him coming"; etc.

$V_j$ is a class of special interest because it introduces a distinction between be\textsuperscript{en} and the other elements of the auxiliary phrase. Alongside of (59) we also have

\[(60)\] $VP_1 \rightarrow V_j^{NP} \langle ing \rangle \ \text{be}^{\text{en}}^{\text{VP}_1}$

giving sentences as

\[(61)\] I don't like to see people \{ be \}

\[(62)\] \{ being \} intimidated

\[(63)\] " " " " " " accused without evidence

But we cannot have either (63) or (64):
(63) I don't like to see people \{ be \} drinking \{ being \} (13)

(64) " " " " " \{ have \} \{ having \} drunk

Thus be\(^{\text{en}}\) can occur in this position, but neither be\(^{\text{ing}}\) nor have\(^{\text{en}}\) (nor, of course, \{ have \} \{ be \} \{ ing \}, etc.) can occur here.

A way to avoid the necessity of treating (60) as an exception is suggested by the fact that (65) is a grammatical sentence.

(65) I don't like to see people \{ be \} \{ argumentative without cause \} \{ being \}

This is a case of (59) = \( Y_5 \ P < \text{ing} > \ V P_{1} \), where \( V P_{1} \) becomes be\(^{\text{adjective}}\). But "accused" \( \text{en} \text{accuse} \) (or "climbed") can also be regarded as an adjective. If it is, then (60) becomes a special case of (59), paralleling (65). This suggests that the analysis can be simplified if be\(^{\text{en}}\) is dropped from \( V P_{A2} \), so that a passive such as "John was accused" is treated as a special case of be\(^{\text{adjective}}\), just like "John was sad" or "John was tired". (14)

There are other examples of a distinction between be\(^{\text{en}}\) and the other elements of \( V P_{A2} \) that lend further support to the analysis of passives as be\(^{\text{adjective}}\). For instance, we see that (66) and (67) are acceptable sentences, but not (68).

(66) he likes to attend meetings and be applauded (by his supporters)

(67) " " " " " " " friendly (with the delegates)

(68) " " " " " " voting (for his friends)
Here again be en V is treated as a VP₁ with the main verb be and a predicate adjective en V, rather than as VP₂ en V. In this case, the evidence that it is in fact a VP₁ is given by the conjunction criterion, since "attend meetings" is certainly a VP₁.

We will find additional support for this analysis below. It appears to be the case that this analysis of passives as be adjective is actually forced on us by considerations of simplicity. However, it is not an intuitively acceptable analysis, as it stands, since it fails to account for the fact that in "John was accused", "accuse" has, intuitively, a 'verbal force' not shared by "sad" in "John was sad", "tired" (which is also en Verb) in "John was tired", or "surprising" in "his success seems surprising". The failure of this analysis to account for intuition becomes even clearer in the case of "John was accused by his enemies", analyzed as John was adjective PP, like "John was successful on the stage."

There are also systematic difficulties with this analysis, since the subcategory of adjectives consisting of true passives (cf. fn.14) fails to share certain distributional features of predicate adjectives, e.g., they do not occur after 'stative' verbs — we do not have "he seems accused". This stands as an unmotivated exception at the present level of analysis. Actually neither of the possible alternative analyses seems quite right on this level, a fact which suggests that our theory is inadequate. This is a conclusion which has already
appeared several times, and to which we will be led repeatedly throughout this discussion of phrase structure. We will continue to record these inadequacies, and proceed with the analysis, returning to them later with more adequate means.

62.3. The analysis of transitive verbs depends on how fine an analysis is made of nouns, and both questions thus refer back to the grammaticalness considerations of chapter IV. In this analysis of English phrase structure, we assume that, among others, the following distinctions have been established.

(69) \[ N_{\text{anim}} = \text{John, boy, dog, } \ldots \]
\[ N_{\text{inan}} = \text{table, book, } \ldots \]
\[ N_{\text{ab}} = \text{sincerity, truthfulness, justice, companionship, } \ldots \]

Certainly many further distinctions must be recognized, but this will suffice for present purposes.

Transitive verbs \( V_T \) can be classified by the subjects and objects that they take. We will recognize, for the time being, the following subclasses of \( V_T \).

(70) \[ V_{t1} = \text{appoint, feed, sue, scold, } \ldots \]
\[ V_{t2} = \text{like, recognize, admire, desire, seek, } \ldots \]
\[ V_{t3} = \text{frighten, surprise, bore, entertain, interest, thrill, } \ldots \]
\[ V_{t4} = \text{carry, scratch, see, throw, } \ldots \]

We can add to the specification of \( VP_1 \) a statement based on (71).

\[ VP_1 \rightarrow \left\{ \begin{array}{c}
V_{t1} \ldots N_{\text{anim}} \\
V_{t2} \ldots N_{\text{inan}} \\
V_{t3} \ldots \end{array} \right\} \text{ in env. } N_{\text{anim}} \overset{VPA}{\hookrightarrow} \]

(71)

\[ VP_1 \rightarrow V_{t3} \ldots N_{\text{anim}} \]
Thus we have "John appointed the boy", "John likes sincerity", "John carried the table", "sincerity frightens John", etc.

We can also distinguish among the intransitives $V_1$, between those that have animates and inanimate subjects, thus recognizing the classes

$V_{11} =$ sleep, run, bark, ...
$V_{12} =$ occur, break, ...

(71) and (72) will play a crucial role in the following discussions. Since these rules of the grammar state how the choice of the verb is determined by the nature of the subject and the object, we will refer to them as rules of verbal selection. We will extend this name to refer to any form that these rules may take under subsequent formulations of the grammar, and to extension of these rules to cover other positions where $V_T$ and $V_I$ may occur. This account is much oversimplified. Actually, the rules of verbal selection will be much more complex in a detailed and adequate grammar. Note that we are building a system of phrase structure only for first order grammatical sentences, a category that presumably excludes such semi-sentences as "sincerity appointed the table" and "John frightens sincerity". Given the grammar, these partially grammatical sequences can be recovered from the system $C$. Note further that the fact that an element is in one class does not, of course, mean that it cannot also be a member of other classes.

If we proceed with the system of (56) one major qualification must be added to (71). This rule will hold only if $V_P$ does not
contain be\^en. If be\^en does appear in \( \text{VP}_A \), then a transitive verb can occur even with no following noun phrase, and a separate rule of verbal selection must be given, since in the passive, subject and object are interchanged. This is a major distinction between be\^en and the other elements of the auxiliary phrase. This consideration lends support to the proposal of \( \text{66}.2 \) that be\^en be dropped from \( \text{VP}_A \) and that passives be treated as be\^adjective. If this is done, then no qualification need be added to (71), though it is still necessary, naturally, to give a statement of the contextual conditions on the occurrence of transitive verbs in the context be\^en. Note further, in this connection, that the major restriction on the development of \( \text{VP}_A \), namely, (46i), can also be dropped if we follow this suggestion. We will proceed now, accepting this revision. be\^en is dropped from \( \text{VP}_A \) in (56), (i) is dropped from (46), (60) can be dropped from the grammar, and we will find it easier to account for sentences like (66). We are left with a grammatical statement that is somewhat simpler, but that does not succeed in grounding certain clear intuitions about linguistic form, since the apparent verbal force of the passive is unexplained. We will return to the case of the passive from the different point of view which will be developed in the next chapter, where it will appear that one dimension of the analysis has been left out. We must now state, as part of the analysis of the adjective phrase AP

\[(73) \quad \text{AP} \rightarrow \text{an}^\circ \text{V}_T\]
(74) gives a partial analysis of the verb phrase with be as the main verb.

\[
V_P \rightarrow \text{be} \left\{ \begin{array}{l}
\text{NP} \\
\text{AP} <\text{PP}> \\
\end{array} \right\}

\text{(PP = prepositional phrase)}
\]

Having accepted this solution for the passive, we can rephrase (48) somewhat more adequately as

\[
N_{P1} \rightarrow \left\{ \begin{array}{l}
\text{to} <V_{P2}> \\
\text{ing} <\text{have}^\text{n} \text{en}> \\
\end{array} \right\} \text{VP}_1
\]

The extra restriction given below (48) is now incorporated into rule (75).

62.4. The further analysis of \( V_{P1} \) gives such forms as

\[
V_{e} <\text{that}> \text{Sentence} \quad \text{(I knew}\langle\text{that}\rangle\text{he would come)}
\]

\[
V_{e} ^{\text{NP}} \text{to} ^{\text{a}} \text{be} \left\{ \begin{array}{l}
\text{NP} \\
\text{AP} \\
\end{array} \right\} \quad \text{(I believe him to be a creative thinker)}
\]

\[
V_{f} ^{\text{NP}^2} \text{NP} \quad \text{(I gave him three books)}
\]

\[
V_{g} ^{\text{NP}^2} \text{NP} \quad \text{(they elected him president)}
\]

\[
V_{h} ^{\text{NP}} \left\{ \begin{array}{l}
\text{NP} \\
\text{AP} \\
\end{array} \right\} \quad \text{(they considered him a creative thinker)}
\]

\[
V_{e} = \text{know, believe, think, discover, ...} \\
V_{f} = \text{give, ask, refuse, make, ...} \\
V_{g} = \text{elect, choose, ...} \\
V_{h} = \text{consider, think, make, ...}
\]

\( V_{f} \) and \( V_{g} \) differ in that \( V_{g} \) requires agreement in number between the succeeding noun phrases.
There are still further forms of the verb phrase, but if we were to continue to analyze these we would apparently be led to quite counter-intuitive analyses. We will return to the more complex forms later on in chapter IX, when English grammar will be approached from a different point of view.

We now return to the investigation of the validity of the proposal of the last paragraph of §64.1 that "they want to come" be analyzed as $\text{NP-V-NP}_1$ (15) and "they want him to come" as $\text{NP-V-NP}_1^2\text{NP}$, with the final NP in each case being $\text{NP}_1$ of (75). This proposal then offers as part of the analysis of $\text{VP}_1$

\[
(77) \quad \begin{array}{c}
\text{NP}_1 \\
\text{NP}_2 \\
\text{NP}_3 \\
\text{NP}_4 \\
\text{NP}_5 \\
\text{NP}_6
\end{array}
\]

where $\text{NP}_1 \rightarrow \text{NP}_1$ when the verb is not $V_1$, $V_3$, or $V_5$. There are two overriding considerations that rule out this analysis. One concerns the rules of verbal selection, and one a certain rule of agreement in number.

We have analyzed sentences like

(78) John wants to read the book
as though "wants" were the main verb, and "read" part of the succeeding nominal phrase. But alternatively, we might consider "read" to be the main verb, and "wants to" to be a special type of preposed auxiliary. Actually, the latter analysis seems to be necessary at this point. To see this consider the rule of verbal selection given above in (71). Exactly the same rule applies for the environment (79) as for (80).

(79) \( \text{NP} \xrightarrow{\text{VP}_1} \tilde{\text{V}} \xrightarrow{\text{NP}_1} \quad \text{to} \quad \text{NP} \quad (15^\circ) \)

(80) \( \text{NP} \xrightarrow{\text{VP}_1} \quad \text{NP} \quad \)

Thus we have (81) but not (82).

(a) The law covers these cases

(b) The law fails to cover these cases

(82)

(a) The law eats lunch

(b) The law fails to eat lunch

A more detailed analysis of transitive verbs than the one we have given would also show the possibility of (83) but not (84)

(a) The law applies to theft

(b) The law fails to apply to theft

(83)

(a) John applies to theft

(b) John fails to apply to theft

(84)

It follows that the simplest way to give this rule is to cover both cases in a single statement, i.e., paralleling (71), as

\( \text{NP}_1 \rightarrow \tilde{\text{V}}_1 \xrightarrow{\text{NP}_1} \quad \text{in env.} \quad \text{NP}_x \xrightarrow{\text{VP}_1} \tilde{\text{V}}_x \xrightarrow{\text{to}} \quad \text{NP}_y \quad \rightarrow \quad \),

where \( \text{NP}_x \) and \( \text{NP}_y \) are specifications of the various relevant types of noun phrases, and \( x, y = 1, \ldots, 4 \).
or perhaps

\[(86) \quad V_T \rightarrow V_t \text{ in env. NP}_x \,
\wedge \,
\text{VP}_A \, \langle V_a \,^{to} \rangle \,
\rightarrow \,
\text{NP}_y\]

But if we develop the grammar on the basis of (77), it will be impossible to give the same rule for both the case with \(V_a \,^{to}\), and the case without it. The reason for this is that in the reduced grammar, the statement which gives the analysis of \(\text{NP}_I \,^{to} \text{VP}_1\), i.e., (75), must occur after (77), since \(\text{NP}_I\) (which becomes \(\text{NP}_I\)) is introduced in (77). Hence the \(\text{VP}_1\) within \(\text{NP}_I\) will not be developed in running through the grammar the first time, since the statement that develops \(\text{VP}_1\) is (77). The result of running through the grammar once will thus be, in the case of (78),

\[(87) \quad \text{John} \,
\wedge \,
\text{wants-to} \,^{\text{VP}_1}\]

and only by running through the grammar again will \(\text{VP}_1\) become \(\text{read} \,^{\text{the} \,^{\text{book}}.}\) But (85) or (86) will not apply (in the form given) to this \(\text{VP}_1\) in running through the grammar the second time, since \(\text{NP}_x\) in (85) or (86) will have been fully developed in running through the grammar the first time, and the form of \(\text{NP}_x\) which appears in these rules will thus no longer be present as a conditioning context. Thus if we consider "want" as the main verb in (78), we must rephrase (85) or (86) with a double characterization of \(\text{NP}_x\), once in only partially developed form to determine the choice of the main verb "read" in such sentences as "John reads the book", and once in fully developed form (with a departure from optimality in this case, cf. fn.16) to determine the choice of the verb in the following to-phrase in sentences like (78). Various attempts to avoid this run aground as long as (77) is retained as the basic
A second and related difficulty concerns agreement in number. Consider the sentences

(a) John wants to be an officer
(b) they want to be officers

If "wants" is regarded as the main verb, as in (77), then the same difficulties will arise in explaining the possibility of (88) alongside of the impossibility of

(a) John wants to be officers
(b) They want to be an officer

If "be" is considered part of the following noun phrase, it will not be developed until the second run-through of the grammar, necessitating a complex statement of agreement in number for just the reasons we saw above in the case of verbal selection.

The rule which has just been discussed, as well as all subsequent rules of similar type, will be referred to as rules of agreement in number. The two major considerations in rejecting (77), then, are the complications which result for the rules of verbal selection and agreement in number. The effect of the latter is considerably less compelling, however, since agreement in number could be handled by a mapping, somewhat similar in form to (39). In investigating the further effect of these rules we will concentrate on the problem of selection and show how considerations of number parallel this. Such citation of the rules for number does lend some support to the conclusions reached in terms of the rules of selection,
since even in the alternative treatment of number which has just been suggested, we should like to keep the mapping from becoming too complicated.

63.2. These considerations lead us to set up a second auxiliary verb phrase $VP_B$ containing $V_a$. Whatever solution is adopted for $V_a$ will hold as well for $V_x$, $V_2$ and $V_y$. Thus $VP_B$ will contain these elements as well. The simplest way to accomplish this is to incorporate $V_a$ into $V_B$, $V_C$, and $V_\alpha$ into $V_\beta$, $V_\gamma$. taking $V_a$ as the overlap of $V_B$ and $V_C$ and $V_\alpha$ as the overlap of $V_\beta$ and $V_\gamma$. $VP_B$ will then contain $V_2$ and $V_\gamma$, and also sequences $V_2^\wedge V_C$, $V_C^\wedge V_\gamma$, etc., since in a sentence like

(90) John wanted to try smoking a different brand

"smoke" must be treated as the main verb for the reasons that we have seen above.

We have, then, as possible analyses of $VP$, both $VP_A^\wedge VP_B^\wedge VP_1$ and $VP_A^\wedge VP_1$. The correct way to analyze the first of these strings seems to be as $VP_A^\wedge VP_B^\wedge VP_1$. As the initial part of the analysis of the verb phrase, then, we will have

(91) $VP \rightarrow VP_A^\wedge VP_1$

$VP_1 \rightarrow \langle VP_B \rangle VP_2$

This analysis is necessitated, on the one hand, by the fact that $VP_B^\wedge VP_2$ occurs in the same positions as $VP_x$ (e.g., in "I saw him trying to open the door" alongside of "I saw him opening the door"), and, on the other hand, by the conjunction criterion, since we have, e.g.,
(92) he should stop wasting time and get down to serious work

A complicating factor is that $VP_B$ can be followed by $VP_{A2}$ (where $VP_{A2}$ can be only have$^enk$ after ing), e.g.,

(93) he wanted to have been introduced

This possibility, with exactly the correct restriction, would have been taken care of automatically by (75) if the analysis of (93) as NP-wanted-NP$_1$ had in fact been accepted, but in the solution we have just adopted, it must be given separately. This would seem to shed some doubt on our analysis, but we will see below, in \[\text{\textit{[text]}},\] when we come to actually sketch the grammar, that these two restrictions can still be stated together, even with our present solution. We will overlook this apparent complication here, omitting sentences like (93), but in presenting the grammar below we will include them again. Cf. statements 10, 21, 47.

In the light of this discussion, we amend (56) as follows

\begin{align*}
(94) & \quad \textbf{Sentence} \rightarrow \text{NP}^n \text{VP} \\
& \quad \text{VP} \rightarrow \text{VP}_A^n \text{VP}_1 \\
& \quad \text{VP}_1 \rightarrow \langle \text{VP}_B \rangle \text{VP}_2 \\
& \quad \text{VP}_A \rightarrow \text{VP}_A \langle \text{VP}_A^2 \rangle \\
& \quad \text{VP}_A \rightarrow \{C \langle M \rangle \} \\
& \quad \text{VP}_A \rightarrow \langle \text{have}^enk \rangle \langle \text{be}^ing \rangle \quad (18) \\
& \quad \text{VP}_B \rightarrow \langle Z_1 \langle Z_2 \langle \ldots \langle Z_n \rangle \ldots \rangle \\
\text{where each } Z_i \text{ is one of the } V_{c}^\text{to, } V_{y}^\text{ing} \quad (19)\end{align*}
Coalescing several steps, the derivation of (78) will be

\[(95) \text{Sentence} \]
\[\text{NP}^c \text{VP} \]
\[\text{NP}^c \text{VP}_A \text{VP}_B \text{VP}_1 \]
\[\text{NP}^c \text{VP}_A \text{VP}_B \text{VP}_2 \]
\[\text{NP}^c C \text{VP}_B \text{VP}_2 \]
\[\text{NP}^c C \text{want} \text{to} \text{VP}_2 \]
\[\text{John}^c \text{want} \text{to} \text{read} \text{the} \text{book} \]

62. The solution embodied in (94) incidentally has a more satisfactory intuitive correspondence than the solution (77) which we have now rejected, since "John" does seem intuitively to be the subject of "read" in (78). Further evidence that we are on the right track comes from a closer investigation of \(V_{\alpha}\) and \(V_{\gamma}\). We find that as these classes are defined in (58) (recall now that we have assigned \(V_{\alpha}\) to both \(V_{\beta}\) and \(V_{\gamma}\), as the overlap of these classes), only a part of them is subject to this analysis.

We have seen that such verbs as "want" are properly assigned (in certain of their uses) to an auxiliary phrase \(VP_B^c\). The criterion applied in this assignment was the following.

\[(96) v \text{ is assigned to an auxiliary phrase } v^c \text{to}, \text{ rather than being regarded as the main verb (with object } to^c \text{VP}_1)\text{, if it meets the condition that for each noun phrase } np, \text{ and for each verb phrase } vp, \text{ if } np^c v^c \text{to} ^c vp \text{ is grammatical, then so is } np^c vp. \text{(20)}\]

If this condition is met, we found that the rules of selection
and number were simplified significantly by this analysis. In fact, all members of $V_a$ and $V_c$ meet this condition. Investigating $V_a$ and $V_c$, however, we find that this is not the case. $V_y$, it will be recalled, is the class of verbs that occur in the environment $\text{---ing}^{\text{VP}_1}$. For a verb $v$ to fail to meet the analogue for $V_y$ of (96), it must be the case that

(97) there is a noun phrase $np$ and a verb phrase $vp$ such that $np^v\text{---ing}^\epsilon vp$ is grammatical, but $np^v vp$ is not.

But (97) is in fact satisfied for certain verbs $v$ in $V_y$. e.g., $v=\text{forbid}$. Thus we have (98) but not (99).

\[\begin{cases}
  (a) \text{That law forbids eating in the park} \\
  (b) \text{Decency forbids mentioning that incident}
\end{cases}\]

\[\begin{cases}
  (a) \text{That law eats in the park} \\
  (b) \text{Decency mentions that incident}
\end{cases}\]

The same is true of "prevent", "concern", "cover", etc. This makes it necessary to distinguish between those members of $V_y$ that meet (96), and those that do not. Suppose that we continue to denote by $V_y$ those that meet (96), thus preserving (94). Those that do not meet (96) we can then extract from $V_y$, and set up now as a new subclass $V_T$ of $V_T$, occurring with 'abstract' objects $NP_I$. Thus we add to (71)

(100) $\text{VP}_2 \rightarrow V_{t_5}^{\text{---NP}_I}$ \hspace{1cm} (21)

This gives a distinction between the sentence forms (101) and (102)

(101) $\text{NP}-V_{t_5}^{\text{---NP}_I}$

(102) $\text{NP}-VP_B-\text{VP}_1$
with (98) as instances of the former and

(a) John likes eating in the park
(103)
(b) John avoided mentioning that incident

as examples of (102). We see then that in pursuing the
program of describing grammatical sentences in as simple a
way as possible, we are led to assign quite different
analyses to (98) on the one hand, and (103) on the other,
as, intuitively, should be the case.

Continuing in this manner, we see that, with one qualification
which we give below, the solution to which we have been led
permits a certain interesting case of constructional homonymy
in the case of a verb v which is a member of both \( V_7 \) and \( V_{15} \).

Given (101) and (102), suppose that \( \text{NP}_I \rightarrow \text{ing}'\text{VP}_I \)
in (101), and \( \text{VP}_B \rightarrow v^\prime \text{ing} \) (in (102)). Suppose further
that \( V_{15} \rightarrow v \) in (101), and that \( V_7 \rightarrow v \) in the further
development of (102). I.e., v is a member of both \( V_{15} \) and
\( V_7 \). Then we may have a sentence of the form

(104) \( \text{NP} v^\prime \text{VP}_A \rightarrow v^\prime \text{ing} v^\prime \text{VP}_I \)

analyzed into both (105) (from (101)) and (106) (from (102)).

(105) \( \text{NP} v^\prime \text{VP}_A \rightarrow v^\prime \text{ing} v^\prime \text{VP}_I \)
(106) \( \text{NP} v^\prime \text{VP}_A \rightarrow v^\prime \text{ing} \rightarrow \text{VP}_I \)

where '-' marks constituent break.

There are actual instances that, intuitively, are subject
to this ambiguity. Thus with \( v = \text{stop} \), we have

(107) The policemen must stop drinking after midnight

understood as analogous in construction to (98) or (103).
There is one difficulty here, however. We have given the criteria for membership in $V_\gamma [(96)]$ and $V_{t5} [(97)]$ as contradictories, and this excludes the possibility of homonymity. Thus we have no basis for putting "stop" into both $V_\gamma$ and $V_{t5}$. Any verb meeting (97) (as does "stop") is simply in $V_{t5}$. This difficulty is only apparent. It is due to an insufficient account of grammatical sentences. A more detailed study would reveal that verbs in $V_\gamma$ appear in certain contexts not shared by verbs in $V_{t5}$, and this will lead us to replace (96) by a fuller characterization which is not the contradictory of (97). For example, we find that for verbs in $V_\gamma$, e.g., "try", we have

(a) John tried reading to himself
(108)
(b) try reading to yourself

"suggest", etc.

But for verbs such as "forbid", in $V_{t5}$ we cannot have the analogous forms

(a) John forbade reading to himself
(109)
(b) suggest reading to yourself.

The intuitive explanation for this need not concern us. The grammaticalness of (108) but not (109) shows that for a verb to be in $V_\gamma$ it must meet the positive criterion that sentences like (108) be grammatical. Verbs in $V_{t5}$ fail to meet this condition. But "stop" is in $V_{t5}$, because of

(110) this new law will stop drinking (but not "this new law will drink")

and it is in $V_\gamma$, because of

(a) John stopped reading to himself
(111)
(b) stop reading to yourself.
Thus if we can establish the grammaticalness of sentences like (108), but not (109), then (107) does appear as a case of constructional homonymity.

Another such case of constructional homonymity arises from the fact that "is to", as in

(112) John is to come tomorrow

meets (96), and hence is assigned to $VP_B$. But $is$ as a main verb can have an $NP_I$ object (cf. §60.5), as in

(113) The important thing is to be happy

Our analysis thus leads to a distinction between these intuitively quite different cases. (114), like (55), is a case of constructional homonymity, both intuitively and by our analysis. (22)

(114) his suggestion was to become famous

Such intuitive and semantic correspondences cannot be used directly to support the analysis, but they do support the conception of syntax upon which the analysis was grounded by indicating that these grounds lead to intuitively correct results. By the same token, intuitive inadequacy of the results is an argument against the soundness of the underlying conception. In the case of (98) and (103), it seems likely that the intuitive correspondence can be demonstrated by a detailed investigation; in the case of (107), it seems perhaps less so. Cases which we meet below will present strong reasons for doubting the adequacy of our underlying conceptions.

It will be recalled (last paragraph of §50.2) that the adopted analysis of the auxiliary phrase was strictly incompatible
with the conjunction criterion, because of the treatment of
the affixes 'ing', etc. The same difficulty arises here
with regard to 'ing' and 'to'. Thus we have

(a) John likes reading novels and playing tennis
(115)
(b) " " to read novels and to play tennis

although the adopted analysis puts the constituent break
after 'like' 'ing' and 'like' 'to'. We will continue to disregard this
difficulty for the moment, returning to it below.

64-3. In 46A we have seen how the rules of verbal selection
and agreement of number play a crucial role in determining
the structure of the verb phrase. These furnish criteria
for the analysis in exactly the same sense as does conjunction.
Since at best these rules are quite complex, we set up the
analysis in such a way as to avoid compounding this
complexity by requiring these rules to be stated twice,
once with the conditioning contexts partially developed,
to just the relevant forms (i.e., in the case of selection,
the forms of (69); in the case of number, the two primes
$NP_{\text{plural}}$ and $NP_{\text{singular}}$, or something similar), and once
with the conditioning contexts fully developed. In fact,
the desirability of avoiding the latter formulation alone
suggests an important criterion of analysis because of the
great complexity of this formulation and the consequent
departure from our conditions of 'optimality' for grammars,
as pointed out in fn.16. We may now investigate the
effect of these criteria on the further analysis of the
noun phrase.

Referring to the adjoining diagram, we will encounter the distinction brought out above
between $\Theta$ $\Pi$ and $\Pi_2$. That is, we will start from consideration such Verb-to-Verb
constructions as 'I want to come', 'I want to try to come', etc.
One form that can be assumed by the noun phrase is
given by (116)

(116) $NP \rightarrow NP_\alpha \, ^{ing}V_P$

E.g., the noun phrase subject of (117a), or the object of (117b)

$$\begin{array}{l}
(a) \text{The people holding these slips—may enter} \\
(b) \text{I know the man standing at the bar}
\end{array}$$

The expression on the right in (116) is an instance of
$NP_\alpha NP_P$, i.e., of a compound noun phrase, and the subscript 'α' in $NP_\alpha$ is given purely for reference. Although there are
certain restrictions on the NP's that can occur here, we will
not concern ourselves with them for the moment.

Consider now the problem of analyzing such noun phrases,
that is, the problem of introducing (116) into the reduced
grammar. The statements of the grammar which are relevant in
this connection are the following:

(118) 1. The statement analyzing $\phi$ into $V_P^\alpha NP$, $be^\alpha NP$, etc.
    We will call this statement the "$V_P^\alpha$ analysis".

2. The statement analyzing $NP$ into the terms relevant
    for the rule of verbal selection, i.e., essentially, the
    analysis into the left hand terms of (69). We
    will call this the "NP-analysis" (more specifically,
    the "selectional NP-analysis").

3. The rule of verbal selection

4. The statement analyzing $NP$ into the terms relevant
    for the rule of agreement in number, essentially,
    a statement of the form:
\[ \text{NP} \rightarrow \begin{cases} 
\text{NP}_{\text{sing.}} & \text{in env., etc.} \\
\text{NP}_{\text{pl.}} & \text{"" etc.} 
\end{cases} \]

or something similar.

5. The rule of numerical agreement.

6. The rule which completes the development of the noun phrase to specific nouns.

Consider now the relative order of (116) and 1, 2, 3, 6 of (118). It is first of all clear that 1 precedes 2 precedes 3. Otherwise the rule of selection will not apply properly to simple NP-V-NP sentences. We must assume further that 6 follows the rule of selection, or the conditions of optimality will immediately be violated as pointed out above and in fn. 16. We may assume further that both NP-analyses, (118-2) and (118-4), apply to NP₂ from (116) as well as to NP, since there is no need here to distinguish between the nouns that do and do not occur in the NP₂ position. To simplify the discussion, we refer here only to transitive verbs.

Let us say that a selectional relation holds between a certain noun and a certain verb if the choice of the verb by the rule of verbal selection depends on the form of the given noun. This can be given much more precisely as a relation between 'positions' in strings, but the notion involved is clear.

If we are to meet the criteria posed by the rule of verbal selection, then in any case where such a selectional relation holds, the verb must appear as \( V_T \) and the noun as \( N_{\text{anim}}, N_{\text{inan}}, \) or \( N_{\text{ab}} \) (cf. (69)) at the point in the derivation where the rule of verbal selection applies to the verb in question.
This is to say that the rule of verbal selection cannot apply to the verb in question before the application of the NP-analysis to the NP in question, or after the application of (118-5) to the NP in question. Let us call this requirement the condition C.

We now investigate the selectional relations in sentences of the type (117) with the purpose of determining where (116) must be placed relative to 1, 2, and 3 of (118) in order for condition C to be met. Since \( \text{NP}_a \overset{\text{ing}}{\rightarrow} \text{VP}_1 \) can be either subject or object, we have the following selectional relations:

(119) 1. The relation between the \( \text{NP}_a \) and the main verb \( \text{V}_T \), where \( \text{NP}_a \overset{\text{ing}}{\rightarrow} \text{VP}_1 \) is the subject of the \( \text{V}_T \).
   (e.g., "the man standing at the bar broke a glass")

2. The relation between \( \text{NP}_a \) and the main verb \( \text{V}_T \), where \( \text{NP}_a \overset{\text{ing}}{\rightarrow} \text{VP}_1 \) is the object of \( \text{V}_T \).
   (e.g., "he approached the man standing at the bar")

3. The relation between \( \text{NP}_a \) and the \( \text{V}^* \) into which the \( \text{VP}_1 \) of \( \text{NP}_a \overset{\text{ing}}{\rightarrow} \text{VP}_1 \) is developed.
   (e.g., "he approached the man standing at the bar")

Suppose that (116) is placed after the NP-analysis (118-2). Then the NP analysis will apply to \( \text{NP}_a \) only in running through the grammar the second time. Suppose that \( \text{NP}_a \overset{\text{ing}}{\rightarrow} \text{VP}_1 \) is the object of the main verb \( v \), and that \( np \) is the subject. But the rule of verbal selection cannot apply to \( v \) in running through the grammar the first time, or condition C will be violated for the \( v-\text{NP}_a \) relation (119-2), since \( \text{NP}_a \) will not yet have been developed by the NP-analysis. Nor can this
rule apply to \( v \) in running through the grammar after the first time, or condition C will be violated for the \( np \)-\( v \) relation, since \( np \) will have been fully developed by (118-6). The same is true if \( \text{NP}_g \text{ ing} \text{ VP}_1 \) is the subject of \( v \).

Suppose that (116) precedes the \( \text{VP}_1 \)-analysis (118-1). But suppose again that \( \text{NP}_g \text{ ing} \text{ VP}_1 \) is the object of the main verb \( v \), and that \( np \) is its subject. Thus the \( np \) which is to become \( \text{NP}_g \text{ ing} \text{ VP}_1 \) is introduced by the \( \text{VP}_1 \)-analysis, and hence cannot be developed by (116) until the second run-through of the grammar. But \( np \) will be developed by (118-6) in the first run-through, so that condition C is violated exactly as before.

Suppose then that (116) is placed after (118-1) and before (118-2). Then the \( \text{NP}_g \) of \( \text{NP}_g \text{ ing} \text{ VP}_1 \) will be developed by the \( \text{NP} \)-analysis (118-2) and by (118-6) in running through the grammar the first time, thus before the \( \text{VP}_1 \)-analysis (118-1) can apply to the \( \text{VP}_1 \) of the phrase, \( \text{NP}_g \text{ ing} \text{ VP}_1 \), since this phrase is introduced subsequently to the \( \text{VP}_1 \)-analysis. This contradicts condition C for the relation 3 of (119).

We see then that (116) cannot be introduced into the grammar without conflicting with condition C and thus complicating the rule of selection in just the way we have been trying to avoid. We see further that the relations (119-2) and (119-3) are alone sufficient to preclude the introduction of (116). This is important below.

Exactly the same situation results from the consideration of the rule of numerical agreement, i.e., from the consideration of the relative order of (116), (118-1), (118-4), (118-5), and
In this case we consider that the verb be replaces \( \mathbf{V_t} \) as the main verb of the sentence, and as the verb of the
\( \mathbf{VP_1} \) of \( \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \). The selectional relations, which now hold
between noun phrases, are the following:

(120) (1) the relation between \( \mathbf{NP_a} \text{ of } \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \text{ and } \mathbf{NP^*} \)
where \( \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \) is the subject, \textit{be} the main
verb, and \( \mathbf{NP^*} \) the predicate noun phrase.

\textit{(e.g., "the man standing at the bar is my friend")}

(2) The relation between \( \mathbf{NP_a} \text{ of } \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \text{ and } \mathbf{NP^*} \)
where \( \mathbf{NP^*} \) is the subject, \textit{be} is the main verb, and
\( \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \) is the predicate.

\textit{(e.g., "he is a young student working his way through college")}

(3) The relation between \( \mathbf{NP_a} \text{ of } \mathbf{NP_a} \{ \text{\textit{ing}} \} \mathbf{VP_1} \text{ and } \mathbf{NP^*} \)
where \( \mathbf{VP_1} \) becomes \( \langle \mathbf{VP_B} \rangle \text{ be } \mathbf{NP^*} \).

\textit{(e.g., "he interviewed all students expecting
to be PhD candidates in June")}

The order that must be maintained is (118-1) = (118-2) =
(118-5) = (118-6), and considerations analogous to those above
show that (116) cannot be introduced \textit{in conformity with the
criteria stated in the first paragraph of \textcolor{red}{\textit{[631]}}.}

\textit{This is reminiscent of the situation that we faced
above in \textcolor{red}{\textit{[559]}} in the case of sentences of the form

(121) \( \mathbf{NP_x} - \mathbf{V_c} \{ \text{\textit{to}} \} - \mathbf{y} - \mathbf{NP_y} \)

Since the selection of \textit{v} was determined by both \( \mathbf{NP_x} \) and
\( \mathbf{NP_y} \), it \textit{im} was necessary to have these noun phrases in}
For clarity, let us now restate the results of this section, concentrating on transitive verbs. We have considered such sentences as

(121) I know the man reading the book.

From our earlier discussion of simpler English sentences, in particular, we are led to make the following statements about the phrase structure of (121)

(122) (a) I is an NP
(b) I is an N\_anim
(c) know the man reading the book is a VP\_1
(d) know is a \(V\_1\)
(e) know is a \(V\_12\)
(f) the man reading the book is an NP
(g) the man is an NP
(h) read the book is a VP\_1
(i) read is a \(V\_1\)
(j)\# read is a \(V\_14\)
(k) the book is an NP
(l) man is an N\_anim
(m) book is an N\_inan

In other words, we give, in part, the following analysis of (121)
In detail, this goes slightly beyond the analysis to which we have been led, but not in respects relevant to the present discussion.

We have also arrived at a certain condition on the associated grammar which we have called the condition C. Condition C can be restated in the following terms: in that derivation of (121) which is directly provided by the associated proper linear grammar, the string

\[(124) \quad N_{\text{anim}} \rightarrow V_T \rightarrow \text{the} N_{\text{anim}} \rightarrow V_T \rightarrow \text{the} N_{\text{inan}}\]

must appear as a representing string. This is a way of rephrasing the requirement that the noun phrases relevant to the selection of a given verb must be in parallel (and sufficiently advanced) states of development at the point when the rule of verbal selection applies to the verb in question.

We have seen that (121) and (124) jointly lead to the consequence that the conversion rule (116) cannot be introduced into the grammar. More precisely, the consequence is that the VP₁-analysis must be given twice in the grammar. The reason for this can easily be seen from (123). In fact, the derivation of (121) given by the grammar must be essentially (123), read from bottom to top. And we see that in forming such a derivation the conversion: \(VP₁ \rightarrow V_T \rightarrow NP\) (i.e., the VP₁-analysis) is applied twice before any conversion of \(N_{\text{anim}}\) (into I, John, man, etc.) Otherwise, condition C will be violated. But this is impossible in a proper linear grammar unless the VP₁-analysis is stated twice. By induction, we can show
that the $V_{P_1}$-analysis must be stated infinitely many times, since we must also be able to account for sentences of the type

(125) I know the man watching the man watching the man...

with an obvious extension of (124), to cover such cases. But a proper linear grammar must be finite. Hence we have here outlined the construction of a set of derivations which cannot be produced by a proper linear grammar. And since (122) and condition C are not cotenable, we have been led to a system of phrase structure which cannot be given by a proper linear grammar (or a grammar of any other type that we have considered) without a departure from optimality.

61.2. This is reminiscent of the situation that we faced above in §62 in the case of sentences of the form

(126) $NP \xrightarrow{A} V \xrightarrow{C} v - NP$

Since the selection of $v$ was determined by both $NP_A$ and $NP_C$, it was necessary that these noun phrases be in
parallel states of development when the rule of verbal selection applies. But \( NP_x \) is completely developed in running through the grammar the first time, and if the solution of (77) is adopted, \( v \) and \( NP_y \) will be developed in the second run-through, so that condition C is violated. But here there was the simple solution of taking \( v \) as the main verb.

Assuming for the moment that condition C must be met, we might try to avoid the problem we now face by dropping the simple linear sequence of grammatical statements. Thus we might add to the English grammar a requirement that every time \( VP_1 \) is introduced into a derivation by some statement of the grammar, then we must return to the \( VP_1 \)-analysis and proceed from that point down the list of statements. With this revision the present difficulty can be avoided, but the general statement of the form of grammar will become much more complex.

We can achieve the same result without dropping the simple ordering of grammatical rules if we add to every statement following the \( VP_1 \)-analysis the condition that the statement does not apply if \( VP_1 \) appears in the string under consideration, whether in the conditioning context for this statement or not. (23) It would be sufficient to add this condition to certain rules, e.g., to (118-6). This would guarantee that all relevant \( NP \)'s be in parallel states of development when the rule of verbal selection is to apply, by making it impossible to develop any \( NP^* \) beyond the relevant form until all verbs to which \( NP^* \) might bear a selectional relation have already been selected. (Exactly what secondary effects this will have on the
grammar it is difficult to determine without a detailed investigation — let us now assume that there are no complicating effects) One immediate difficulty with this approach is that it requires an infinite specification of conditioning contexts, since we have to give as the conditioning context for e.g., (118.6), a list of everything that can occur as a context for the elements being developed and that does not contain \( VP_1 \). However, this is not an insuperable difficulty. We might hope to bypass it by developing further our stock of notations for grammar. \( \text{ equation } \)

\( \text{Equation 2.3} \): Before attempting to develop such additional modes of expression for the grammar, let us investigate what their effect will be in related instances. Let us now suppose that we have developed some means of qualifying a certain rule

\( \text{Equation 129) } b \rightarrow b' \text{ in env. } a \rightarrow c \)

so that the development of \( b \) to \( b' \) does not take place if a certain element \( d \) appears anywhere in the string in which \( a^2b^2c \) occurs. Let us refer to this as the qualification \( Q \). In particular, the development of \( N_{\text{anim}} (=b) \) to \( \text{John} (=b') \) will not take place if \( VP_1 (=d) \) appears anywhere in the string in question. We note first that the line of reasoning followed in \( \text{Equation 2.3} \) is no longer valid. That is, given a sentence of the form (121), \( NP_x \) need no longer be developed completely in running through the grammar the first time, and \( NP_x \) and \( NP_y \) can appear in parallel states of development when the rule of verbal selection applies to \( y \) in running through the grammar the second time. Thus we are forced to accept the analysis (77) which analyzes "John wants to come" as \( N-Y_T-N \)
(paralleling John-wants-a"book) and "John wants him to come" as N-V-T-N-N (like John-gave-him-a"book). Consequently, we lose the intuitive correspondences of §42.3, and we base our analysis on an intuitively invalid formal analogy.

An intuitively much more serious consequence comes from noting that the whole discussion of §62 applies in a perfectly analogous way to sentences of the form

\[(128) \text{NP}_x - V_y \text{ing} - \text{v} - \text{NP}_y\]

for instance,

\[(129) \text{John} - \text{likes} \text{ing} - \text{read} - \text{the} \text{book} \quad ("\text{John likes reading the book}\")\]

But we can carry this one step further. The discussion of §62 also carries over to sentences of the form

\[(130) \text{NP}_x - \text{is} \text{ing} - \text{v} - \text{NP}_y\]

for instance,

\[(131) \text{John} - \text{is} \text{ing} - \text{read} - \text{the} \text{book} \quad ("\text{John is reading the book}\")\]

which is perfectly analogous to (126) in all relevant respects. It is obvious that the analysis of (126) as a sentence of the form NP - be - NP, paralleling

\[(132) \text{John} - \text{is} - \text{a} \text{politician}\]

with "be" as the main verb, and "reading the book" as a predicate noun phrase is completely counter-intuitive. The obviously correct solution, which we adopted above, is the analysis with "be" as part of the auxiliary, and "read" as the
main verb, but the only considerations militating against
the analysis as NP - be - NP are those adduced against (77),
e.g., the fact that if we accept this analysis, then NP and
NP of (139) will not be in parallel stages of development
when it comes to selecting the qualification Q which we are now assuming to be available
bypasses this objection, as we have seen in the first paragraph
of (139). In fact we are now forced to accept the completely
counter-intuitive analysis of (139) as NP-be-NP (i.e., as
a sentence of the same form as (124)) by considerations of
simplicity (cf. fn. 24). In its favor are the fact that
"reading the book" must anyway be assigned to NP because of
(139) reading the book will only take a few hours
and that choosing this analysis, we arrive at the sentence
form NP - be - NP which is already familiar on independent
grounds. In fact, since the predicate NP in the analysis of
(139) is now NP, we can drop from the analysis of NP in
the grammar the condition that NP cannot become NP after
N-anim...be. If we permit the suggested qualification, then,
it appears that we are required to accept a completely
counter-intuitive analysis of sentences like (139), at the
same time dropping being from the auxiliary verb phrase,
just as earlier we dropped been (and, incidentally, losing
the intuitive correspondences pointed out in 15.4, (112-4)).
Not only does the qualification Q lead to intuitively
unacceptable analyses, as we have just seen, but it also has
- certain systematic consequences for the form of grammars which
we may find it difficult to accept. We have seen that if we
extend the machinery available for grammatical description to the point where qualification \( Q \) becomes formulable, what we have really done is equivalent to rejecting the linear ordering of grammatical statements. More exactly, if the context \( a\ldots c \) contains all of the information relevant to the development of \( b \) to \( b' \) (instead of to \( b'' \)), then the qualification \( Q \) 'holds back' the development of \( b \) to \( b' \) in env. \( a\ldots c \), if there is some element (which may be outside of \( a\ldots c \)) to which \( b \) bears a certain kind of selectional relation. In other words, it may be the case that: (i) there is a rule \( S_i \) of the form \( b\rightarrow b' \) in env. \( a\ldots c \) \( (26) \); (ii) \( a^\beta b^\gamma c \) appears at the stage when \( S_i \) is to apply in running through the grammar the first (or \( n^{th} \)) time; (iii) \( S_i \) does not apply to convert \( a^\beta b^\gamma c \) into \( a^\beta b'^\gamma c \), because of qualification \( Q \); (iv) in running through the grammar the second time (or \( n+k^{th} \) time), even though \( a\ldots c \) has not been changed, \( S_i \) does apply to convert \( a^\beta b^\gamma c \) into \( a^\beta b'^\gamma c \), since \( Q \) may no longer apply.

But this situation in itself can be considered a departure from the optimal form of a grammar, since where it arises, we cannot develop a certain element at the point where its entire conditioning context has been developed to the relevant degree relevant to the development, which is the possible alternative development could take place.

Reviewing the situation, it seems that we are faced with a set of unacceptable alternatives. If we retain the conception of grammars that we have developed, we cannot incorporate such sentences as (117) without a serious departure from optimality, i.e., without violating the criteria stated at the outset of \( \frac{1}{6} \). If we try to alter this conception of
But this situation in itself can be considered a departure from the optimal form of a grammar, since where it arises, we cannot convert a certain element at the point where its entire conditioning context has been developed to the degree relevant to the determination of which of the possible alternative conversions may take place.

63.5. We might try to escape the dilemma posed in §62.1 in a different manner by rejecting condition C, i.e., rejecting the requirement that (124) appear as a representing string in the derivation produced by the grammar. We saw that condition C can be disregarded only at the exorbitant cost of adding to the grammar a rule determining the selection of the verb by each particular noun (instead of just the classes \( N_{\text{anim}} \), \( N_{\text{ninan}} \), etc., or whatever classes are taken to be relevant to the selection of verbs). But the reasoning which led to this conclusion can be circumvented if we revise certain earlier steps in our analysis of English phrase structure.

We have been assuming here a grammar of roughly the following form:

\[
\begin{align*}
(134a) \text{Sentence} & \rightarrow NP \lor VP_1 \\
(b) VP_1 & \rightarrow V_T \lor NP, \text{ etc.} \\
(c) NP & \rightarrow \begin{cases} N_{\text{anim}} \\
N_{\text{ninan}} \\
N_{\text{ab}} \end{cases} \\
(d) V_T & \rightarrow V_{\text{ll}} \text{ in env. } N_{\text{anim}} \rightarrow N_{\text{anim}}, \text{ etc.}
\end{align*}
\]

The crucial point here is that the rule of verbal selection (134d) presupposes a prior analysis of the subject and object nouns. If we give the rule of verbal selection in this form, then condition C appears to be an inescapable requirement. But we might reformulate the grammar along the lines of (71a) so that the choice of the verb is determined by the choice of the subject, and the choice of the object is determined by the particular verb that has been selected. That is, instead of (134), we might have: \( a \)
(135) (a) Sentence \[ \rightarrow NP \ V P_1 \]

(b) \[ NP \rightarrow \begin{cases} N_{\text{anim}} \\ N_{\text{ninan}} \\ N_{\text{ab}} \end{cases} \]

(c) \[ VP_1 \rightarrow \ V_{t1} \ NP \text{ in env. } N_{\text{anim}} \text{, etc.} \]

(d) \[ NP \rightarrow N_{\text{anim}} \text{ in env. } V_{t1}, \text{ etc.} \]

or something of this kind. Such a revision would entail many other alterations in the grammar which we need not discuss here. The crucial point is that the selection of the verb as \( V_{t1} \), etc., no longer presupposes a prior development of the surrounding noun phrases, so that the reasoning which led to stipulating condition C no longer holds. We can now introduce the conversion rule (116) into the grammar without the unfortunate consequences discussed in \( \text{62,1} \), by ordering the conversions as follows in the proper linear grammar:

(136) (i) (135a)

(ii) (135b)

(iii) (135c)

(iv) \( N_{\text{anim}} \rightarrow \ I, \ John, \text{ etc.} \)

(v) (116)

(vi) (135d)

By means of the grammar (136) we can produce a derivation of the form of (123) (read from bottom to top), not containing the string (124).

This grammar already represents a certain departure from optimality, since the NP-analysis must be given in two separate statements((135b,d)), once for the subject and once for the object, if we are to be able to derive simple 

Noun - Verb and Noun-Verb-Noun sentences with no recursions by running through
the grammar once. But a closer examination shows even more disturbing consequences.

Suppose that we are generating such sentences as (121) (= "I know the man reading the book") by means of the grammar (136). The derivation will look something like this:

(137) 1. Sentence
   2. NP \( \sqsupset V_P_1 \)
   3. \( N_{\text{anim}} \sqsupset V_P_1 \)
   4. \( N_{\text{anim}} \sqsupset V_{\text{bl}} \sqsupset \text{NP} \)

\[ \text{step 1c} \]

5. \( I \sqsupset V_{\text{bl}} \sqsupset \text{NP} \)
6. \( I \sqsupset V_{\text{bl}} \sqsupset \text{NP} \sqsupset \text{ing} \sqsupset V_P_1 \)
7. \( I \sqsupset V_{\text{bl}} \sqsupset N_{\text{anim}} \sqsupset \text{ing} \sqsupset V_P_1 \)
8. \( I \sqsupset \text{know} \sqsupset N_{\text{anim}} \sqsupset \text{ing} \sqsupset V_P_1 \)

\[ \text{step 1d} \]

9. \( I \sqsupset \text{know} \sqsupset N_{\text{anim}} \sqsupset \text{ing} \sqsupset V_{\text{bl}} \sqsupset \text{NP} \)
10. \( I \sqsupset \text{know} \sqsupset \text{the} \sqsupset \text{man} \sqsupset \text{ing} \sqsupset V_{\text{bl}} \sqsupset \text{NP} \)
11. \( I \sqsupset \text{know} \sqsupset \text{the} \sqsupset \text{man} \sqsupset \text{ing} \sqsupset V_{\text{bl}} \sqsupset N_{\text{nan}} \)
12. \( I \sqsupset \text{know} \sqsupset \text{the} \sqsupset \text{man} \sqsupset \text{ing} \sqsupset \text{read} \sqsupset N_{\text{nan}} \)

\[ \text{step 1e} \]

13. \( I \sqsupset \text{know} \sqsupset \text{the} \sqsupset \text{man} \sqsupset \text{ing} \sqsupset \text{read} \sqsupset \text{the} \sqsupset \text{book} \)

We note that in this derivation, the conversion of \( N_{\text{anim}} \) to \( \text{the} \sqsupset \text{man} \) takes place after the conversion of \( V_{\text{bl}} \) to \( \text{know} \). But note that in certain cases, the conversion of \( N_{\text{anim}} \) is dependent directly on the subject of \( V_{\text{bl}} \). E.g., if the subject is \( I \), as in this case, then the object of \( \text{know} \) could be \( \text{myself} \), but not \( \text{himself} \), etc. Hence the rule which applies \text{in some form} \text{to form} \text{step 10 in (137)} must read in part:

(138) \( N_{\text{anim}} \rightarrow \text{myself} \) in env. \( I \text{know} \rightarrow \), etc.
But in the conditioning context for this conversion it will be necessary to list every individual verb along with know, since the \( V_t \)'s have already been developed. But this is clearly an intolerable complication. The reader will note that we are now troubled by a difficulty which is a special case of a more general characteristic of grammar: the type of grammar that we have developed. Thus I refer to the fact, discussed above in a footnote, that these conversions in these grammars do not take into account the past history of the elements undergoing conversion, but only the present shape of these elements. If we could introduce past history into the statement of the we could replace the list of all verbs in (138) by the conditioning context we have introduced/general introducing statement that the conversion in question takes place whenever an element derived from an earlier \( V_t \) (or \( V_t \) for \( i=1,2,\ldots \)) is present. But we have not yet discussed or developed means for such statements, or for evaluating grammars in which they appear.

The intolerable elaboration of (138) is a consequence of the fact that the conversion of \( N_{anim} \) to the man in (137) takes place after the conversion of \( V_t \) to know. Therefore we must inquire into the possibility of reordering the conversion rules in question. But note that the conversion of \( VP_1 \) to \( V_t \cdot NP \) in (137) (step 9) must precede the conversion of \( N_{anim} \) to the man, or the rule (136iiii)\(=(135c) \) will have to list every animate noun instead of just \( N_{anim} \) as the conditioning context. Hence if we accept the reordering in question here, the rule (136iiii) (the \( VP_1 \)-analysis) will have to apply twice before (136vii) applies even once, and we know that this is impossible in a proper linear grammar unless the \( VP_1 \)-analysis is actually stated twice in the grammar. But this is a productive construction. Hence by reasoning analogous to that of the last paragraph of \( \S \) 63.1 we can show that the \( VP_1 \)-analysis will have to be stated infinitely often, if this course is taken. It thus appears that the damage cannot be repaired, and that (135) is not an acceptable alternative.

It is important to notice that the argument which we have presented against (135) does not apply with the same force against (134), the grammar
which meets condition C. In (134) subject and object are introduced and
developed simultaneously before the conversion of \( V_T \) to \( V_{t1} \) ... to
\( \text{know} \). We can thus elaborate (134c), introducing \( I, \text{you}, \text{myself}, \text{etc.} \) as
additional forms of \( \text{NP} \), and stating the relation between them by means of
conversion statements in which only \( V_T \) (and not all instances of transitive \text{verbs})
\( \text{must appear. This is of course a rather unhappy solution for this}
\) problem. Not only is (134c) further complicated, but it is now necessary to
list separately the elements \( N_{\text{anim}}, N_{\text{inan}}, I, \text{you, myself, etc.} \) in later
\( \text{rules of conversion where previously only } N_{\text{anim}}, N_{\text{inan}}, \text{or } N_{\text{ab}}
had to be listed. But this complication, \( \text{though undesirable, is still much}
\) less objectionable that the elaboration of (138) which would be necessary, as we
\( \text{saw above, if (135) selected in preference to (134). It appears, then,}
\) that rejection of condition C \( \text{simply leads to an extension of the}
\) difficulties which appeared in \( \text{62.1.} \)

\( \text{63.5.} \) Reviewing the situation, it seems that we are faced with a set of
If we retain
unacceptable alternatives. \( \text{The conception of grammars that we}
\) have developed, \( \text{we are apparently unable to incorporate}
\) such sentences as \( \text{(117) and (121) without a serious departure from optimality,}
\) i.e., without violating the criteria stated at the outset of \( \text{62.}(\text{If we try}
to revise earlier steps in the analysis so that these criteria no longer
apply, we are faced with other equally serious difficulties, as we
have seen in \( \text{63.5.} \). If we try to alter our present conceptions of the form of
grammar so as to make room for such sentences as (117) and (121), we find that
we are led to such completely unacceptable results as those of
grammar so as to make room for such sentences, we find that we are led to such completely unacceptable results as those of...

in other cases, where previously we did apparently reach a satisfactory solution, and that in fact, the grammar fails to have certain other desirable properties. It appears then, that when we begin to go beyond sentences of the very simplest construction, the difficulties begin to mount so rapidly that grammars, in the form that we have outlined, become inordinately complex. If we were to continue to investigate more and more complex sentences, the overwhelming difficulty of stating the structure of \( \mathbb{P} \) rigorously and completely would become continually more evident, and the essential clarity and systematic organization of the structure of the simple sentences would be completely obscured.

We will therefore exclude from consideration such sentences as (117), that are based on (116). We have no real warrant for this arbitrary limitation from any general considerations that have been developed up to this point. This poses a theoretical gap to which we will return below.

We note, incidentally, that it is intuitively quite easy to explain why the construction of a system of phrase structure should begin to break down just at the point when we begin to investigate sentences like (117). These are the first instances of sentences that seem intuitively to be compounded out of other more simple sentences. That is, a sentence like "I know the man standing at the bar", we really find incorporated the sentence "the man is standing at the bar", in a certain sense. And it is not surprising that
at just this point the network of selectional relations, which now in a way cross over a kind of 'sentence boundary', should become so complex as to make the possibility of an optimal grammar appear remote. But this intuitive reasoning has no analogue in our linguistic theory, as so far developed. This theory approaches 'simple' and 'complex' sentences in a neutral way, and the notion of one sentence 'incorporated' in another has no significance in our terms. This suggests a way of circumventing these problems which we will explore below.

It is important to remember that we have not literally proven that optimal grammars cannot be constructed to cover these more complex sentences. Our criteria of optimality are much too vaguely stated for any proof to be possible. It would be an important and non-trivial project to rephrase these criteria more precisely, and to carry out a much more serious formal investigation of the kinds of selectional relations that lead to violation of optimality, or to the breakdown of some rigorously stated form for grammatical description. We have at most suggested an outline for such a demonstration. But this discussion does shed serious doubt on the possibility of describing phrase structure in a way that is both reasonably simple and organized, on the one hand, and comprehensive, on the other.

65. In §63 we completed the analysis of sentences like "John wanted to read", hence generally of $V_a$ and $V_y$. We now consider the adequacy of the proposal, given in (77), that (129) and (130) be analyzed as cases of (131).
In §57 we initiated an investigation into the limitations of particular types of grammar from a formal, abstract point of view. In this section we have approached the same problem from the other end, investigating the empirical consequences of a strict attempt to construct grammars of the prescribed form for actual language material. In our earlier abstract investigation we noted that certain sets of derivations which are in fact associated with a system of phrase structure may not be producible by any proper linear grammar. In this section we outlined the construction of a certain set of derivations (to which we were led by a sequence of observations about simple sentences, each of which seemed reasonable in itself) which is in fact not producible by a proper linear grammar. It appeared to be the case, furthermore, that even if we drop the requirement of proper linearity for grammars, we are not able to present a satisfactory description of the system of phrase structure in question, or to find an alternative phrase structure for the sentences under consideration which could be described in a satisfactory way. It is important to remember that we have not literally proven that optimal grammars cannot be constructed to cover these more complex sentences. For one thing, we have obviously not exhausted all possible alternative analyses. For another, our criteria of optimality are much too vaguely stated for any proof to be possible. It would be an important and non-trivial project to rephrase these criteria more precisely and to carry out a much more serious formal investigation of the kinds of selectional relations that lead to violation of optimality, or to the breakdown of some rigorously stated form for grammatical description. Coupled with a much more intensive study of the empirical consequences of applying strictly certain notions of grammar, such an investigation could fill in the gap still remaining between the investigations of §57 and those of §62, and could lead to a conclusive answer to the question of the formal adequacy (the question of intuitive adequacy would still remain open -- a grammatical theory must meet conditions of simplicity and intuitive adequacy, as we have pointed out in §7 and elsewhere)
of description in terms of phrase structure. We have at most suggested an outline for a demonstration of a negative answer to this question. But this discussion does, I think, shed serious doubt on the possibility of describing phrase structure in a way that is both reasonably simple and organized on the one hand, and comprehensive, on the other.

64. In §63 we completed the analysis of sentences like "John wanted to read", hence generally of \( v \) and \( v_t \) (see §61.1). We now consider the adequacy of the proposal, given in (77), that (139) and (140) be analyzed as \( n \) instances of (141).
(139) John-wanted-him-to come

(140) The guard-caught-him-trying to escape

(141) NP-V-NP-NP

There is of course no possibility here, as there was in (62), of the verb from NP being the main verb since its selection is determined by the second, not the first NP. The selectional relations in sentences of the form (141) are (among others) between the second NP and the verb of NP, as we can see from the grammaticalness of (142) but not (143).

(142) (a) John wanted Bill to read
(b) John wanted justice to prevail

(143) John wanted justice to read
and between V (the main verb) and the second NP, as we can see from the grammaticalness of (144), but not (145).

(144) (a) John asked Bill to read
(b) John wanted justice to prevail

(145) John asked justice to prevail

But we have seen in (62) that with these selectional relations it is impossible to introduce

(146) VP₁ → Vᵀ NP

NP → NP⁺ ing ᵃ NP₁

into the grammar without serious complications. By exactly the same reasoning it is impossible to introduce

(147) VP₁ → Vᵀ NP⁺NP₁

directly. Hence the analysis of (139) or (140) as (141) cannot be
introduced. These sentences must be excluded from consideration just as the sentences based on (116) were excluded.

Actually an even more unpleasant conclusion seems to be forced on us if we attempt to account for these sentences despite the resulting complexity. There seems to be no reason not to consider (118) to be analogous to (117), i.e., to be a simple sentence of the form NP-\(V_T^*\)NP*, with NP* developed by (116). Thus we may take \(V_\beta\) to be simply a subclass of \(V_T\), and we may have, as a part of the grammar,

\[
1. \text{Sentence} \rightarrow \text{NP} \wedge \text{VP}_1 \\
2. \text{VP}_1 \rightarrow \text{VT} \wedge \text{NP} \\
3. \text{NP} \rightarrow \text{NP}_a \left( \text{ing} \wedge \text{VP}_1 \right) \\
4. \text{VT} \rightarrow \text{VT}_1 \cdot \text{V}_\beta
\]

The derivations of both (117) and (118) would be basically

\[
\text{Sentence} \\
\text{NP} \wedge \text{VP}_1 \\
\text{NP} \wedge \text{VT} \wedge \text{NP} \\
\text{NP} \wedge \text{VT}_1 \cdot \text{NP}_a \cdot \text{ing} \wedge \text{VP}_1 \\
\text{etc.}
\]

We cannot really show that this is the best analysis, since we do not know how to include (117) in the grammar, but, speculating, this seems like the most reasonable approach.

Even with this analysis, \(V_\beta\) will have to be retained as a special subclass of \(V_T\) with certain distributional peculiarities. For instance, \(V_\beta\) operates somewhat differently in the environment \(V_\beta\) -- then in other environments. Thus we have
(140) They caught him trying to escape \[ \text{catch} \ \hat{\nu}_\beta \]

but not

(a) him trying to escape was a mistake

(b) I know him trying to escape \[ \text{know} \ \hat{\nu}_\beta \]

Furthermore, we can have

(142) Imagine the dog having jumped over that fence

\[ \text{imagine} \ \hat{\nu}_\beta \]

but not

(143) Recognize the dog having jumped over the fence

\[ \text{recognize} \ \hat{\nu}_\beta \]

Thus the form \( \text{ing} ^{\gamma} \text{have} ^{\gamma} \text{en} ^{\gamma} \hat{\nu}_1 \) permitted by (75) can occur only after \( \hat{\nu}_\beta \). Again, all transitive verbs not in \( \hat{\nu}_\alpha \) can have objects with both forms of (132-3). Thus we have

(144) I know the man alongside of (117), etc. But some verbs in \( \hat{\nu}_\beta \) do not have this property. E.g., we do not have, alongside of (142),

(145) Imagine the dog

This does not distinguish \( \hat{\nu}_\beta \) from the rest of \( \hat{\nu}_T \), however, since many members of \( \hat{\nu}_\alpha \) also have this property.

Another special statement distinguishing \( \hat{\nu}_\beta \) from other members of \( \hat{\nu}_T \) will be added to the rule of verbal selection for \( \hat{\nu}_T \), for the context \textit{en} -- . That is, we will have

(146) \( \hat{\nu}_T \rightarrow \hat{\nu}_\beta \) in env. \textit{en} -- \( \hat{\nu}_F \)

This will be one of a number of selectional statements for \( \hat{\nu}_T \) in this position. It accounts for the possibility of (147) but not (148).
The man was caught trying to escape

This particular case will turn out to be of special importance later on.

It seems fairly clear that the simplest way to describe this situation will be, essentially, (148), with a statement of special conditions for the case where the main verb is in $V_3$. If this is correct, then we have a situation here which contrasts sharply with that of §163. There we saw that the analysis led to certain intuitively appealing results. Sentences that were intuitively different in structure received different analyses, and certain cases of intuitive homonymy were paralleled by constructional homonymy which was a consequence of the application of formal criteria. But in §164 we have found exactly the opposite situation. Certainly (150) differs from (117), and (159) differs from (160) in much the same way that "John likes eating in the park" [*(103a)*] differs from "That law forbids eating in the park" [*(98a)*].

I can't imagine the dog climbing that tree

" " " recognize the man " " "

but in these cases we have been led to a single analysis. Furthermore,

I can't catch the dog climbing that tree

is intuitively a homonymous construction (with the analysis of (150) on the one hand, and (160) on the other), just as is "The policemen must stop drinking after midnight" [*(107)*]. But here we have been led to assign only a single $P$-marker.
We will return to this intuitive inadequacy below.

The analysis of (129) poses a somewhat different problem, since there is no analysis of NP related to (129) the way (116) is related to (19). That is we do not have "I know the man to stand at the bar", etc. Nevertheless, the simplest way of handling (129) is probably by an elaboration of (147), continuing along the lines suggested above. In all of these cases an infinitive phrase is involved, so this elaboration to include (129) may actually have the effect of removing a restriction, rather than adding a condition.

Summing up the discussion of $V_B$, $V_2$, $V_3$, $V_Y$, then, we see that $V_2$ and $V_Y$ can be successfully incorporated into the grammar without excessive complexity and with intuitively appealing results. But $V_b$ and $V_3$, which introduce one additional selectional relation, can be incorporated into the grammar only at the cost of a departure from desirable formal properties on the one hand, and a counter-intuitive description of phrase structure on the other. Since $V_a$ and $V_\eta$ have been assigned to $V_b$, $V_2$ and $V_\alpha$, $V_Y$, as their respective overlaps, this concludes, for the time being, the discussion of the elements of (57), (58). In chapter IX, we will be led to give quite a different analysis for these constructions.

The case of $V_\delta$ (cf. § 62.2) and sentences like "I saw him come" should be similar. There is a selectional relation between "him" and "come", but it is difficult to find a clear case, at least on the level of grammaticalness
with which we are now working, of a selectional relation between "see" and "him". If the latter cannot be established, then \( V_{i} \) can actually be introduced. Because of the parallel between this case and the others of this section, however, we will also drop this case from the grammatical sketch presented below, returning to it later with a different approach.

\[ \text{651. We may now survey the cases to which the rule of verbal selection will apply, after the exclusions of the preceding section. In running through the grammar the first time, the only selectional relations relevant to this rule are those of subject-verb and verb-object. Considering in this account only } V_{1}, \ldots, V_{4} \text{ and } V_{a}, V_{b} \text{ (cf. (70), (72), §64.2), we have as the relevant parts of the conditioning contexts:} \]

\[
\begin{array}{|c|c|c|}
\hline
 & \text{Initial conditioning context} & \text{Final conditioning context} \\
V_{1} & N_{\text{anim}} & N_{\text{anim}} \\
V_{2} & N_{\text{anim}} & N_{\text{anim}}, N_{\text{inan}}, N_{\text{ab}} \\
V_{2} & N_{\text{anim}}, N_{\text{inan}}, N_{\text{ab}} & N_{\text{anim}} \\
V_{4} & N_{\text{anim}} & N_{\text{anim}}, N_{\text{inan}} \\
V_{a} & N_{\text{anim}} & N_{\text{anim}}, N_{\text{inan}} \\
V_{b} & N_{\text{anim}}, N_{\text{inan}}, N_{\text{ab}} & \\
\hline
\end{array}
\]

In other words, \( V_{1} \) can appear only in env. \( N_{\text{anim}} \), etc. If we treat \( V_{a} \) as the subclass of \( V_{T} \), that occurs when there is no object, then the \( VP_{1} \)-analysis (cf. (118-1)) will contain a statement \( VP_{1} \rightarrow V_{T} < NP > \), and the rule of verbal
selection will include, for the case of (162), such statements as

\[ V_T \rightarrow V_{t1} \text{ in env. } \overline{N_{anim}} \cdots \cdots \cdots \overline{N_{anim}} \]

\[ V_T \rightarrow V_{t2} \quad " \quad \overline{N_{inan}} \cdots \cdots \cdots \]

etc.

There are four other major cases to which the rule of verbal selection applies, all having to do with the verb which appears in the development of \( NP_I \) (see (75)), hence all occurring in running through the grammar a second time.

First we have the selectional relation between \( V_T \) and \( NP \) when \( NP_I \) is developed, e.g., into \( V_T \np \), e.g., "to write a novel is his ambition". This is the same as the relation between verb and object. Then we have the selectional relation between \( NP^* \) and \( V_T \) where \( NP_I \) is, e.g., \( ing \ V_T \np \), and \( NP_I \) is preceded by a possessive adjective \( NP^* \np \) (e.g., "John is flying those big planes is something I don't approve of"). (27)

This is the same as the relation between subject and verb.

Since the possessive adjective may or may not occur, we have

| \( V_{t1} \) | Initial conditioning context | Final conditioning context |
| \( V_{t2} \) | \( \langle \overline{N_{anim}}^* S_1 \rangle \) | \( \overline{N_{anim}} \) |
| \( V_{t3} \) | \( \langle \overline{N_{anim}}^* S_1 \rangle \) | \( \overline{N_{anim}} \) |
| \( V_{t4} \) | \( \langle \overline{N_{anim}}^* S_1 \rangle \) | \( \overline{N_{anim}} \) |
| \( V_{t5} \) | \( \langle \overline{N_{anim}}^* S_1 \rangle \) | \( \overline{N_{anim}} \) |
| \( V_{t6} \) | \( \langle \overline{N_{anim}}^* S_1 \rangle \) | \( \overline{N_{anim}} \) |
Thus the rule of verbal selection, applying in the second run-through of the grammar, will include, for example, such statements as

\[(134) \ V_T \rightarrow V_{t1} \ \text{in env.} \ \langle N_{\text{anim}} ^ S_1 \rangle \ldots \ldots N_{\text{anim}} \]

etc.

The third case comes from a development of NP into NP_{1}^c Prep Phrase, where the prepositional phrase is of NP*. An ordinary example would be "the men of England", etc. If NP_{1} \rightarrow NP_{1}^c, we have a phrase

\[(155) \ NP_{1}^c \ \text{of} \ NP^* \]

In this case, NP_{1} can only be developed into the special cases \[\{ \text{to} \ \\ \text{ing} \} \ V_T, \] and there is a selectional relation between V_T and NP* (e.g., "the screeching of brakes," "reading of good literature"). When V_T \rightarrow V_{t1}, \ldots, V_{t4}, this relation is that of verb-object, when V_T \rightarrow V_{i1}, V_{i2}, it is the relation of verb-subject, as is evident from the examples. When V_T \rightarrow V_{t1}, \ldots, V_{t4}, we have as a fourth case, the possibility of a possessive adjective NP** S_1 preceding the noun phrase (135a) and we then have a selectional relation between NP** and V_T, the same relation as subject-verb (e.g., "John's reading of good literature has convinced him that...".) Coalescing these two cases we have
<table>
<thead>
<tr>
<th>V, t</th>
<th>Initial conditioning context</th>
<th>Final conditioning context</th>
</tr>
</thead>
<tbody>
<tr>
<td>V, t1</td>
<td>( N_{\text{anim}}^{s_1} )</td>
<td>of ( N_{\text{anim}} )</td>
</tr>
<tr>
<td>V, t2</td>
<td>( N_{\text{anim}}^{s_1} )</td>
<td>of ( N_{x} )</td>
</tr>
<tr>
<td>V, t3</td>
<td>( N_{x}^{s_1} )</td>
<td>of ( N_{\text{anim}} )</td>
</tr>
<tr>
<td>V, t4</td>
<td>( N_{\text{anim}}^{s_1} )</td>
<td>of ( N_{\text{anim}} ), of ( N_{\text{inan}} )</td>
</tr>
<tr>
<td>V, t6</td>
<td>( N_{\text{anim}} )</td>
<td>of ( N_{x} )</td>
</tr>
</tbody>
</table>

Thus the rule of verbal selection will include, for (157), such statements as

\[
V \rightarrow V_{t1} \text{ in env. } \langle N_{\text{anim}}^{s_1} \rangle \ldots \text{ of } N_{\text{anim}}
\]

e tc.

Clearly (162), (157), and (156) can be consolidated into a single generalized form. But the rule of verbal selection will remain quite a complicated statement.

'Selectional relation' and 'same selectional relation' have been used here only as suggestive terms. Actually, all that we know of the relation between noun and verb is given in the charts (162), (154), and (156). We have developed no theory of selectional relations which would permit us to state that the relation of "brakes" to "screech" is the same in "the screeching of brakes" as in "the brakes screech".

We could perhaps develop such a theory within \( P \). This would give us an important new approach to grammatical relations, different from that of \( \frac{5}{4} \), though not unrelated to it. (29)
In this discussion we have seen enough examples of the use of the term 'selectional relation' to indicate roughly what the content of such a development would be. We see, for one thing that the notion of selectional relation is relative to a given way of stating the grammar. It may be that there is a string $Z = X_1^a X_2^b X_3^c$ in which the choice of $X_3^c$ is partially determined by the choice of $X_1^a$, (30) but yet it will not necessarily be the case that a selectional relation in the sense that we have been using the term holds between $X_1^a$ and $X_3^c$. For a selectional relation in our sense to hold, it is necessary that the rule of the grammar that states the analysis of $X_3^c$ contain forms of $X_1^a$ as a relevant part of the conditioning context. If the simplest grammar states the selection of $X_2^b$ in terms of $X_1^a$, and of $X_3^c$ in terms of $X_2^b$, then there is no selectional relation between $X_1^a$ and $X_3^c$, but only between $X_1^a$ and $X_2^b$, and between $X_2^b$ and $X_3^c$. (31) This property is crucial if we wish to develop a significant notion of grammatical relations from selectional considerations. Certainly among the major grammatical relations are subject-verb and verb-object, but we might, on some level of grammaticalness find that the choice of subject partially determines the choice of the object. We would not want to set up, on the basis of this, a third relation between subject and object, and in our present sense of 'selectional relation', this unwelcome possibility would be excluded for the reasons just sketched.

However, there will be certain difficulties in constructing a theory of grammatical relations in selectional terms. Thus on our present level of grammaticalness, there
are verbs which occur only in the environment $N_{anim} \ldots N_{anim}$. Here the grammatical relations subject-verb and verb-object correspond to the same selectional relation. The relation of noun to verb is the same in both cases. But here, as opposed to the case of $NP^0V^1$ and $V^1_{\text{of}}^0NP$ ("brakes screech" and "screeching of brakes"), we would not want to say that the same grammatical relation is involved. On the other hand, if certain verbs can take any noun as object, then in selectional terms the verb-object relation would, in such cases, simply not appear. The further development of this theory is thus not obvious.

We will not go on to try to develop such a theory. The results which we might hope to obtain from it will appear as a by-product of the constructions of the next chapter.

With this much introduction, we can proceed to sketch the outlines of a grammar of English phrase structure in reduced form (but cf. §58), so that derivations can be given and the underlying phrase structure reconstructed. In discussing the form of grammars (cf. §51) it was emphasized that a distinction must be made between statements which are obligatory and those which are merely optional. We can simplify this grammatical sketch considerably by giving a single general rule to determine when a statement of the grammar is obligatory, so that no qualification to this effect need actually be incorporated in the grammar.

The grammar in reduced form is a sequence $\Sigma_1, \ldots, \Sigma_n$ of statements numbered $1, 2, \ldots, n$, each of which can be expanded into a sequence of elementary statements $S_{i,j}$ of the form $\alpha \rightarrow \beta$. 
The grammar is in the first place a sequence of elementary statements, each of the form $\alpha \rightarrow \gamma \beta$. When we apply the 'notational transformations' of chapter III, we collapse this sequence to a reduced sequence $\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_n$ of statements containing $n$ brackets and angles, these statements being numbered $1, 2, \ldots, n$.

Suppose that the $i$th numbered statement $\tilde{\epsilon}_i^i$, when expanded fully into elementary statements, is

\begin{equation}
S_{i_1}^i: \quad \alpha_1 \rightarrow \gamma_1 \\
S_{i_2}^i: \quad \alpha_2 \rightarrow \gamma_2 \\
\vdots \\
S_{i_m}^i: \quad \alpha_m \rightarrow \gamma_m
\end{equation}

$\alpha_1, \alpha_2, \ldots, \alpha_m$ need not be distinct, and it may be the case that for some $i, k \leq m$, $\alpha_i$ contains $\alpha_k$ as a proper substring (i.e., $\alpha_k \subsetneq \alpha_i$).

The grammar will be presented as the reduced sequence $\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_n$ of numbered statements, and each of these numbered statements corresponds, as in (169), to a certain sequence of elementary statements. The formation of the reduced, generalized grammar $\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_n$ thus imposes a grouping on the sequence of elementary statements $S_{i_1}^i$ of the form $\alpha \rightarrow \gamma \beta$. We can make use of this grouping to formulate a general rule for the occurrence of obligatory statements among the $S_{i_1}^i$'s.

We first impose the following condition on grammars:

**Condition 1**: Suppose that the $i$th numbered statement $\tilde{\epsilon}_i^i$ is reduced from (and expandable to) (169). Suppose that for $i, k \leq m$, $\alpha_i$ contains $\alpha_k$ as a proper substring (i.e., $\alpha_k \subsetneq \alpha_i$). Then $i < k$. I.e., $S_{i_k}^i$ precedes $S_{i_1}^i$ in the sequence (169).

Suppose that (170) in constructing a derivation we have already applied statements $\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_{i-1}$ and we are about to apply $\tilde{\epsilon}_i^i$ which is expanded into (169). Suppose that the
Suppose that in constructing a derivation we have already applied statements \( Z_1, \ldots, Z_{1-1} \), and we are about to apply \( Z_1 \) which is expanded into (169). Suppose that the last string in the initial part of the derivation which has already been constructed is \( Z \). That is, after application of \( Z_1, \ldots, Z_{1-1} \) we have arrived at a string \( Z \) to which \( Z_1 \) is now to be applied. Suppose further that \( Z = \ldots x_1 \ldots \), where \( x_1 \in [\) (for \( 1, k \leq m \)). That is, \( Z \) contains two of the left hand terms of (169), one within the other. The motivation for Condition 1 is that we do not want \( x_k \) to be converted by \( x_1 \) before \( x_1 \) is converted. That is, in carrying out a conversion we always want to consider the maximum stretch of relevant context. (Cf. 12.2, where the rule for expansion of angles was motivated by the same consideration — note that a weaker but more complex formulation than Condition 1 would actually suffice for this purpose).

In order to make sure that the purpose behind Condition 1 is fulfilled, we formulate the following rule for the occurrence of obligatory statements in (169):

**Rule 1:** Suppose that the \( i^{th} \) numbered statement \( Z_1 \) is reduced from (and hence expandable to) (169). Then \( S_1, \ldots, S_1 \) is obligatory if and only if for each \( k \) such that \( 1 < k \leq m \), \( x_k \neq x_1 \).

In other words, if in (169) there are several conversions \( S_1, \ldots, S_1 \) which convert a string \( x \) into some string \( y \), then only the last of these is obligatory. This method of utilizing the grouping of conversions in the completely reduced grammar \( Z_1, \ldots, Z_n \) to determine automatically the place of obligatory conversions in the sequence of conversions is the same as that employed in the morphophonemic study in ..., and is in accord with the general motivation for distinguishing obligatory and optional conversions, as discussed in ...

As an example of the functioning of this rule, suppose that \( Z_1 \) is:

\[
Z_1: x \rightarrow \left\{ \begin{array}{l}
\varepsilon \\
U
\end{array} \right. \] in env.

The unique expanded form of (170) is
\[ (171) \quad S_{11} : \alpha \delta \varepsilon \rightarrow \beta \delta \varepsilon \]
\[ S_{12} : \alpha \delta \rightarrow \beta \delta \]
\[ S_{13} : \alpha \delta \rightarrow \varepsilon \delta \]

Suppose that in constructing derivations D, D', and D'', we arrive, respectively, at the strings \( Z, Z', \) and \( Z'' \) after having applied \( Z_1, \ldots, Z_{1-1} \).

\[ (172) \quad Z = \ldots \alpha \delta \varepsilon \ldots \]
\[ Z' = \ldots \alpha \delta \eta \ldots \quad (\eta \neq \delta, \varepsilon) \]
\[ Z'' = \ldots \alpha \gamma \ldots \]

We note first that the augment \((171)\) of the grammar meets condition 1. Furthermore, by \( S_{11} \) and \( S_{13} \) (but not \( S_{12} \)) are obligatory. Hence the result of applying \( Z_1 \) in the case of the derivation D will be \( \ldots \beta \delta \varepsilon \ldots \); in the case of D', it may be either \( \ldots \beta \delta \eta \ldots \) or \( \ldots \beta \gamma \ldots \); in the case of D'' it will be \( \ldots \alpha \eta \ldots \).

Note that the fact that \( \alpha \) appears on the left in \((171)\) does not mean that \( \alpha \) must be eliminated by application of \( Z_1 \). The rule applies only to the expanded sequence \((171)\). This rule causes occasional complications (statements 16 and 17, below), but its total effect is a great simplification, since it is no longer necessary to present along with each substatement \( S_{11} \) an indication as to whether or not it is obligatory. This indication would be difficult to include in the reduced grammar \( Z_1, \ldots, Z_n \). We will see below that the occasional complications disappear at a higher level of analysis.

**66.2. Phrase Structure of English.** In accordance with the plan of §58, many statements of restriction and many possible conversions are omitted in this sketch.
1. Sentence $\rightarrow$ NP $^\ast$ VP

2. VP $\rightarrow$ VP $^\ast_a$ VP $^\ast_1$

3. VP $^\ast_1$ $\rightarrow$ $<D_2>$ $<$VP $^\ast$ B$>$

\[
\begin{align*}
&\begin{cases}
V_T \\
V_E
\end{cases} \\
&\begin{cases} NP \end{cases} \\
&\begin{cases} \text{Sentence} \end{cases} \\
&\begin{cases} \text{be} \end{cases} \\
&\begin{cases} \text{NP} \end{cases} \\
&\begin{cases} \text{Predicate} \end{cases} \\
&\begin{cases} V_T \end{cases} \\
&\begin{cases} \langle NP \rangle \end{cases} \\
&\begin{cases} \langle \text{PP} \rangle \end{cases}
\end{align*}
\]

$D_2$ is a special class of adverbs containing "certainly", "quickly", etc. $NP = \text{propositional phrase}$.

4. NP $\rightarrow$ \begin{cases} NP $^s$ \\
NP $^p$
\end{cases} except in env. $V_E \, NP$ (33)

NP $^s$ is the singular noun phrase, NP $^p$, the plural.

5. Predicate $\rightarrow$ \begin{cases} \text{AP} \langle \text{PP} \rangle \\
\{ \}
\end{cases} $\begin{cases} \text{AP} \text{ phrase} \end{cases}$ $\begin{cases} \text{PP} \text{ phrase} \end{cases}$

6. NP $\rightarrow$ \begin{cases} NP $^s$ \\
NP $^p$
\end{cases} in env. \begin{cases} NP $^s$ \\
NP $^p$
\end{cases} ... ... where \begin{cases} \text{contains no} \end{cases} $NP_s$, NP $^p$ (33)

7. $\begin{cases} NP $^s$ \\
NP $^p$
\end{cases} \rightarrow$ T $\langle \text{AP} \rangle$ N $\{ \}$ $\{ \text{PP} \}$
8. \[ N \rightarrow \begin{cases} N_{\text{anim}} \\ N_{\text{inan}} \end{cases} \]

The conditioning environment \( S_{1} \) in 8 will only appear in running through the grammar the second and later times, \( S_{1} \) being here the possessive suffix introduced in statement 17, below. The context \( S \) appears the first time, where \( S \) is the plural morpheme. \( N_{A} \) is a generalized class of abstract nouns, containing \( N_{ab} \) and \( N_{P} \). (cf. (69) and (75)). Many other restrictions can and should be added here, e.g., we cannot have \( N_{\text{anim}} \), i.e., \( N_{A} \), only \( N_{\text{anim}} \) can follow \( V_{f} \), etc.

Statement 9 gives the information contained in (152), (153), and (156). I.e., it is the rule of verbal selection. It is quite a complex statement, though a good deal of generalization is possible. (34) The part of statement 9 corresponding to just (162), i.e., the statement of the rules for the subject-verb and verb-object relation in simple \( N-V \) and \( N-V-N \) sentences, can be given as follows:

\[
\begin{align*}
9^{*} & : V_{T} \rightarrow \begin{cases} V_{t1-2} \\ V_{t1-4} \end{cases} \text{ in env. } N_{\text{anim}} \\
& \begin{cases} V_{t2} \\ V_{t2} \end{cases} \text{ in env. } \begin{cases} N_{\text{inan}} \end{cases} \
& \begin{cases} V_{l2} \\ V_{t2} \end{cases} \text{ in env. } \begin{cases} N_{\text{inan}} \end{cases} \\
& \begin{cases} S \\emptyset \end{cases} \langle \text{PP} \rangle V_{P} A \langle D_{2} \rangle \langle V_{P} B \rangle \langle T \rangle \langle A P \rangle \begin{cases} N_{\text{inan}} \\ N_{A} \end{cases} \\
& \begin{cases} \ldots \end{cases}
\end{align*}
\]

We will see in chapter IX that \( 9^{*} \) (with some simplification) suffices alone for the grammar of English. (153) and (156) will
We will see in chapter IX that $g^*$ (with some simplifications) suffices alone for the grammar of English. (164) and (167) will
be derived from 9* by methods to be developed below. We will therefore not trouble to give 9 in full detail here.

10. \( V_{PB} \rightarrow Z_1 <Z_2 <\ldots <Z_n >\ldots > \)

where \( Z_1 \) is one of the elements of the form:

\[
\left\{ \frac{V_c}{V_x} \right\}_{\text{Inf}} <V_{PA2}>
\]

Actually a distinction should be made between subclasses of \( V_c \) and \( V_x \) in terms of the selection of subject, but we will not go into this refinement.

11. \( \frac{N_{\text{anim}}}{N_{\text{p}}} \rightarrow \{ N_{\text{u}} <\text{in env.} -- S > \} \)

\( N_{\text{p}} \) is the class of proper nouns, \( N_{\text{ac}} \), animate common nouns. Many further subdivisions and much more detailed restrictions can be stated.

12. \( PP \rightarrow P^0NP \)

13. \( P \rightarrow \text{of, in, by, } \ldots \)

\[
\begin{cases}
\emptyset <\text{in env.} -- \left< \frac{AP}{N_{\text{p}}} \right> \\
\emptyset <\text{the}>
\end{cases}
\]

14. \( T \rightarrow \{ \text{the} \} <\text{in env.} -- \left[ \frac{\text{N_{ac}}}{\text{N_{inan}}} \right] S \\
\text{the}<\text{in env.} -- \left[ \frac{\text{N_{ac}}}{\text{N_{inan}}} \right] \emptyset >
\]

This formulation permits "a sincerity of manner", "a screeching of brakes", and "a certain sincerity", but not "a sincerity", "a screeching", (Cf. fn. 33). This statement is
This formulation permits "a sincerity of manner," "a screeching of brakes," and "a certain sincerity," but not "a sincerity," "a screeching." (Cf. fn. 23). This statement is
only partially adequate, but here, too, we will not give the further detail necessary. "#" signifies sentence boundary. (C

15. \[ N^a \rightarrow \left\{ \begin{array}{l} V_T \\ \text{Inf} \left\{ \begin{array}{c} \left\langle V_{PA2} \right\rangle V_{P1}, \text{except in env.} \\
\text{the} \left\langle \text{AP} \right\rangle \end{array} \right\} \end{array} \right\} \]

Thus Inf-phrases as abstract nouns are divided into two types, Inf\(^n\)\(V_T\) and Inf\(\left\langle V_{PA2} \right\rangle V_{P1}\). Only the former can occur before of or after the article. Thus (with Inf \(\rightarrow\) ing) we have "screeching of brakes", "the screeching", "an unpleasant screeching", but not "playing the piano yesterday of...", "the playing the piano yesterday", "an unpleasant playing the piano yesterday", with a full Verb Phrase. This analysis is correct as far as it goes (that is, some detail is omitted), but it certainly gives little insight into the processes at work here.

16. \[ \text{Inf} \rightarrow \left\{ \begin{array}{l} \text{to in env.} \\
\text{V}_c \\
\text{ing < in env.} \end{array} \right\} \]

17. \[ \text{AP} \rightarrow \left\{ \begin{array}{l} \text{be} \\
N_{ac} \\
N_{inan} \\
N_{ab} \end{array} \right\} \text{ } \left\{ \begin{array}{l} \text{be} \\
\text{ing} \left\langle V_{P1} \right\rangle \end{array} \right\} \]
The possessive adjective $NP^S_1$ deserves some comment. In a phrase like "the boy's companions", "the" goes with "boy", not with "companions". Thus the phrase is not of the same form as "the strange companions". This is proven by the impossibility of "the boy's and strange companions", by the possibility of "a man's children" (but not "a children"), "John's book is..." (but not "book is..."), etc. (38) All of these cases show that the possessive suffix is not added to a noun which appears in the adjective position, between an article and a noun, but to a noun phrase which appears in the adjective position with a $\emptyset$ article (or no article, in an alternative treatment). Only certain $NP^i$'s can appear in this position (cf. statement 8, above). To determine whether a noun phrase of the form Noun$^P$Prepositional Phrase can appear in this position it is necessary to make a decision as to the grammaticality of "the man from Philadelphia's car", etc. I have noted many such instances in normal conversation, and would thus be inclined to admit this possibility, which is in fact allowed for in this grammatical sketch (cf. statement 7, above). A more detailed study than this, taking into consideration adjective sequence and selection, would result in a more satisfactory analysis of the character of the possessive phrase.

18. A rule of verbal selection suitably restricted to apply only to the $V_T$ introduced in statement 17 (the passive). This
rule is related to statement 9, with initial and final conditioning contexts interchanged, and other alterations. We do not give it in detail, since it is quite complex and subsequent methods will permit its elimination.

A rule of selection should also be given for the \( V_{I1}, V_{I2} \) introduced in 17. We omit this too.

19. \( VP_A \rightarrow VP_{A1} < VP_{A2} > \)

20. \( VP_{A1} \rightarrow \{ed\} < M > \)

21. \( VP_{A2} \rightarrow < have > < being > except in env. ing -- > (39) \)

22. \( D \rightarrow \{D_1\} \)

23. \( D_1 \rightarrow \) very, rather, ...

\( D_2 \rightarrow \) quickly, certainly, ...

\( A \rightarrow \) old, red, ...

\( N_p \rightarrow \) John, ...

\( N_{eb} \rightarrow \) sincerity, truthfulness, ...

\( N_{ac} \rightarrow \) boy, dog, ...

\( N_{inan} \rightarrow \) table, store, ...

\( M \rightarrow \) will, can, shall, may, must ...

\( V_{t1} \rightarrow \) appoint, feed, ...

\( V_{t2} \rightarrow \) recognize admire, ...

\( V_{t3} \rightarrow \) frighten, surprise, ...

\( V_{t4} \rightarrow \) carry, see, ...

\( V_{II1} \rightarrow \) sleep, laugh, ...

\( V_{I2} \rightarrow \) flourish, occur, ...
\( V_C \) → want, decide, ...
\( V_y \) → prefer, urge, ...
\( V_e \) → know, believe, consider, ...
\( V_f \) → give, ask, ...
\( V_g \) → elect, choose, ...
\( V_h \) → consider, think, make, ...
\( V_i \) → interest, surprise, excite, ...
\( V_k \) → tire, drink, bore, ...

68.3. This gives us an analysis into elements of \( P \) (cf. 47.3)
It remains to characterize the relation between the level \( P \) and the level \( W \), i.e., to set up the mapping \( \Phi^P \), which in 68.3 we have considered to be broken down into components \( \Phi_1, \Phi_2, \Phi_3 \).
It will be recalled that these are mappings of \( P \)-markers into strings in \( W \). Hence in stating these mappings we may refer to the constituent structure of the mapped string, i.e., to any stage in its 'history', and not just to its present form, (cf. 47.3)

we did not go into the nature of these mappings very deeply, and here we will simply give an informal characterization of them, to which we will return below.

As mappings we have (cf. (35), (39)):

\[ \Phi_{11}: \text{To be added in chapter IX,} \]
\[ \Phi_{12}: \text{...of... goes into...} \]
\[ \Phi_{13}: \left( \begin{array}{c}
\text{NP}_S \\
\text{NP}_P
\end{array} \right) \text{ goes into } \left( \begin{array}{c}
\text{NP}_S \\
\text{NP}_P
\end{array} \right) \]
\[ \Phi_{14}: \text{be } \rightarrow \text{were in env. } \text{NP}_P \text{—ed} \]
$\Phi_{15}$: Let $K = \{z, e, s, e, e, f, u, i, m, n\}$

Suppose that $Z$ is of the form:

$$X_1^{k_1}d_1^\gamma v_1^\epsilon X_2^{k_2}d_2^\gamma v_2^\epsilon \cdots X_n^{k_n}d_n^\gamma v_n^\epsilon X_{n+1}^\gamma,$$

where $X_i$ is a string not containing $k_i^\gamma d_i^\gamma v_i^\epsilon$

$k_i \in K$

d_i is a $D$ or is $U$

$v_i^\epsilon$ is a verb, an $M$, have, or be.

Then $\Phi_{15}(Z)$ is $X_1^{\gamma}d_1^\gamma v_1^\epsilon X_2^{\gamma}d_2^\gamma v_2^\epsilon \cdots X_n^{\gamma}d_n^\gamma v_n^\epsilon X_{n+1}^{\gamma}$

\section{\nonumber \Phi_{16}, \Phi_{17}}: To be added in chapter IX

$\Phi_{18}$: $\emptyset$ goes into $U$

$\Phi_2$: Let $Z = X_1^{\gamma} \cdots X_n^{\gamma}$ (where $X_i$ is a prime). Let $K$ be as in the description of $\Phi_{15}$. Let $X_{\alpha_1}, \ldots, X_{\alpha_m}$ ($1 \leq \alpha_1 < \ldots < \alpha_m \leq n$) be all the primes of $Z$ not followed by a $k_i^\gamma d_i^\gamma v_i^\epsilon$ directly by some $k_i^\gamma d_i^\gamma v_i^\epsilon$ or by $S_1$. Let $X_{\alpha_0} = U$

Then for each $i$, $1 \leq i \leq m$, $X_{\alpha_{i-1}}^{\gamma} d_{i-1}^\gamma v_{i-1}^\epsilon X_{\alpha_i}^{\gamma}$ goes into a single word, i.e., a single prime of $W$.

$\Phi_{19}$ is (39). $\Phi_{14}$ takes care of column IV, table I, §60.1.

$\Phi_{15}$ is (35). For $\Phi_{18}$, cf. §60.1. $\Phi_2$ is an operation of placing word boundaries in strings of morphemes (cf. §58.1, §58.12 and comments). It must still be demonstrated that this informal characterization which refers to constituent structure has some clear and precise significance.
66.4. We present two sample derivations to illustrate the functioning of the grammatical sketch. These are given in the style of App., chap. V. Steps are coalesced where a single rule applies several times (e.g., step 6, derivation 1). The number of the rule used in forming a given line of the derivation appears in brackets to the right of that line. The mappings are added at the end as separate steps. Dashes indicate repetition of the material immediately above the dashes.

Derivation 1.

1. Sentence
2. NP*VP
3. NP*VP_A*VP
4. NP*VP_A*V_T*NP
5. NP*VP_A*V_T*NP
6. T^N*Z^VP_A*V_T*V_T^AP*N^S
7. T^N*anim*Z^VP_A*V_T*V_T^AP*N^inanimate*S
8. ----------------------V_T^4----------------------
9. T^N
10. Z^N*P*Z^VP_A*V_T^4*Z^AP*N^inanimate*S
11. ----------------------P^ing^v^i*V_T^animative*S
12. --------VP_A1*VP_A2
13. ------C
14. ------C*have*en
15. ----------------------P_1
16. Z^John*Z^C*have*en*witness*Z^very*ing*surprise*event*S
17. -------S
18. -------have*S*witness*en*-------surprise*ing-------
19. John*have*S*witness*en*very*surprise*ing*event*S
20. John*have*S*witness*en*very*surprise*ing*event*S
21. John has witnessed very surprising events

In step 16, hyphens indicate word boundaries.

By morphological rules
Derivation 2.

1. Sentence
2. NP^VP
3. NP^VP_A^VP_1
4. NP^VP_A^P_2^V_T^PP
5. NP^------------
6. T^N^S^ VP_A^P_2^V_T^PP
7. T^N_{anim}^S-----------------
8. --------------- V_I1----------
9. T^N_{sc}^S-----------------
10. ------------------------- P^NP
11. -------------------------- of^NP
12. the---------------------
13. the^N_{sc}^S^ VP_A^P_1^P_2^V_T^of^NP
14. ---------- ed ----------
15. the^voter^S^ed^sincerely^approve^of^NP
16. -----------------------NP_S
17. ---------------------------- T^AP^N^∅
18. --------------------------- N_A^∅
19. --------------------------- ∅-----------------
20. -------------------------------- Inf^VP_1
21. -------------------------------- ing -------
22. -------------------------------- ∅^NP_S^ing^VP_1
23. -------------------------------- be^Fred
24. -------------------------------- ∅^NP_S
25. -------------------------------- AP
26. -------------------------------- ∅^T^N^∅^S_1^ing^be^Fred^Ap
27. -------------------------------- N_{anim}^∅^S_1^ing^be^Fred
28. -------------------------------- N_p
29. -------------------------------- ∅^∅-------------------
30. the voter's ed sincerely approve of $\mathcal{Z} \mathcal{V} \mathcal{N}_2 \mathcal{Z} \mathcal{S}_1$ ing be\textsuperscript{en} V \[17\]

31. \[V_l \[18\]

32. \[\text{John elect} \[23\]

33. \[\text{sincerely approve ed being elect en} \[\mathcal{Z}_1 \[5\]

34. the voter's sincerely approve ed of John's being elect en \[\mathcal{Z}_1 \[8\]

35. the voter's sincerely approve ed of John's being elect en \[\mathcal{Z}_2 \[2\]

36. The voters sincerely approved of John's being elected

We have taken "of John's being elected" to be a prepositional phrase, but we might consider an alternative analysis with "approve of" as a transitive verb. This has intuitive appeal, particularly in sentences like "I approve of John", but whether or not it would turn out to be the simplest analysis is an involved question. We will see below, however, that from a point of view different than the one developed so far, the latter analysis is in fact dictated by very simple and obvious considerations.

In we found that (assuming a certain extension of the analysis of grammaticalness) strict application of criteria of simplicity compelled us to assign several P-markers to certain utterances which happened simultaneously to be instances of two independently established patterns, so that constructional homonymy resulted in a manner which, in that instance, was in accord with strong intuitive judgment. Investigating the grammatical sketch just presented, we can find several other cases where even our limited account of grammaticalness leads to
intuitively correct \textsl{random} instances of constructional homonymy.

Statement 17 gives $\text{ing}^i V_T$ as one analysis of the adjective phrase. Thus with $V_T = \text{bark}$, we have

\begin{equation}
(162) \text{barking dogs never bite}
\end{equation}

as an instance of \textbf{Adjective Phrase - plural Noun - Verb Phrase} (\textit{AP-N^pS-VP}).

Statements 15 and 16 give $\text{ing}^i VP_1$ as one analysis of the noun phrase. Thus with $VP_1 = \text{play 'musical' instruments}$, we have

\begin{equation}
(163) \text{playing musical instruments is his chief joy in life}
\end{equation}

as an instance of \textbf{abstract Noun - be - Noun Phrase} ($N_A^\text{be-NP}$). That the abstract noun is singular is evident from the singular "is". Since the verb is singular, we know that the subject of

\begin{equation}
(163)
\end{equation}

cannot be an instance of the construction \textit{AP - plural Noun}, as is the case in

\begin{equation}
(163)
\end{equation}

Given a verb that is both transitive and intransitive (e.g., "fly", which can occur in "planes fly" and "they fly planes"), we can therefore construct a sentence which is simultaneously an instance of \textit{AP-Noun-VP} and $N_A^i-VP$ (where $N_A \rightarrow \text{ing}^i VP_1$); for instance,

\begin{equation}
(164) \text{flying planes can be dangerous}
\end{equation}

which can be understood in the sense of either (165) or (166).

\begin{equation}
(165) \text{flying planes is a dangerous sport for an untrained pilot}
\end{equation}

\begin{equation}
(166) \text{flying planes cast strange shadows (40)}
\end{equation}

\begin{equation}
(166a) \text{is ambiguous, e.g., (165) and (166), (167) and (168)}
\end{equation}

are not, since the number of the verb is indicated.

\begin{equation}
(167) \text{flying planes is dangerous}
\end{equation}

\begin{equation}
(168) \text{flying planes are dangerous}
\end{equation}
This again is a case where the results of strict application of our criteria correspond to our intuitive judgment about constituent analysis and dual interpretation, and thus where formal considerations of the kind we have adduced suggest a basis for this intuition.

In §62.4 we saw that

\begin{equation}
(169) \text{The police stopped drinking}
\end{equation}

could be analyzed either as "the police - stopped - drinking" \((\text{NP} - V_T - \text{NP})\) or "the police - stopped\(^{\ddagger}\)ing - drink" \((\text{NP} - V_B - \text{VP})\).

A third interpretation is now provided for certain sentences by the possibility of forming an adjective phrase \(\text{ing}^*V_1\) (cf. statement 17). Thus we have

\begin{align*}
(170) & \text{rats leave sinking ships} \quad (\text{NP} - V - \text{AP}^*\text{N}) \\
(171) & \text{John likes reading books} \quad (\text{NP} - \text{like}^*\text{ing} - \text{VP}, \text{i.e.}, \text{NP} - V_B - \text{VP}) \\
\end{align*}

Both interpretations are possible in

\begin{equation}
(172) \text{Bombardiers like sinking ships.}
\end{equation}

There should, then, be three possible ways of construing certain sentences of the form \(\text{N-V-}\text{ing}-\text{V-N}\). It is difficult, however, to discover a clear and natural instance of a triple homonym. (41)

§2.1. One extremely serious deficiency of this grammar, and of the conception of grammar on which it is based, is that we really have no good way to introduce the rule for conjunction as a statement of the grammar. But the simplification of the conjunction rule was one of the fundamental criteria for the determination of constituent structure. Hence if the grammar
cannot incorporate this rule, the proposed approach to a demonstration of validity is undermined, and there is considerably less justification for the particular form that the grammar has assumed. Of course, inability to state the conjunction rule is itself a serious defect, irrespective of the fundamental character of this rule as a criterion for the establishment of constituents.

Roughly, the conjunction rule asserts that \((\text{173})\) is an optional conversion.

\[-(\text{173}) \quad \text{.X.} \rightarrow \text{.X'and'X.}, \quad \text{where X is a prime}\]

But our framework has no place for such a statement as \((\text{173})\). It will be recalled that each rule of the grammar operates on the forms as they appear in the last step of the derivation in question, at the point when the rule is to be applied. Our definition of generation of derivations from grammars does not provide for the possibility of taking into account the history of the elements that appear in this last step of the derivation in question. But if \((\text{173})\) were introduced into the grammar as a statement, it would be necessary to know the history of the elements that appear in the last step of the derivation at the point where \((\text{173})\) is to be applied. That is, we should have to know which substring is represented by a single prime, i.e., is a constituent, and in fact, what sort of constituent it is.

This suggests that \((\text{173})\) be stated as a mapping \(\Phi_3\), rather than as a statement of the grammar, since the mappings are defined on \(\text{P}\)-markers, and \(\text{P}\)-markers do give us the constituent structure of a string. But clearly this idea would involve a
radical revision of our whole notion of levels as systems of representation. The function of the mappings of $\mathbb{P}$ into $\mathbb{W}$, as this conception of linguistic structure has been developed, is to provide the information that, e.g., the representation of "John was here" on the level $\mathbb{P}$ is a certain abstract element $\Gamma^1$ (called a $\mathbb{P}$-marker) from which the constituent structure of "John was here" can be derived in the manner described in chapter VI, just as its representation on the level $\mathbb{W}$ is a certain string of words, namely, John was here. It makes no sense to say that $\Gamma^1$ is also the representation on the level $\mathbb{P}$ of "John and Bill were here", as would be the case if the conjunction rule were construed as part of the mapping $\Phi^P$. A further difficulty is that some notion of non-obligatory mapping is implied here, and this has as yet no meaning.

68.2. Another approach would be to generalize our conception of the form of grammars so that the history of an element can be utilized in applying a rule to a step of a derivation. This would invalidate a good part of chapter VI, and seems an unfortunate step to take in view of the simplicity and naturalness of the conception of grammatical form developed there, and in view of the inadequacy of this conception of grammar for limited description such as that of $\mathbb{B}^2$ (or the appendix to chapter V). On the other hand, in the preceding sections we have noted a considerable number of inadequacies in this conception. Some of these might be remedied with a more elaborate development of the conception of grammars.

We will see that a third approach can be developed which will enable us to construct the conjunction rule in accordance
with the suggestion of §62.1 (final paragraph), but avoiding the objections raised there; to resolve the problems which have arisen in applying the conceptions which have been developed without any alteration of these conceptions or any broadening of the basis on which they have been constructed; and finally, to extend the scope of syntactic analysis considerably beyond the bounds imposed by the system of levels so far developed.

70. Appendix to chapter VII. This section is to contain a sketch of a syntactic description of Modern Hebrew along the lines that have been developed. This syntactic statement will provide the framework within which the morphophonemic study of the appendix to chapter V is developed. That is, the derivations of the appendix to chapter V can be appended to the derivations of this section so as to provide derivations of strings of phonemes from the representation Sentence. This then, gives a sketch of a complete grammar in the sense of chapter III. The major difference between the system of phrase structure described here and that described in §52 is in the extensive use here of long components.
Footnotes - Chapter VII

(1)(p.274) We have also noted that when our theory is sufficiently developed, the notion of 'systematic behavior' or 'grammaticalness' should become refined with the establishment of $\mathbb{E}$ and with the establishment of $\mathbb{F}$ itself.

(2)(p.275) Naturally, if the corpus does not contain a large number of occurrences of "and" in various positions in normal sentences, then we could not make these distinctions on the basis of the corpus. But this is just the tautology that if we have no data, we have nothing to describe. All of our discussion is based on the assumption that the data has been collected -- that the grammar is based on an adequate corpus. We have not discussed the very important linguistic question of how a corpus is put together and how the linguist obtains the information about linguistic behavior. Cf. Lowbury, "Field Methods and Techniques in Linguistics", Anthropology Today, Harris and Voegelin, *Eliciting in Linguistics*, etc.

(4)(p.278) One interesting fact, in this connection, is that there is a certain subclass $K$ of grammatical sentences that has an independent significance and for which this rule apparently comes much closer to being adequate. The importance of this remark will only become clear later, when we see that $K$, which we will later call the kernel of the language, can be isolated by purely syntactic techniques on the higher level of transformational analysis, quite independently of any consideration of suprasegmentals. We will then argue, on independent grounds, that only the elements of $K$ need be assigned a phrase structure.


(3) This rule is necessary anyway, to account for such sentences as "John has been" (the answer to "who has been absent?"). In discussing grammaticalness we noted several times that the most frequent words might be taken as defining diagnostic environments for categories. But this had a highly ad hoc flavor since there is no natural break between more frequent and less
frequent. A linguistically more significant and less arbitrary version of this approach would be to select those words which are always unstressed (except for contrastive stress) as defining diagnostic environments. This is a natural and closed category with independent linguistic status, and all the important high frequency words would in fact be included.

(6)(p.280) But see below, §6.3, where passives are analyzed as verb+adjective.

(7)(p.283) Note that "I" and "you" are 'plural' from the point of view of this rule of 'disagreement in number'.

(8)(p.283) That these rules are morphophonemic, not phonemic, is evident from such pairs as "peers" - "pierce", "raisins" - "essence", etc.

(9)(p.284) Actually, it must still be shown that sung must be taken as sing'en, not, e.g., as take'en, etc. In order to show that the intuitively correct analysis of the irregular verbs actually follows necessarily from the available linguistic notions, it would be necessary to show that the rules for forming irregular verbs, the statement of verb distribution, etc., are actually simplified with this analysis. It would be interesting to determine to what extent the intuitively correct analysis can be validated on distributional grounds. Cf. also §24.1 and fn. 4, chap. IV.

(10)(p.287) Note that we cannot have f(X, Y). Thus a notational convention must be added that in cases like the analysis of VPa2 where all elements appearing in the analysis are 'angled', at least one of them must be developed, i.e., only 2n-1 of the 2n ways of developing such statements permitted by Def. 4, chapter III (§19.2) can actually be permitted.

(11)(p.287) I.e., we must assume that the methods of chapter IV (which are stated in terms of words) lead to the construction of a category of intransitive participles ("barked", "slept", etc.), which never occur in the context ("John was", etc., alongside of a category of intransitive 'finite' verbs ("bark", "sleep", etc.). The methods of morphological analysis, which we have not discussed, must be such as to lead to the identification of these
sets element by element. I.e., we must show that the grammar is simplified in certain ways by this identification, or we must develop independent means to reach the same result. Obviously, these problems of morphology deserve very serious study, and failure to undertake this is a serious gap in this theoretical development. We might hope that the correspondence between these two categories need not be 1-1 in order for linguistic method to carry out the identification. This leads into the problem of projection on the level $M$, which we have also not discussed. Note incidentally that because of the problem of morphemic alternation, projection on the level $M$ presents quite special problems. Thus we could not have known, if "slept" had not appeared in the corpus, that this is the phonemic form of $\text{sleep}^{\text{en}}$. It is likely then that projection on the morphological level will be mostly a matter of dropping parts of $\text{Tr}(W)$, rather than extending its limits. Cf. 443.

(12) (p. 293) But note that (59) as it stands is very likely to lead to a violation of Axiom 7, 50.2. We cannot rewrite "$\text{VP}_2$" on the right as "$\text{VP}_2$" because in running through the grammar a second time, this element must itself be subject to (59), since we can have "I saw him watching them coming down the road", etc. To avoid this difficulty we might add an extra step between the left and the right side of (59), thus complicating the grammar. We will continue the analysis overlooking this potential trouble spot, merely noting it as one of the many difficulties involved in full scale recursive production, i.e., in attempting to present system of constituent analysis that completely covers grammatical sentences.

(13) (p. 294) We must be careful about choice of examples here. Thus we might have $A_1$ = "I don't like to see people be$^{\text{ing}}$ intimidating", but not $A_2$ = "I don't like to see people be$^{\text{ing}}$ intimidating others". But the grammaticality of the form is irrelevant to the point at issue. We know that this is a case of an adjectival, not verbal use of "intimidating" because if $A_1$ is acceptable, then so is $A_2$ = "I don't like to see people
be<ing>very intimidating". Hence $A$, even if judged grammatical, does not contain an instance of the auxiliary be<ing>, as does (63).

(14)(p.294) But note that "accused" will still be in a different subcategory of adjectives than "sad" or "tired", since it cannot be preceded by "very".

(15)(p.300) Actually, as $NP^oVP_A^wV^wNP$. In discussing various alternative analyzes we often will omit mention of the auxiliary phrase, where this plays no role in the choice between them. Major constituent breaks will often be marked by a dash in informal discussion.

(15') (p.301) The following discussion applies to both $V_A$ and $V_B$. (16)(p.302) Note that it is important to place the rule of verbal selection before the rule that gives the full development of $NP$ in the sequence of rules that forms the reduced grammar. Otherwise, all the specific forms into which $NP$ will be developed will have to be listed as conditioning contexts in the rule of selection, and this is of course a vast list. An optimal grammar gives each rule at the point where the conditioning contexts have been developed just to the degree relevant for correct application of the rule. Cf. §2.1.3.

(17)(p.303) For a different, but related possibility for distinguishing between the rules of number and selection in terms of their systematic significance, cf. §5.0.4.

(18) (p.305) Cf. fn.10.

(19)(p.305) We can give this directly, without the intermediate $Z_i$, by using indefinitely many types of brackets. Cf. §19.2, 2.6.4.

We have not provided for statements like this in the construction of the form of grammars, but such a recursive extension of the notations could be carried out. There is no need for this however, since, as we will see below, this statement can be eliminated completely. Cf.

(20)(p.306) Failure would be treated as a case of selection of the $V$ by the $NP$ subject, and need not concern us here.
(21)(p.307) Note that what we denoted by $VP_1$ in (71) has now been renamed $VP_2$ in (94).

(22)(p.310) Note that we have not shown that (112) and (113) are not cases of constructional homonymy, and that they in fact have only one interpretation. It may be that this can be shown by a more detailed development of the possibilities for describing grammatical sentences in terms of phrase structure, or it might require the methods of the next chapter which will enable us to make use, in support of the correct analysis, of the fact that we have, e.g., "to be happy is the important thing", but not "to come tomorrow is John".

(23)(p.317) Recall that the conditioning context has been reduced to the point where it contains just those elements that determine how the rule in question operates.

(24)(p.318) Our reasons for not accepting it are now invalid, and in favor of this analysis is the fact that an extra sentence form and an extra element ($VP_b$) are dropped, and that we have the intuitively unacceptable but formally compelling parallels given in (77).

(25)(p.318) We might, incidentally, suspect that the problem of introducing such sentences as (117) could be solved by taking these to be derived from sentences of the form "...who is..." by an optional grammatical rule "who is $\rightarrow \emptyset\emptyset $", counting on the word "who" (as opposed to "which" or "that") to carry the selectional information even when $NP_a$ is developed beyond the relevant forms. But the incorrectness of this view becomes quite clear when we note that inanimate and abstract nouns cannot be distinguished in this way, so that, e.g., "the sincerity scratched by John was...", "the table manifested by John was...", etc., would have to be admitted as fully grammatical, even though (69) does permit us to exclude such semi-grammatical material as "John scratched sincerity", "John manifested the table", etc. We will see later (cf.) that there are other reasons why this approach is not possible.
(26) (p. 321) This may even be an obligatory rule. We are not here making the distinction between optional and obligatory. Cf. $\mathcal{Q}_{21}, \Sigma_{12}, 2.2$

(27) (p. 330) The possessive morpheme $S_1$ cannot be identified with the noun plural and verb singular morpheme $S$. Despite similar morphophonemic effects, they differ as we can see from "wives" (plural) but "wife's" (singular possessive).

(28) (p. 330) We use $\mathcal{N}_{X}$ when any of the three forms of the noun that we are considering can appear.

(29) (p. 332) There are also other approaches which we have not considered. E.g., one might approach grammatical relations in terms of the study of the domain of long components, government, etc. These approaches should be integrated into a single theory of grammatical relations.

(30) (p. 333) That is, it may be the case that if $X_1 \rightarrow Y_1$, then $X_3$ must be developed into $W_1$, whereas if $X_1 \rightarrow Y_2$, then $X_3$ becomes $W_2$.

(31) (p. 333) Of course, we might be able to develop a significant notion of primary and derivative selectional relations.

(32) (p. 334) Note that it is really illegitimate to simplify a given grammar by a rule of the sort we are about to give unless we are willing to generalize this rule to all grammars, that is, unless we are willing to regard it as a reduction in the sense of $\eta 5 \eta$. But the study of the form of grammars is at such an early stage that it is worthwhile to investigate ways of simplifying given grammars even if we are not yet prepared to generalize. Cf. $\eta 7.2$.

(33) (p. 337) In this and several other places we give statements in a form that is literally inadmissible, since we have not introduced formally the notions used in formulating the restriction. But in all such cases, the restriction will appear below to be eliminable.

(34) (p. 338) Note that in stating (152), (153), and (156) the statement of that part of the conditioning contexts introduced in statements 1-8 was omitted.

(37) (p. 340) This is one case where the convention of §61 increases the complexity of the statement.

(38) (p. 341) We also have "a children's book", but only with the stress pattern of a compound noun (like "lighthouse"). These (as in the appendix to chap. IV), we are continuing to treat as single words, so "a children's book" is a construction like "a lighthouse" or "a tree".

(39) (p. 342) Cf. fn. 10.

(34) (p. 346) These instances are not normally marked by difference in stress, as are "hunting dogs" and "hunting lions" in "hunting lions is a dangerous pastime" and "hunting dogs must be carefully trained".

(41) (p. 349) In fact, it is difficult to find even a double homonym which is natural in the case of N-V-ing-V-N. Thus the interpretation of "the police stopped drinking hard liquor" in the same sense as "the law forbade drinking hard liquor" is forced, though both interpretations are natural when the object of "drink" is dropped, as in (159).
Chapter VIII - Transformational Analysis

21. In a broad sense, the problem of syntactic analysis is to determine the membership of the class \( \Lambda^w \) of grammatical strings of words. The goal of a grammar is to provide a method for constructing exactly this set. Given \( \Lambda^w \), there are trivial ways to meet this requirement. For instance, the set of sentences of books in the New York Public Library (with obvious qualifications) is probably a fairly good approximation to the set \( \Lambda^w \). But the purpose of a linguistic grammar is to rebuild the vast complexity of the language more elegantly and systematically by extracting the contribution to this complexity of the various linguistic levels. And in linguistic theory, we face the problem of constructing each level in an abstract manner, so that we can systematically develop a characterization of \( \Lambda^w \) for any given language by interpreting the abstract formalism of linguistic theory for the corpus of observed sentences of this language. The system of levels of linguistic theory not only provides the means for giving a simplified description for this enormously complex set of grammatical sentences, but also for determining the bounds of this set in the first place (cf. \( b_{32}, b_{45} \)). Thus the New York Public Library contains an approximation not to \( \Lambda^w \) itself, but only to that part of it that the linguist might ever come in contact with. But a linguistic grammar must answer further questions which cannot be dealt with by this trivial "grammar," e.g., how can a speaker generate new sentences? (Cf. \( b_{24}, 2 \)).

Clear examples of this systematic and piecemeal reconstruction of the complexity of language have appeared in the preceding chapters. A straightforward characterization
of $\mathcal{P}^W$ could be achieved if the level $\mathcal{P}$ of phrase structure was taken as simply $\mathcal{W}$, with the element $\text{Sentence} \ (= \mathcal{P}_0)$ adjoined, and if a derivation was defined as a sequence $(\text{Sentence}, X)$, where $X$ is a member of $\mathcal{P}^W$. The set of such derivations would provide a complete grammar. But it is clear that an enormous simplification in the characterization of $\mathcal{P}^W$ is achieved when we develop a notion of derivation in terms of which $\mathcal{P}^W$ can be developed stepwise, with the higher level similarities between members of $\mathcal{P}^W$ marked in terms of phrase structure. Similarly, we developed a more systematic and workable notion of constituent by requiring non-overlap of constituents (relative to a given interpretation of a sentence), but we recovered the original complexity by assigning alternative $\mathcal{P}$-markers ('interpretations') to certain strings, where special and stateable circumstances dictate this multiple assignment. We have seen that when we interpret the abstract levels in the prescribed manner so as to lead to the simplest grammar, there are, in specific instances, correspondences with strong intuition about language structure. This result may be interpreted as giving an explanation, in terms of a theory of linguistic structure, for these intuitions of the native speaker.

In other words, suppose that we had constructed only one syntactic level, namely $\mathcal{W}$. We would then be able to represent utterances only as strings of phonemes, morphemes, and words. The result of attempting to construct a grammar in accordance with this limited theory of linguistic structure would have been an enormously complex grammar of completely unmanageable proportions. Furthermore, we would have found that many strong intuitions of the native speaker would be
quite unaccounted for. These considerations would have led
us to construct a new and higher level \( P \), as a part of
linguistic theory.

Having constructed \( P \), we must investigate the extent of
its success in resolving these two difficulties. That there
is a considerable degree of success is evident from all that
has been said above. But we have seen in the course of
working out the interpretation of \( P \) for English that both
types of inadequacy remain. All but the simplest sentences
had to be excluded, at least if we wished to achieve anything
near an 'optimal' grammar in which each rule applies when
just the relevant specification of strings has been given
and in which what is essentially a single rule of selection
need not be repeated in several different forms for various
cases. We saw that any attempt to introduce more complex
sentences not only led to overwhelming complexities (which,
incidentally, threatened to distort seriously the simple and
adequate picture of the simple sentences) in the formulation
of the grammar, because of the tremendous number and involved
types of recursions, but also appeared to favor analyses that were in sharp contradiction to strong intuition.

We might, then, seek to amend \( P \), perhaps along the lines
suggested above in \( H_{2} \) and \( H_{21} \). Alternatively, for
exactly the reasons that led to the establishment of \( P \) in
the first place, we might attempt to construct a new
level of linguistic structure in terms of which utterances
can be described. An investigation of the specific
shortcomings of our present theory will, I think, favor the
latter course. We will now proceed to survey some of the major problems that this theory leaves unresolved.

22.1. There are cases where similar strings have intuitively quite different interpretations, but where we can discover no grounds, on any of our present levels, for assigning different markers to them. For instance,

(1) This picture was painted by a real artist
(2) This picture was painted by a new technique
are quite different sorts of sentences. (3) lends itself to either interpretation.

(3) John was frightened by the new methods

This can mean, roughly, "John is a conservative -- new methods frighten him." Or it can have the sense of "new methods of frightening people were used to frighten John" (an interpretation which would be the normal one in the case of "John was being frightened by the new methods"). Introspecting, the two interpretations seem to involve a difference of construction. In the first case, "methods" is the 'subject' (as is "artist" in (1)); in the second, it seems to be the noun of a prepositional phrase expressing 'means' (as is "technique" in (2)).

(4) differs intuitively from (5) in the same way that (1) differs from (2).

(4) The escaped prisoner was caught by the police

\[(a)\] The escaped prisoner was caught by the railroad tracks
\[(b)\] The escaped prisoner was caught by ten o'clock

(6), much like (3), is ambiguous, being subject to the interpretations of either (4) or (5a).

(6) The man was killed by the car
In the intuitive conception, then, (1) and (4) are classed together as opposed to (2) and (5), the distinction being some feature of construction. (A subsidiary three-way distinction within the group (2), (5a), (5b) apparently has to do with the meaning of "by", and need not concern us in our investigation of the adequacy of a grammatical theory.) (3) and (6) seem intuitively to be cases of constructional homonymity, cases of overlap of opposed structural patterns. Yet (1)-(6) are all instances of the pattern NP - was - A - PP (cf. §64.2-2 and statements 3, 5, 17, and 18, §65.2). A theory of selectional relations might be of some help here. A detailed account of statement 18, §65.2, would reveal that the PP in (2) and (5) is of a type that need not be stated as a conditioning context in statement 18, (1) whereas the PP of (1) and (4) must be stated there. But even if this distinction can be developed in some systematic way, it will apparently class (3) and (6) along with (2) and (5), rather than as ambiguous sentences. Furthermore we have seen (cf. §64.1) that there are many potential trouble spots in this approach. Finally, such an approach will provide no explanation for the 'verbal force' of "paint", "frighten", "catch", "kill" in (1)-(6), or of "tire" in (7) John was tired by the unusually hard work as compared with the non-verbal character of "tire" in (8) or of "bore" in (9)

(8) John was tired by evening
(9) John was bored by that time

Note further that such sentences as (10) are intuitively cases of constructional homonymity, in a different way than are (3) and (6).
(a) John was frightened
(b) John was surprised
(c) John was bored

E.g., in (10a), "frightened" can be 'verbal' as in either interpretation of (3) or in "he was once frightened by a mad dog", or it can be 'adjectival' as in the interpretation of "John was obviously very frightened". But in all of these cases, we are compelled simply to classify these \( Y \)\( \alpha \)\( en \) forms in a single way, as a certain subclass of adjectives.

There is thus a complex of intuitively quite different structural patterns, overlapping in cases of constructional homonymy, but with no counterpart in the rigorous application of our present theory.

There are other instances of intuitively felt difference in structure that are not properly accounted for in the application of the theory of syntactic structure as so far developed. Consider the difference in our feeling for the structure of (11) and (12)

(11) The growling of lions...
(12) The reading of good literature...

Or, with either interpretation,

(13) The rearming of Germany...
(14) The shooting of the hunters...

In the case of (11), "lions" is intuitively the subject. In the case of (12), "literature" is intuitively the object. In the case of (13) and (14), the noun of the PP can be understood as either the subject or the object. But these comments, although intuitively obvious, are beyond the range of our theoretical development. The phrases in (11)-(14) are all similarly constructed in our terms. All are instances of NP-PP (cf. the discussion relating to (3)). In terms of the grammatical sketch of (12), (13) and (14) are in fact cases of constructional homonymy, since the verbs "rearm",

"shoot" can be either $\mathbb{W}_1$ or $\mathbb{W}_2$. But the relevant distinction is not this, but rather the relation between the verb and the noun of the following prepositional phrase. For some cases, an elaboration of the notion of selectional relation might give a basis for this felt difference, but for others (including (13) and (14), on the present level of grammaticality), it would not, unless the difficulties cited in are somehow resolved.

In (15), we came across another example of unexplained dual interpretation ((159)-(161)). We saw there that,

(15) I found the boy studying in the library
can be interpreted as having the same construction as

(16) I know the boy studying in the library,

but that it can also be interpreted, even more naturally, in quite a different way, as having the same construction as

(17) I found him studying in the library.

This distinction is an obvious and clear one, as we can see by adding, e.g., "not running around in the streets" to these three sentences. In the case of (16) and one interpretation of (15), "the boy studying in the library" is the complex noun phrase object of the main verb. In the case of (17), and a second interpretation of (15), it seems more natural to regard "found ... studying in the library" as being, somehow, a complex discontinuous verbal element with "the boy" and "him" as its 'objects' (though even this is not satisfactory, since "the boy" and "him" appear at the same time to be 'subjects' of "study"). In any event, we have no grounds for establishing
or describing the difference in construction between (16) and (17), or the ambiguity of (15). In our analysis, the phrase structure of all of these sentences seems to be identical.

Another fairly subtle but nonetheless real case of dual interpretation arises in sentences such as

(18) I don't approve of his drinking

(4)

In one sense, this might be paraphrased roughly by "I don't think he ought to drink" (i.e., he's too young), in another sense, by "I don't like the way he drinks." Similarly

(19) I don't approve of his cooking

may indicate that I think his wife should cook, or that I think he uses too much garlic.

Introspecting, the difference between the two interpretations in both cases seems to depend on the extent to which we regard "his" as the subject of "drink" (and "cook"), or as an adjective modifying "drinking" (and "cooking"). But we can find no grounds for a dual interpretation within the study of phrase structure.

The cases cited in this section exemplify one type of intuitive inadequacy of our present theoretical framework. That is, these are examples of sentences with dual interpretations, but where we have no grounds for establishing constructional homonymity in the intuitively relevant manner.

22.2. The opposite deficiency would occur if we were sometimes led to assign several F-markers to sentences that have, intuitively, only one analysis. We can also find cases of this kind. Consider the sentence

(20) The dog is barking.
In accordance with the analysis presented in chapter 17, we analyze (20) as

(21) The dog is barking, i.e., NP VP A V I

as is intuitively correct. But "barking" is also an adjective (cf. statement 17, ⑩⑩⑩.2) as in

(22) barking dogs never bite

and there is a sentence form NP VP A (cf. statements 5, 5, ⑩⑩, ⑩⑩⑩.2) as in

(23) The dog is noisy

Thus by a completely different route we arrive at the analysis of (20) as

(24) The dog is barking, i.e., NP A V I

making (20) analogous in construction to (23). But this analysis has no intuitive support.

This, then, is a case where we have too much constructional homonymy. (5)

22.2. We have come across cases where the distinction between what are intuitively quite differently constructed sentences is not properly marked, and where distinctions are characterized that are not intuitively felt. A third problem arises when the correct number of A-markers are assigned, but where the considerations so far available lead to the assignment of what is intuitively the wrong marker. The analysis of (17) as I found him studying in the library (⑩⑩.12) [NP VP A] is one such case. In ⑩⑩.3, we found that if we were to attempt to present a description of phrase structure that would
cover all grammatical sentences, then there would be another and more serious case of this nature.

To recapitulate briefly, it seems intuitively clear that in a sentence such as

(25) John was eating a sandwich

"was eating a sandwich" is a verb phrase, analyzeable either into the auxiliary "was" and the verb phrase "eating a sandwich" (the latter being further analyzeable into the verb "eating" and the noun phrase "a sandwich"), or into the verb phrase "was eating" and the noun phrase "a sandwich". But under one assumption (which we will state directly), the grounds available to us at this point apparently necessitate the different and counter-intuitive analysis into the main verb "was" and the noun phrase "eating a sandwich", so that the sentence (25) is of the form NP-was-NP, like

(26) John was a politician.

The reason for this analysis is formal similarity and simplicity. Since we must anyway assign some occurrences of "eating a sandwich" to NP because of

(27) eating a sandwich takes only a few minutes

it is clearly more economical to assign it to NP in this instance too, particularly since we arrive at a sentence form NP-was-NP, which is already familiar on independent grounds because of (26). We actually manage to drop one restriction now in the analysis of this form, i.e., the requirement that NP cannot become ing\textsuperscript{VP} in the context
The assumption under which this counter-intuitive analysis follows is that we add to the grammar certain notational devices which permit us to adjoin to certain statements of the grammar a restriction which in we called the qualification \( Q \). But we saw in that we could not provide an analysis for certain sentences (without serious departures from certain desirable properties that we would like a grammar to have) unless these notational devices were available. Hence if we construct linguistic theory in such a way that the grammar can present a phrase structure for every sentence directly, within the segment of the grammar which is reduced (in the sense of \( \delta \)) from the level \( E \), then this counter-intuitive analysis of (25) as analogous to (26) will follow. Thus we must either arbitrarily limit the grammatical description to only the simplest sentences (in a sense which cannot be independently characterized, at this point), or we must accept a strongly counter-intuitive analysis of phrase structure.

Summing up this discussion, then, we see that one type of difficulty is that rigorous application of the methods we have so far sketched apparently leads us to analyses that are either too many, too few, or simply incorrect.

23.1. A second class of difficulties concerns the distinction between various different kinds of sentences. It would be hard to disagree with the judgment that "John!", "come here!", "John was here", "was John here?" are in some sense sentences of quite different kinds. Are there, in our terms, general formal grounds for this distinction? (6)

In chapter VII we found that the rule for conjunction serves as a criterion for determining sameness or difference of structure.
If \( X \) and \( Y \) are sentences of different 'types', then we would expect that \( X \bowtie \text{and} \bowtie Y \) should be ungrammatical, just as instances of \text{Sentence} \bowtie \text{NP}, \ldots \text{VP} \bowtie \text{PP} \ldots \) are ungrammatical. This is in fact the case. We have "I saw him yesterday and I lent him a book", but not "I saw him yesterday and can you come?", or "John! and I saw him yesterday." This indicates that the component sentences in these compounds must be considered as different 'constituents', i.e., in this case, as different kinds of sentences, since they are the maximal constituents of themselves. In other words, had we given a more complete grammar in \( \mathcal{G}_0.2 \), the first statement of this grammar would have had to be

(28) \[ \text{Sentence} \rightarrow \begin{cases} \text{Imperative Sentence} \\ \text{Interrogative Sentence} \\ \text{Declarative Sentence} \end{cases} \]

However, the conjunction criterion is not entirely satisfactory as a basis for setting up sentence types. We have not discussed the conjunction rule in anywhere near sufficient detail, but in our present context we can easily see that any formulation of it will fail to make exactly the distinction that we require. We can formulate this criterion so that \( X \) and \( Y \) are sentences of the same type if either \( X \bowtie \text{and} \bowtie Y \) or \( Y \bowtie \text{and} \bowtie X \) is grammatical, or only if both \( X \bowtie \text{and} \bowtie Y \) and \( Y \bowtie \text{and} \bowtie X \) are grammatical. If we choose the first alternative, then imperatives and future declaratives are not distinguished from one another, since we have

(29) \( \text{Come here and I will tell you a secret.} \)
If, noting that (30) is not grammatical, we choose the second alternative, then we will not be able to set up a class of declarative sentences, since (31) is no more grammatical than (30).

(30) I will tell you the secret and come here
(31) I will tell you the secret and I am here

22.2. Furthermore, there are subtypes of sentences within these basic types, and these naturally cannot be distinguished by the conjunction criterion if the main types can be distinguished in this way. Within the general type of interrogatives we must distinguish such subclasses as those to which (32) and (33) belong.

(32) Was he here?
(33) Who was here?

If we decide that such sentences as

(34) did he come and who saw him?

are grammatical, then we have no way of distinguishing (32) and (33) as separate subtypes. If, on the other hand, (34) is considered ungrammatical, then we cannot set up interrogatives as a sentence type. Either way, the result is unsatisfactory, since (32) and (33) are clearly more similar to one another than are declaratives to interrogatives, but less similar than, e.g., two sentences of the 'form' (32).

There is also an intuitively clear subclassification of declaratives. Most grammars of English set up passives as a special sentence type. But there seems to be no compelling formal reason to do this. Why is "John was seen at the lunch counter" more different in form from "John stopped at the
than is "John was eating at the lunch counter", or "John was tired in the morning"? Why does "John was seen at the lunch counter", alone among these sentences, require special treatment in the grammar? Such sentences seem intuitively to hold an intermediate position between "John was here" and "was John here." While they have the basic subject-predicate form, we are nonetheless inclined to treat them separately. We have as yet provided no motivation for this special treatment.

We have a classification of sentences into

\[
\text{Declarative} \begin{cases}
\text{active} \\
\text{passive}
\end{cases} \\
\text{Interrogative} \begin{cases}
(35), \text{ etc.} \\
(33), \text{ etc.}
\end{cases} \\
\text{Imperative}
\]

This is not an exhaustive classification. It has good intuitive support (as well as support in the practice of grammarians), but, as yet, no formal grounds in terms of the system we have constructed. It appears that to distinguish sentence types properly, some criterion additional to conjunction and other criteria of the level \(P_m\), some completely new approach, is required. (8)

73.3. In the same connection, it seems that certain sentences of presumably different types are related to one another. Thus "he was here" and "was he here", or "John hit Bill" and "Bill was hit by John" seem related in a way in which such pairs as "John hit Bill" and "Bill hit John", "John hit Bill" and "John was hit by Bill", "he was here", and "he will be here",
or "John hit Bill" and "John was hitting Bill", are not.

Exactly what this relation is is not clear. It certainly
has some connection with meaning. On the other hand, it is
not synonymy or logical equivalence. This is clear enough
in the case of question and answer, but it is also true in
the case of active and passive, as we have seen in §5.8.
We have at present no way of explaining such relations.

23.4. A related phenomenon is that in each of these pairs of
related sentences, one seems somehow more basic than the
other and more central as far as the structure of the language
is concerned. A study of the arrangement of English words
in sentences will normally treat first such 'basic' patterns
as subject-predicate (actor-action), using as examples such
simple declarative sentences as "John was here" or "I like
John", and will discuss passives, questions, imperatives,
sentences with relative clauses, etc., only as subsidiary
and derived phenomena.

There seems to be no obvious formal justification for
this procedure, though if it were not to be followed, the results
of grammatical analysis might prove to be quite strange. Suppose,
for instance, that the investigation of the phrase structure
of English was initiated on the basis of such examples as
(36) whom have they nominated.

Investigating various analyses of (36) in two parts, etc.,
in accordance with the procedures of constituent analysis, it
seems that we would not arrive at such conceptions as that of
a basic actor-action relation at all. In fact, there seems to
be no reasonable way to even begin to construct an intuitively
satisfactory \$\$-marker for this sentence. Nevertheless, the
subject-verb relation does, clearly, appear in (36), with
"they" as the NP subject, and "Whom have...nominated" as the
verb phrase, with the object "whom", and the verb "have...nominated". But the search for general formal grounds in P for such an
analysis seems futile and formally unmotivated. It seems
reasonable to consider the possibility of denying any P-marker
to (36), and of somehow deriving a partial constituent
analysis for (36) by considering its relation to some more
'basic' sentence which does have a P-marker, e.g., "they
have nominated John".

The significance of this class of basic sentences, the
manner of its construction, and the nature of the relations
between basic sentences and 'derived' sentences of non-central
sentence types remains to be explored. The process of construction
of derived sentences is not an unfamiliar one. It makes
intuitive sense to form the passive of a given active sentence,
or to construct a question corresponding to a given declarative,
active or passive. But we see at once that these processes
of derivation cannot be commuted or compounded freely.
Thus we cannot form the passive of a question.

The study of these cases has no place within the
framework of linguistic theory as we have so far outlined
its development (though these notions are discussed in
traditional grammar). The considerations of this section
thus suggest that this theory is too limited in scope to
reconstruct completely our pre-systematic conception of
linguistic form.

24. A third class of difficulties has to do with the complexity
of the grammatical statement when this is unlimited in scope.
We discovered in investigating English grammar with the tools so far available that when declarative sentences of other than the simplest type are considered, the grammar becomes unwieldy and loses its 'optimal' character (cf. in particular §62-4). And the application of criteria of simplicity, though still formally possible, ceases to have much meaning. The difficulties become even more extreme if we try to state the structure of other sentence types, e.g., the full range of interrogatives, within the same grammatical framework (and there is no systematic justification for not doing this). Although it is clear that the same selectional relations hold for these sentences as for the simple declaratives, the inversions make a single statement of verbal selection quite involved. We have already observed departures from optimality in the case of statement 18, §66.2; and in statements 4, 6, 8, 10, 14, 15, 21 we were forced to use devices which are really unwarranted --- cf. fn.s. 19, 33, 37, chapter VII. Furthermore, we saw in §68 that even the fundamental rule on which much of the validation of the grammar was based could not be introduced into the grammar. We noted in §65.1 (see also statement 9*, §66.2) that the rule of verbal selection becomes extremely complex even if we limit ourselves to declarative statements, and that we are as yet unable to make use of some striking similarities in selectional relations to simplify it.

Several other theoretical gaps have been exposed incidentally in the course of our discussion. Thus in the final paragraphs of §60.2 and §62.4 we noted that many affixes are apparently associated with the wrong elements, with consequent difficulties, for example, in stating the rule for conjunction. See also fn. 12, chapter VII. A more far reaching inadequacy is that we have no means for stating the mapping which carries P-markers into strings of words. Cf. chapter VI and §66.2. The fundamental difficulty in stating is that we have no way to take into account the 'history' (i.e. constituent structure) of a string in converting it further. We noted several times that this same difficulty recurs within the English phrase structure.
This suggests that the difficulty of describing English phrase structure completely and our inability to characterize $\mathcal{D}$ may have the same (or a similar) source. It will be noted, incidentally, that our inability to state the mapping $\mathfrak{F}$ in sufficient generality leaves the status of discontinuous constituents in doubt, since these cannot be introduced in the sequence of conversions that gives the internal structure of $\mathcal{P}$.

In the grammatical sketch developed in chapter VII, and presented in §66.2, we avoided the enormous number of recursions and the excessive complication of formulation caused by more complex sentences (and by different types of sentences), by simply excluding all but the simplest sentences from consideration. In this way we were able to develop a fairly simple and intuitively adequate picture of phrase structure for a limited class of sentences. But we have no real warrant for this arbitrary limitation.

75. In the abstract development of the level $\mathcal{P}$ of phrase structure we left open the possibility that images of $\mathcal{P}$-markers under the mapping $\mathfrak{F}^\mathcal{P}$ will not exhaust the set $\mathcal{S}^\mathfrak{P}$ of grammatical strings of words. That is, we left open the possibility that certain grammatical strings of words will not be generated by the sequence of conversions corresponding to the level $\mathcal{P}$ and will thus not be provided with a constituent analysis by this segment of the grammar. The reason for this
will not receive a constituent analysis. The reason for this was to avoid making the level \( P \) overcomplicated. Thus it was intended that derivations and constituent analysis be provided for sentences only where it is profitable to do so, where an organized and simple system of phrase structure results. If we do deny \( P \)-markers to certain strings in \( \text{Gr}(W) \), then we must provide some other way for characterizing these unmarked sentences not derived by \( P_1 \)-derivations. In the light of all that has been said, it seems most natural to characterize these sentences in terms of some notion of grammatical transformation, regarding these sentences as transforms of certain sentences which are derived on the level \( P \) and which do have \( P \)-markers.

We are thus led to develop a new level of syntactic analysis, the level \( T \) of transformations, and to assign \( T \)-markers to strings of words as markers of their 'transformational history'. That is, the \( T \)-markers of a string of words will tell us how this string is derived from a certain kernel of sentences which have \( T_1 \)-derivations and \( P \)-markers. In terms of previous levels we can represent each sentence as a string of phonemes, words, syntactic categories, and, in various ways, as strings of phrases. Now we will be able to represent a sentence as a sequence of operations by which this sentence is derived from the kernel of basic sentences, each such sequence of operations corresponding to a \( T \)-marker.

We will try to construct the level \( T \) in such a way that cases of constructional homonymy and difference of interpretation will have their formal analogues in the assignment of different \( T \)-markers, i.e., different sequences of operations originating from the same or different kernel sentences, and resulting in a given string. The notion of 'relation between sentences' may
also find an explanation in transformational terms. Thus we may say that a sentence \( X \) is related to a sentence \( Y \) if, under some transformation set up for the language, \( X \) is a transform of \( Y \) or \( Y \) is a transform of \( X \). Centrality of structure can be explained wherever we can show that certain transformations are irreversible, i.e., that although \( X \) can be derived from \( Y \), \( Y \) cannot be derived from \( X \). We can identify sentence types and subtypes with particular transformations and subsidiary transformations of transforms. Finally, we may hope that the residual complexity of the level \( P \), caused essentially by the complicated network of selectional relations in a complex sentence and by the inversions, etc., characteristic of different sentence types, can be eliminated by adjoining this new level which will enable us to construct new sentences out of already existing sentences with the selectional relations already 'built in'.

These are the goals of transformational analysis. In the remainder of this chapter and in the following chapter, I will try to show that a theory of transformations can provide a unified and quite natural approach to all of the problems mentioned in \( 4.72-4 \), and that it can result in syntactic description which is considerably more economical and revealing. Furthermore, it will appear that this theory can we developed on the same basis as is already required for the familiar levels. That is, no new notions, semantic or otherwise, need be introduced to establish the level of transformations and the procedures of transformational analysis. The motivation, the form, and the basis for the level of transformational analysis are those which are familiar from the levels that we have already established.
25. The problem before us is to develop a set of grammatical transformations which will apply in a certain set of cases (e.g., those of which instances were given in §72-4) in an interesting way. This dual program of constructing a certain potentiality of description and then demonstrating that it applies in just the right cases is by now a familiar one. On each level \( L \) we have tried to state the general structure of \( L \), and to give a rigorous account of the central notions of \( L \), and have then tried to develop a procedure whereby, for a given language, various interpretations of this formal structure can be evaluated, and the best of them selected.

We turn first to the technical problem of developing an adequate abstract notion of grammatical transformation. This technical discussion must make a certain set of available transformations for grammatical description. Before actually presenting the construction, I will try to give a fairly thorough account of the reasoning behind the particular formulation which will be adopted. That is, I will try to show how certain conditions are imposed on this construction by the nature of the linguistic material, by our general conception of linguistic theory and of linguistic levels, and by systematic considerations of simplicity and convenience. It is important to show just how the construction is motivated (particularly, in this very early stage in the discussion of transformational analysis) so that if further research shows its inadequacy to linguistic material, it will be clear at what points the construction can best be modified or recast. As elsewhere, the construction of transformations must be regarded as tentative --- as an attempt to generalize from the
problems that have arisen in limited investigation of linguistic materials (such as those reported on in chapter IX).

§ 77-8 will be devoted to a step-by-step development of the notion of transformation which we will finally adopt, along with a statement of the motivation for each step. In §79 we present a more concise and systematic account of this development, with some further refinement. In §§80-1 we discuss some particular transformations of special interest for transformational analysis, and show that in a certain sense these serve as a basis for the whole system. This discussion is carried further in §§82-3 where we investigate the problem of assigning constituent structure to transforms. In §84 we apply the notions that have been developed to clarification of the status of the mapping $F$. In §§85 and 87 we develop transformations as a linguistic level of the form discussed in chapter II and we generalize these notions so that transformations can apply to sets of strings. This development is necessarily somewhat technical and detailed. We summarize it in §82, the first section of chapter IX, which is devoted in full to application of these transformational analysis. The reader who wishes to bypass the technical discussion might then proceed directly to §89, taking it for granted that there are reasons for the fact that the description of transformations, as presented there, is so involved.
problems that have arisen in limited investigation of linguistic materials (such as those reported on in chapter IX).

§ 72 will be devoted to a step-by-step development of the notion of transformation which we will finally adopt, along with a statement of the motivation for each step. In § 79 we present a more concise and systematic account of this development, summarizing §§ 77, 78. This summary is presented as a technical discussion in § 79.1 and 79.2, and as an informal review in § 79.3. The development in §§ 77, 78 is necessarily somewhat technical and detailed. The reader who wishes to bypass the technical discussion might then proceed directly to § 79.3, taking it for granted that there are reasons for the fact that the description of transformations, as presented there, is so involved.

terminology

We now review briefly some of the relevant drawn from earlier chapters, and define certain new terms and notations which will be useful below.

We are concerned with the level \( \mathcal{W} \) of words, the level \( \mathcal{P} \) of phrase structure, and the level \( \mathcal{T} \) of transformations. Suppose for the sake of simplifying this informal exposition we assume that \( \mathcal{W} \) is actually embedded in \( \mathcal{P} \). That is, we assume that strings of words are also strings in \( \mathcal{P} \), in fact, strings in \( \mathcal{P} \), which it will be recalled, is the set of 'lowest level' strings in \( \mathcal{P} \), strings which bear \( \mathcal{F} \) to no string. \( \text{(8)} \)

We will use brackets \( [ ] \) to denote sets, and parentheses ( ) in denoting sequences of members. Thus the set containing the elements \( X, Y, Z \) as members can be denoted \( [X, Y, Z] \), and the ordered pair whose first term is \( X \) and whose second term is \( Y \) will be denoted \( (X, Y) \). The two-membered set containing as members the element \( X \) and the ordered pair \( (X, Y) \) can be denoted \( [X, (X, Y)] \). We will also use the customary symbol \( \epsilon \) as an abbreviation for the expression "is a member of". Thus it is true that \( X \epsilon [X, (X, Y)] \) and that \( (X, Y) \epsilon [X, (X, Y)] \). Very often we require a notation \( ** \) to enable us to express concisely the notion "the set of elements \( ** \) meeting the condition \( \mathcal{F} \) ...", i.e., "the set of \( \mathcal{F} \)'s such that \( \mathcal{F} \)." To express this, we use the notation
Thus the set of declarative sentences can be denoted by the expression

\[ \{ \mathbf{x} \mid \mathbf{x} \text{ is a declarative sentence} \} \].

Since sets are identical if and only if they have the same members, it is always the case, for any set \( \mathcal{K} \), that \( \mathcal{K} = \{ \mathbf{x} \mid \mathbf{x} \in \mathcal{K} \} \).

Slightly extending this usage, in the customary manner, we will denote the set containing the members \( x_1, x_2, \ldots, x_n \) by the expression

\[ \{ x_i \mid 1 \leq i \leq n \} \].

Combining the notations for sets and sequences, we will denote the set whose members are the sequences \( (y_1^1, y_2^1, \ldots, y_m^1), (y_1^2, y_2^2, \ldots, y_m^2), \ldots, (y_1^n, y_2^n, \ldots, y_m^n) \) by the expression

\[ \{ (y_i^1, \ldots, y_i^m) \mid 1 \leq i \leq n \} \].

These notations will be very useful, since we will find a heavy use of indices to be necessary.

A transformation (or a mapping) operates on a certain element and converts it into a new element. By the domain of a transformation we mean the set of elements upon which it operates. By the range of a transformation we mean the set of elements which it produces when applied to elements of its domain. Thus in the system \( P \), the domain of the mapping \( T^P \) is the set of \( P \)-markers and its range is a certain set of grammatical strings of \( P \)-words. Cf.

The element produced by application of a transformation \( T \) to an \( x \) element \( x \) of its domain is denoted \( T(x) \). Suppose that the domain of \( T \) consists of, e.g., ordered \( \mathfrak{M} \). Then \( T(x) \) will denote the element of the range of \( T \) which is produced by applying \( T \) to the ordered \( \mathfrak{M} \) of its domain.

These fundamental and customary notations will be modified slightly in the course of the exposition to suit our particular purposes, and the revisions will be stated along with the summary in § 72.
72.1. We are concerned with certain large classes of sentences such as 'answers' and questions, or actives and passives, which are paired in the sense that sentence by sentence, there is a constant formal difference between them; i.e., there is a fixed formal property such that to each sentence $X$ of one class, there is a sentence $Y$ of the other class differing from $X$ in just this formal property. The formal property may be a difference in word order, a fixed added element, the deletion of a certain element, or some combination of these. Thus passives are formed from actives by interchanging subject and object, and placing been before the main verb, and by after it, so that $\text{John} \overset{\text{love}}{\rightleftharpoons} \text{Mary} \overset{\text{love}}{\rightleftharpoons} \text{John} \overset{\text{love}}{\rightleftharpoons} \text{Mary}$ becomes $\text{Mary} \overset{\text{been}}{\rightleftharpoons} \text{love} \overset{\text{by}}{\rightleftharpoons} \text{John} \overset{\text{love}}{\rightleftharpoons} \text{Mary}$ ('Mary is loved by John').

Suppose for the sake of simplicity of presentation that in the informal expository account only transformations defined within the bracket $\text{WR}$ we assume that $\text{WR}$ is actually embedded in $\text{P}$. Thus strings of words are also strings in $\text{WR}$, in fact, strings in $\text{WR}$, which it will be recalled, is the set of 'lowest level' strings in $\text{P}$, strings which bear $\mathfrak{f}$ to no string. (This amounts to developing $\text{P}$ in such a way that $\mathfrak{f}$ is $\text{WR}$, cf. §47125).

The first requirement is that the result of a transformation must be unambiguous. Each transformation must be a single-valued mapping of certain strings in $\text{WR}$ into $\text{P}$. The reasons for this condition are just those behind the requirement that the product of a $\text{P}$-marker be unique. The purpose of transformational analysis, from one point of view, is to provide a substitute
analysis for sentences which are not provided with derivations in P, i.e., which are not the products of restricted P<sub>1</sub>-derivations. Given an analysis (a P-marker or a T-marker) we must know of what sentence it is an analysis.

Suppose that a certain transformation T is basically a permutation which carries a<sub>1</sub>-a<sub>2</sub>-a<sub>3</sub> into a<sub>3</sub>-a<sub>2</sub>-a<sub>1</sub>, for any strings a<sub>1</sub>, a<sub>2</sub>, a<sub>3</sub>. Suppose that T is to be applied to

(37) My friend hit Bill

This is ambiguous. If we interpret (37) as "my friend - hit - Bill", the result of applying T will be

(38) Bill-hit-my friend

If we interpret (37) as "my - friend hit - Bill", the result of applying T will be

(39) Bill-friend hit-my

Clearly these grammatical transformations cannot be taken simply as permutations on strings and the like. There must be a qualification specifying the analysis of the string to which the transformation is applied. This qualification could be given as part of the definition of the transformation. That is, we could define a certain permutational transformation as applying only, say, to strings of the form NP - V - NP, so that in the instance discussed above, only (38) is possible. Alternatively, we can take the domain of a transformation to be the ordered pair of a string and an analysis of the string, and we can allow transformations to be applied freely to such ordered pairs. For the case discussed above, either limitation would be adequate, but we will see that both of these restrictions are necessary for other reasons.
We note first of all that transformations must be limited by definition to certain kinds of strings. The transformation which turns an answer into a question will be based (in part) on a permutation of \( a_1 - a_2 - a_3 \) into \( a_2 - a_1 - a_3 \), since it will convert "John can come" into "can John come." But certainly it will not be the case that in general a string which is an \( X-Y-Z \) can be transformed significantly into \( X-Y-Z \). This will be the case only, e.g., when \( X=NP \), \( Y \) is an auxiliary verb of some sort, etc. So we cannot allow transformations to be unrestricted mappings of \( (X,K) \) into \( Y \), where \( K \) gives the analysis of \( X \). Transformations must be limited by definition to certain kinds of strings.

But this limitation alone is not sufficient, in general. We cannot define transformations with a qualification as to domain and then allow them to apply freely to strings in \( \mathcal{P} \), for how will we know whether a given string \( X \) in \( \mathcal{P} \) has the analysis required by the transformation? To determine this, we must be provided with a \( \mathcal{P} \)-marker of \( X \). But \( X \) may have several \( \mathcal{P} \)-markers (and these different \( \mathcal{P} \)-markers may in fact have one or more steps in common(9)). That is, we may have a string \( Z \) in \( \mathcal{P} \) with the \( \mathcal{P} \)-markers \( K_1 \) and \( K_2 \), and it may be the case that there is a string \( X'Z \) which is a member of both \( K_1 \) and \( K_2 \). And this string \( X'Z \) may in fact correspond to two conflicting analyses of \( Z \) into constituents. Suppose, for instance, that we have the following situation:

\begin{align*}
(40) \quad & f_1(\underbrace{a \circ b \circ \ldots}_{\text{a grammatical string of words}}, \underbrace{X'Z}_{Y}) \\
& f_1(X, a \circ b), f_1(X, a) \\
& f_1(Y, b \circ a), f_1(Y, a)
\end{align*}
Then we have two different P-markers of $a^b c$, which can be diagrammed as follows:

```
Sentence
  X
   a b c
```

```
Sentence
  X
   a b c
```

*Figure 1*

Suppose that $T$ is limited in application to strings of the form $X \cdot Y$. Then $T(a^b c)$ is ambiguous if the P-marker is not specified, may be either $a \cdot b \cdot c$ or $a^2 \cdot c^2$, where $T$ is the permutational transformation that carries $a_1 \cdot a_2$ into $a_2 \cdot a_1$. $T(a \cdot b)$ may be either $a \cdot a \cdot b \cdot b$ or $a \cdot b \cdot b \cdot a$, since both $a \cdot a \cdot b$ and $a \cdot b$ represent divisions of $a \cdot b$ into constituents $X \cdot Y$.

If $T$ is in general to be a single-valued mapping, then, it must apply to pairs $(Z, K)$, where $K$ gives the analysis of $Z$. If the analysis $K$ were always to be a P-marker, we could define $T$ as a mapping of P-markers into strings in $E$, since the product of a P-marker is unique. But there is actually a quite general sense in which a class $K$ of strings may give an analysis of $Z$, even if $K$ is not a P-marker. Thus $E_0$ has been defined in terms of a class $K$ which may or may not be a P-marker (cf. §4.2.1). This will be useful below, since we will also want to apply transformations to strings which are themselves products of transformations, and which do not have $E$-markers although they do have associated with them a unique class $K$ of strings that provides them with a derived analysis. Unless we specify that this class $K$ may contain only one string in $E$, we cannot define transformations on $K$ alone, but only on the pair $(Z, K)$, where $Z \cdot K$. Where $K$ is a P-marker of $Z$, $Z$ is uniquely determined by $K$. (10)
27.2. Summing up these considerations, we have the following conditions on the set of grammatical transformations:

(C1) $T$ is defined on ordered pairs $(Z,K)$, where

(a) $Z, T(Z,K)$ are strings in $F$

(b) $K$, a set of strings in $P$, and $Z$ is a member of $K$.

(T operates on a string $Z$ of $F$ with the analysis given by $K$ and produces a new string in $F$ if $K$ is a transformation $T(Z,K)$).

(C2) $T(Z,K)$ is unique. I.e., $T$ is a single-valued mapping

(C3) The domain of each $T$ is limited to strings of a certain structure. This limitation can be effected by associating with each $T$ a finite restricting class $Q$ of sequences of strings. Suppose that $(W_1, \ldots, W_n)$ is a string, then $T$ can be applied to $(Z,K)$ only if $Z$ is analyzed by $K$ into a $W_1$ concatenated with a $W_2 \ldots$ concatenated with a $W_n$.

These conditions are fundamental. Condition (C1) states the domain and range of the transformation. Condition (C3) requires that each transformation apply only to a certain subpart of this domain. Condition (C2) prescribes that only certain of the mappings from $W$ to $F$ be admissible into the set of grammatical transformations, namely single-valued (functional) mappings (i.e., each ordered pair in $W$ has a unique string).

Each transformation $T$ in the sense of conditions (C1)-(C3) can be represented as a set of ordered triples $\{(Z,K,Z')\}$, where $Z' = T(Z,K)$. Conditions (C1)-(C3) set up certain requirements that a set of ordered triples must meet in order to qualify as a transformation. We must now go on to investigate further requirements.

27.2. If we are to be able to use transformations in grammatical description they must be finitely characterizable.
In particular, we must be able to characterize the domain of the transformation in a finite manner. \(C3\) gives a necessary condition for a set of pairs \((Z, K)\) to qualify as to which the domain of some transformation. It is now necessary to add a sufficient condition.

We cannot simply replace the "only if" of \((C3)\) by "if and only if." The reason for this is that for certain choices of \(Z, K, Q\), it will be the case that \(Z\) can be analyzed by \(K\) in various ways into a sequence of strings that correspond, term by term, to the terms of some sequence in \(Q\). The case discussed in \((40)\) provides an example of this possibility.

\[(41)\] Assume \((40)\). Let \(\bar{Z} = a^n b^m c_n\)

\[K = \{z, x_1, x_2, x_3, \ldots\}\]

\[Q = \{(x, y)\} \text{ are unique members of } Q\]

Then \(\bar{Z}\) is analyzed by \(K\) into \(a-b^m c\) or \(a^n b-c\), each of which is an instance of \(X-Y\).

But suppose that \(T\) is defined as, e.g., the permutation that carries \(X-Y\) into \(Y-X\). And suppose that the domain of \(T\) is exactly the set of pairs \((Z, K)\) such that \(Z\) is analyzed by \(K\) into an \(X\)-followed-by-\(Y\). Then in the case of \((41)\), the result of the transformation will be either \(b^m c-a\) or \(c-a^n b\), depending on whether \(Z\) is analyzed by \(K\) into \(a-b^m c\) or \(a^n b-c\) (each of which is \(X-Y\)), thus contradicting \((C2)\).

We might suspect that the difficulty here is due to the fact that in \((41)\), \(K\) is an inconsistent analysis of \(Z\).\((12)\)

But the same difficulty can arise even in the case of a consistent analysis, as we can see from \((42)\).
(42) Suppose that: \[ Z = a \cdot b \cdot c \cdot d \]
\[ K = \{ X \cdot c \cdot d, a \cdot b \cdot Y, Z \} \]
\[ Q = \{ (X \cdot c \cdot d), (a \cdot b \cdot Y) \} \]
\[ \{ (X, a \cdot b) \}
\[ \{ (Y, c \cdot d) \}

Then \( Z \) can be analyzed by \( K \) into \( a \cdot b \cdot c \cdot d \) or \( a \cdot b \cdot c \cdot d \), each of which corresponds term by term to a sequence in \( Q \).

The simplest way out of these difficulties is to require that for \((Z, K)\) to be in the domain of \( T \) (with restricting class \( Q \)) it is necessary that there be exactly one way of analyzing analyzing \( Z \) by \( K \) so as to conform term by term to some sequence of \( Q \). Preparatory to giving this condition as \((C4)\), we define the notion of \( \text{pr} \) "proper analysis."

**Def. 1.** Let \( Q \) be a set of sequences of strings.

Then \((Y_1, \ldots, Y_N)\) is a proper analysis of \( Z \) with respect to \((wrt)\) \( K, Q \) if and only if

(i) \[ Z = Y_1 \cdots Y_N \]
(ii) there is a \( \bullet \) \((W_1, \ldots, W_M) \in Q \) such that for each \( i \geq 1 \), \( Y_i \) is a \( W_i \) of \( Z \) wrt \( K \)

\[ \text{i.e., such that } \left( Y_i, Y_{i-1}, \ldots, Y_1, W_i, Z, K \right) \]

For example, suppose that \( Q = \{ (NP, verb, NP) \} \); that \( Z = \text{"John caught a fish";} \); and that \( K \) is a \( P \)-marker of \( Z \) with respect to which one constituent analysis of \( Z \) is \text{John} - \text{caught} - \text{a fish}. \) Suppose further that \text{John} and \text{fish} are \( NP \)'s, and that \text{caught} is a \text{verb}. Then in this case the sequence \((\text{John}, \text{caught}, \text{a fish})\) is a proper analysis of \( Z \) wrt \( K, Q \), since both conditions (i) and (ii) of \text{Def. 1} are satisfied.

We can now state condition 4.

(C4) With each \( T \) there is associated a finite restricting class \( Q \) such that
$(\mathbb{Z}, K)$ is in the domain of $T^{(14)}$ if and only if there is a unique proper analysis of $\mathbb{Z}$ wrt $K, Q$.

One particular type of restricting class contains only sequences $x$ terms in length, for some fixed $x$. 
Def. Q is an r-termed restricting class if and only if Q is a finite set of r-ads of strings. I.e.,
\[ Q = \left\{ (w_1^{(1)}, \ldots, w_n^{(1)}) \mid 1 \leq i \leq N_Q \right\} \]

From the fact that the unit element U can appear as \( w_1^{(1)} \), and from Theorem 1, chapter VI, 42.1, it follows that any domain defined by a finite restricting class (as in C4) is also defined by some r-termed restricting class (and vice versa, of course). Hence there is no loss in generality if we replace "finite restricting class" by "r-termed restricting class" in (C4). These terms are not freely interchangeable, however, in all later contexts. From now on we will consider only r-termed restricting classes. A transformation is essentially a certain fixed way of rearranging and otherwise reconstructing a sequence of r terms of a given kind. It seems natural, then, to require that for each grammatical transformation the number of terms on which it operates be fixed.

We can then add as an amendment to (C4),

(C5) A restricting class Q is associated with a transformation T if Q is an r-termed restricting class, and Q characterizes the domain of T in the sense of (C4).

We can now specify the domain of a transformation in terms of a finite set of sequences of strings, each r terms in length. It remains to find some fixed and finite way to characterize T so that knowing \( Z \) and \( K \), we can determine \( T(Z,K) \).
22.4. Consider now the converse of the requirement (C2) that every grammatical transformation be a single-valued mapping. A little reflection shows that we must limit the set of grammatical transformations to exclude all but a very limited kind of functional mapping. One reason for this conclusion is that not every such mapping is finitely characterizable. But we can reach the same conclusion from another approach.

Suppose that \( \Gamma_j \) is the \( P \)-marker of "John came home", \( \Gamma_B \) of "Bill came home", \( \Gamma_M \) of "Mary came home," etc. There is a perfectly good functional mapping \( \Phi \) that carries (John came home, \( \Gamma_j \)) into "a police car was cruising around the neighborhood," (Bill came home, \( \Gamma_B \)) into "have you seen today's paper?", (Mary came home, \( \Gamma_M \)) into "I think so", etc. But such a mapping as \( \Phi \) would clearly be of no interest, and we will certainly want to exclude it from the grammatical description of English. In fact, we must exclude such mappings somehow from the set of grammatical transformations in the general theory, or there will be no hope of applying transformational analysis to any language in an interesting way, since all (functional) relations between classes of sentences will appear as special cases of some transformation. We might hope to exclude \( \Phi \) on the grounds of simplicity of the grammar, since clearly the description of \( \Phi \) for English will be extremely complex. But this will not succeed, within our present framework, for the reason that the strings in English can be enumerated (by 'alphabetical' ordering of the primes of \( P \) -- we assume such an 'alphabetization' below), and such mappings as \( \Phi \) can be defined in completely general terms in linguistic theory in terms of an arbitrary enumeration
of strings. Though the definition of $\Phi$ in general linguistic theory may be extremely complex, its application to a particular grammar, once it is defined abstractly, will not add to the complexity of this grammar, given an alphabetization of $P$. And we have no way to include complexity of linguistic theory as a factor in the evaluation of particular grammars. Thus $\Phi$ and similar mappings must be excluded from the set of grammatical transformations from the start, by an appropriate abstractly formulated condition.

The point here is roughly this. We do not want a transformation to depend on the 'content' of the particular strings into which $Z$ is decomposed for the purposes of transformation, but only on the number and order of these substrings. Only such transformations will reflect general structural relations between classes of strings. Once we have settled on some fixed way of characterizing grammatical transformations in terms of certain structural properties, then it will be possible to take the complexity of this fixed form of characterization into account as a feature in the evaluation of given grammars.

We can give this condition a more precise formulation. From (C1), (C4), we know that a transformation $T$ converts a string $Z$ analyzed in a certain way (say, into $Y_1 \ldots Y_T$) into a second string. The particular $(Y_1, \ldots, Y_T)$ into which $Z$ is analyzed for the purpose of applying $T$ is determined by the relation between the restricting class $Q \overset{\cdot}{\rightarrow} T$ and the class $K$ which gives the analysis of $Z$. This sequence $(Y_1, \ldots, Y_T)$ we have called the proper analysis of $Z$ wrt $K, Q$. Thus a proper analysis of $Z$ is a sequence of terms of $Z$ which figure in some
way in the transformation, which are rearranged, dropped, or developed somehow in the application of $T$.

We can characterize a transformation by stating how the terms of the proper analysis are embedded in the transform.

We require that for each $T$, there be a fixed manner of embedding these terms into a constant and fixed context of strings.

(C6) Suppose that $T$ is a grammatical transformation. Then there is a sequence of strings $C=(Z_1, \ldots, Z_{K+1})$ and a sequence of integers $A=(a_0, \ldots, a_K)$ such that

(i) $k \geq 0$; $a_0 = 0$; $1 \leq a_j \leq \infty$ for $1 \leq j \leq k$,

there is a restricting class $W$ associated with $T$ such that for all $Z_1, Z_2, \ldots, Z_k$,

(ii) for some associated restricting class $W$,

there is $\lambda$ a proper analysis of $Z$ wrt $K, Q$, then

$$T(Z, K) = Z_1^\wedge Y_{a_1}^\wedge Z_2^\wedge Y_{a_2}^\wedge \cdots Y_{a_k}^\wedge Z_{K+1}^\wedge,$$

where $Y_{a_l} = U$ if and only if $a_l = 0$.

We now add a further condition on the notion of "associated restricting class."

(C7) A restricting class $Q$ is associated with a transformation $T$ only if for some $C$ and $A$, $Q$ characterizes $T$ in the sense of (C6) the notion.

(C7), like (C5), is a condition on "associated restricting class". (C6), like (C4), thus states the existence of an associated restricting class for each $T$.

We will call the sequence $C=(Z_1, \ldots, Z_{K+1})$ a constant part of $T$ (or of $T(Z, K)$), and we will call the sequence $A=(a_0, \ldots, a_K)$ a term arrangement of $T$ (or of $T(Z, K)$). If $K=0$, then no term of the proper analysis occurs in the transform, except possibly, for certain $Z$'s, in the constant part.

It follows that each transformation is completely and
uniquely determined by a triple \((Q, C, A)\), where \(Q\) is an \(r\)-termed restricting class, \(C\) is a sequence of strings (the constant part) and \(A\) is a sequence of integers (the term arrangement). \(A\) is a rearrangement (not necessarily a permutation -- thus terms may be dropped or repeated) of the indices of the terms of the proper analysis, whatever these terms may be. Distinct triples \((Q, C, A)\) may determine the same transformation.

Note that not every triple \((Q, C, A)\) determines a transformation. Certain triples may fail to meet condition \((C6)\).

(43) Let \(Q = \{ (\mathbf{W}_{\mathbf{1}}^{(1)}, \ldots, \mathbf{W}_{\mathbf{r}}^{(1)}) \mid 1 \leq i \leq N_Q \} \), \(\mathbf{W}_{\mathbf{i}}^{(1)}\) a string

\[ C = (z_1, \ldots, z_s), \quad z_j \text{ a string} \]

\[ A = (a_0, \ldots, a_r), \quad a_i \text{ an integer} \]

Then a necessary and sufficient condition for \((Q, C, A)\) to define a transformation with \(Q\) as its associated restricting class, \(C\) as its constant part, and \(A\) as its term arrangement, is that

(i) \(s-r+1 > 0 \)

(ii) \(a_0 = 0 \)

(iii) \(1 \leq a_i \leq r \) for \(1 \leq i \leq t \)

(iv) for \(i \leq N_Q\), \(1 \leq j \leq t\), it is the case that \(W_{a_i}^{(j)} \neq U \)

(43) is correct only when we take one of the transformations to be a null transformation, i.e., the null set of triples \((Z, K, T(Z, K))\). This null transformation will turn out to be a sort of identity transformation under an extension to be introduced below in \(\S 29.2\). To say that a triple \((Q, C, A)\) defines the null transformation is not the same as to say that it defines no transformation at all.
Suppose, for example, that we have a transformation \( T \) with an associated restricting class \( Q \) whose sole member is \((W_1, \ldots, W_5)\). Suppose that the associated term arrangement is \((0, 3, 5, 3)\), and that the corresponding constant part is \((Z_1, Z_2, Z_3, Z_4)\). Suppose that the proper analysis of \( Z \) wrt \( K, Q \) is \((Y_1, \ldots, Y_5)\). Then
\[
T(Z, K) = Z_1 \uparrow Y_1 \uparrow Z_2 \uparrow Y_2 \uparrow Z_3 \uparrow Y_3 \uparrow Z_4
\]
Suppose that we have a second string \( Z' \) whose proper analysis wrt \( K', Q \) is \((Y'_1, \ldots, Y'_5)\). Then
\[
T(Z', K') = Z_1 \uparrow Y'_1 \uparrow Z_2 \uparrow Y'_2 \uparrow Z_3 \uparrow Y'_3 \uparrow Z_4
\]
Thus although \( Z \) and \( Z' \) may be completely distinct from one another in 'content,' there is a clear sense in which the structural relation of \( Z \) to \( T(Z, K) \) is the same as the structural relation of \( Z' \) to \( T(Z', K') \). This seems to be a natural way to approach the problems posed at the outset of this section.

27.5. The conditions (C1)-(C7) delimit a certain set \( \mathcal{J} \) of transformations. Each \( T \in \mathcal{J} \) is a single-valued mapping on a subset of \( \{(Z, K)\} \), where \( Z \) is a string in \( P \) and \( K \) is a class of strings containing \( Z \). \( T \) carries the elements of its domain into strings of \( P \) which differ from one another and from their pre-images in the sense of condition (C6). That is, \( T(Z, K) \) differs from \( Z \) in some structural characteristic. A subset \( S \) of \( \{(Z, K)\} \) qualifies as the domain of some transformation if there is a set \( Q = \{(W^{(1)}, \ldots, W^{(1)}) \mid 1 \leq i \leq N \} \) such that (i) for some \( j \leq r \), there is no \( i \) such that \( W^{(i)}_j = Y \), and such that (ii) \((Z, K) \in S \) if and only if there is exactly one sequence \( (Y_1, \ldots, Y_j) \) such that \( Z = Y_1 \uparrow \cdots \uparrow Y_j \) for each \( j \leq r \).

\( Y_1 \) is a \( W^{(1)}_j \) of \( Z \) wrt \( K \). \( \mathcal{J} \) is thus a set of ordered triples

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Transformation will be a 'constant transformation' that carries each \((Z, K)\) into some fixed string \( Z' \).
(Z, K) \in \mathcal{S} \text{ if and only if there is exactly one sequence } (Y_1, \ldots, Y_n) \text{ such that } Z = Y_1 \cdots Y_n \text{ and for some } i \text{ it is the case that for each } j \neq i, Y_j \text{ is a } \kappa(j) \text{ of } Z \text{ wrt } K. \text{ } S \text{ will be the domain of a 'constant transformation' that carries each } (Z, K) \in \mathcal{S} \text{ into some fixed string } Z' \text{ unless the restricting class } \mathcal{Q} \text{ in question meets the further condition that for some } j \neq i \text{ there is no } i \text{ such that } \kappa(j) = Y_i \text{ (of course } S \text{ may be the domain of many transformations, some of which are constant). } \mathcal{Y}_i \text{ is thus a set of sets of ordered triples} \text{.}
(Z, K, T(Z, K)) meeting these conditions, and each \( T \in \mathcal{J}_1 \) can be characterized in a finite way as a triple \((Q, C, A)\), appropriately related as in (43).

22.5. A further requirement for grammatical transformations is suggested by an investigation of the restrictions imposed on the proper analysis of a string by the conditions we have laid down above. (C2) and (C6) jointly impose the following condition on grammatical transformations.

(44) Suppose that \( T \) is a grammatical transformation with \( Q \) and \( Q' \) as associated restricting classes, for a given constant part \( C \). Suppose that \( A_i^Q = (a_0, a_1, \ldots, a_k) \) and \( A_i^{Q'} = (a_0, a_1', \ldots, a_k') \) are the respective term arrangements. Suppose further that for some \( Z, K \), the unique proper analysis of \( Z \) wrt \( K, Q \) is \((Y_1, \ldots, Y_n)\), and the unique proper analysis of \((Z, K)\) wrt \( Q' \) is \((Y'_1, \ldots, Y'_n)\).

Then \( Y_a = Y_a' \); \( \ldots; Y_a = Y_a' \).

We define the proper analysis of a string with respect to a transformation in the obvious way.

**Def. 2.** \((Y_1, \ldots, Y_n)\) is a proper analysis of \( Z \) wrt \( K, T \) if and only if for some \( Q \) which is an associated restricting class of \( T \), \((Y_1, \ldots, Y_n)\) is the proper analysis of \( Z \) wrt \( K, Q \).

From (44) we see that if \((Z, K)\) is in the domain of \( T \), then the proper analysis of \( Z \) with respect to \( K, T \) is uniquely determined up to a certain point. In particular, if, for a given constant part, all terms of some proper analysis of \( Z \) wrt \( K, T \) appear in \( T(Z, K) \) outside of the constant part, then the non-unit terms of every proper analysis of \( Z \) wrt \( K, T \) are uniquely determined. That is, the actual break-up of \( Z \) into terms is unique, given \( K, T, C \).
This, for instance, is the case if $T$ is a permutational transformation. But clearly uniqueness of the non-unit terms is not the case in general, even with a fixed constant part, as we can see from the following example.

\[(45) \text{ Suppose that: } \mathcal{J}(X, b \circ b) \text{ and if } \mathcal{J}(X, Y), \text{ then } Y = a \circ b \]
\[Z = a \circ b \circ c \]
\[K = \{Z, X \circ c\} \]
\[Z, Z_1 \text{ are strings in } \mathcal{F}\]

Let $T$ be the transformation whose single member is $(Z, K, Z_1 \circ c)$. I.e., $Z_1 \circ c = T(Z, K)$. Then both $Q$ and $Q'$ are associated restricting classes of $T$:

\[Q = \{X, c\} \]
\[Q' = \{a, b, c\} \]

and $T$ is defined by $(Q, C, A_Q)$ and by $(Q', C, A_{Q'})$, where $C = (Z_1, U)$, $A_Q = (0, 2)$, and $A_{Q'} = (0, 3)$. But in the first case $(a \circ b, c)$ is the proper analysis, and in the second case $(a, b, c)$ is the proper analysis.

As an example to show that even if all the terms of some proper analysis appear in the non-constant part of the transform (where the constant part is fixed), the proper analysis of $Z$ wrt $K, T$ is still not uniquely determined in full, consider the following.

\[(46) \text{ Suppose that: } Q = \{W_1\} \]
\[C = (Z_1, Z_2), \text{ } Z_1 \text{ and } Z_2 \text{ being strings in } \mathcal{F} \]
\[A = (0, 1) \]
\[T \text{ is the transformation defined by } (Q, C, A). \]

Suppose that: $Q' = \{W_1, U\}$

$T'$ is the transformation defined by $(Q', C, A)$. 
But in this case, $T=T'$. For suppose that $(Z,K)$ is in the domain of $T$. Then $E_0(Z,Z,W_1,Z,K)$; the one termed sequence $(Z)$ is a proper analysis of $Z$ wrt $K,T$; and $T(Z,K)=Z_1Z_2Z_2$.

But then it is also the case that $E_0(U,Z,U,Z,K)$ (cf. Th.1, chapter VI, §42.1). Hence we can analyze $Z$ into the proper analysis $(Z,U)$ wrt $K$, $T'$, and $(Z,U)=Z,K$ is in the domain of $T'$ with $T'(Z,K)=Z_1Z_2Z_2$.

Similarly every member of $T'$ is a member of $T$. Thus $T=T'$, and both $(Z)$ and $(Z,U)$ are proper analyses of $Z$ wrt $K,T$.

There are good reasons for requiring that the constant part of a transformation be unique; distinct constant parts correspond to distinct analyses of those terms of the transformed string that do figure in the transformation. However, there does not seem to be any compelling motivation in the linguistic material for requiring that the proper analysis be uniquely determined beyond those terms that actually appear in $T(Z,K)$ in the non-constant part. Nor is there any apparent gain in the fact results from having the unique constant requirement, extra freedom. Since the terms that appear in the transform should certainly be uniquely determined (and are, in fact, if the constant part is fixed — cf. (44)), and since there is a certain resulting systematic simplification (along with an increase in the power of the system), as we will see in §78, we will go on to require

(C8) The proper analysis of $Z$ wrt $K,T$ is unique (if it exists).

It follows from (C8) that for any transformation which is sufficiently diversified in application to be of any interest, the constant part is unique. This condition of "sufficiently diversified application" is given below, in §78, as (C12). Th.9, §82.5, indicates that for independent reasons we must limit ourselves to transformations that meet (C12).
Conditions (C1)-(C8) delimit a set of elements which we may call $\mathcal{Z}_2$. Each $T^2 \in \mathcal{Z}_2$ can be represented as a set of ordered quadruples $\{ (Z, K, Z', Pr) \}$, where the set $\{ (Z, K, Z', Pr) \} = T^1 \in \mathcal{Z}_1$, and for some fixed $Q$ which is an associated restricting class of $T^1$. $Pr$ is the proper analysis of $Z$ wrt $K, Q$. $Pr$ is thus uniquely determined for each $(Z, K)$ in the domain of $T^2$. $\mathcal{Z}_2$ is more specific than $\mathcal{Z}_1$. That is, each $T^1 \in \mathcal{Z}_1$ corresponds to a subclass of $\mathcal{Z}_2$ consisting of transformations which produce the same transformational effect as $T_1$, but which differ in the way proper analyses are assigned to strings. Clearly each $T^2 \in \mathcal{Z}_2$ is also uniquely determined by a triple $(Q, C, A_q)$ meeting (43), just as is each $T^1 \in \mathcal{Z}_1$. In other words, a triple $(Q, C, A_q)$ provides more information than is necessary to specify a $T \in \mathcal{Z}_1$; while distinct triples may determine the same $T \in \mathcal{Z}_1$. Some of this extra information is utilized in constructing $\mathcal{Z}_2$. (15) While (C6) restricted the set of grammatical transformations by eliminating all but a special kind of functional mapping, (C8) in a sense enlarges the set by giving us several transformations, each imposing a different proper analysis, where before we [only] had one.

76.1. This still does not exhaust the requirements for grammatical transformations. It is necessary to study the internal effects of these transformations on strings in greater detail that we have done so far. The basic reason for this is that we must provide a derived constituent structure for transforms, for one thing, so that transformations can be
compounded. From (C1), \[ 1 \], it follows that transformations can be applied only to analyzed strings. Compounding of transformations will permit the formation, e.g., of a question "was the game won by John" from the passive "the game was won by John", which in turn is derived from the active "John won the game". Cf. \[ 1 \].

To provide a constituent structure for \( T(Z,K) \), it is necessary to mark off constituent boundaries, and then to state how \( \mathcal{E}_0 \) holds for the marked off segments. There are various sources from which this kind of information can be drawn.

Certain relevant information can be supplied by our previous knowledge about the level \( P \). That is, it may be the case that one of the segments of the transform is in fact a \( P_A \), for some prime \( P_A \) of \( P \) (i.e., \( P_A \) bears \( \mathcal{E} \) to this segment). Suppose, for instance, that passives are dropped from the kernel of basic sentences for \( \mathcal{E} \), which \( P \)-markers are provided. Passives must then be introduced by transformation of actives which are derived on the level \( P \) in the normal way. Thus
(47) The game was won by John

is formed from "John won the game" by inverting subject and object, and adding several morphemes, in particular, adding "by" before the former subject to form "by John". But we know from P itself that "by John" is a prepositional phrase. Hence "by John" in (47) can be regarded as a PP. This approach to the structural analysis of transforms merits a much fuller discussion, and we return to it below in §42-3, where we develop the notion of 'derived constituent structure' in greater detail.

Another source for information about constituent structure is in the definition of the transformation itself. Transformations can be defined in such a way as to impose a certain constituent structure on the transformed string, and this is the approach to derived structure that concerns us directly now.

It is in fact possible to define transformations in so detailed a fashion that all information about the constituent structure of the transform is provided by the transformation itself, and that any constituent hierarchy can be imposed on any transform by an appropriate transformation. On the other hand, there are good reasons for limiting the contribution of this source of information. For one thing, such a conception would make the definition of transformations extremely cumbersome. For another, it would not permit the fact that a constituent of $T(z,k)$ is a well-known phrase (as in the example cited above) to be used in support of a transformation. Yet this does seem to be a relevant support. But fundamentally, this conception of derived structure
would be inadequate for the same kind of reason that led to
the framing of condition (C.6) in §77.4. Just as we do not
want every functional relation to serve as a grammatical
transformation, similarly, given $Z$, $K$, and $T(Z,K)$, we do not
want any arbitrary derived analysis of $T(Z,K)$ to be the result
of some transformation on $Z$, $K$. This would be extremely
ad hoc, and we would never be able to justify the assignment
of one derived analysis or another (i.e., the choice of a
transformation giving one analysis or a transformation giving
another) if all were equally available. The question is,
then, to what extent should derived constituent structure
be provided by the transformation itself.

Information about $E_0$ can be brought to bear on the
analysis of the transform only after segments
have been marked off in this string. Hence at least the
initial marking off of segments must be provided by the
transformation itself.

There is a natural way in which an initial segmentation
of the transform can be provided by the operation of the
transformation itself. In developing the elements of $J_1$,
we found it necessary to consider the transformed string $Z$
to be analyzed into a proper analysis $(Y_1,\ldots, Y_p)$ such that
$Z = Y_1 \wedge \ldots \wedge Y_p$ and the $Y_i$'s are the segments of $Z$
which actually figure in the transformation, i.e., which are permuted,
dropped, or to which elements are adjoined. We can characterize
any transformation which imposes an $r$-termed proper analysis
by stating what effect this transformation has on each of
the $r$ terms of the proper analysis of the string $Z$ to which
it is applied. That is, we can associate with each transformation
$T$ an elementar
proper analysis of \( Z \) wrt \( K, T \), then \( T(Z, K) = t(Y_1) \wedge \ldots \wedge t(Y_n) \).
This leads us to define the **proper analysis of the transform** as the sequence of terms \( t(Y_1), \ldots, t(Y_n) \).

For the same reasons that led to condition \( (C_2) \), we stipulate that the proper analysis of \( T(Z, K) \) must be unique. Just as we want the product of a \( S_1 \)-derivation to have a unique analysis associated with it, given the derivation, we would like the analysis of each \( T(Z, K) \) to be unique, given \( Z, K, T \). The terms of the proper analysis of \( T(Z, K) \) can be provided as the basic constituent segmentation for \( T(Z, K) \).

Summing up these remarks, we have the following informal condition on grammatical transformations.

\( (C_9) \) Underlying each \( T \) there is an elementary transformation \( t \).
If \( (Y_1, \ldots, Y_n) \) is the proper analysis of \( Z \) wrt \( K, T \), then there is a unique sequence \( (Y_1, \ldots, Y_n) \) called the **proper analysis** of \( T(Z, K) \) wrt \( Z, K, T \) such that \( Y_1 \) is the transform under \( t \) of \( Y_1 \), and \( T(Z, K) = Y_1 \wedge \ldots \wedge Y_n \).

The requirement that the proper analysis of \( T(Z, K) \) must have the same number of terms as the proper analysis of \( Z \) might seem at first glance to be an overly severe limitation. But actually this is not so. We can add indefinitely many units to either sequence and choose \( t \) so that it converts units into non-unit strings, or non-unit strings into units.

Since \( Z \) and \( T(Z, K) \) are the concatenations of their proper analyses, the effect is that \( Z \) and \( T(Z, K) \) are not necessarily divided into the same number of (non-unit) terms even if their proper analyses are of the same length. This same possibility
permits us, by revision of the restricting class \( Q \), to change
the position of the constituent breaks in \( T(\mathbb{Z}, K) \) freely, even
if the term arrangement and constant part of \( T \) are fixed. Thus
under the interpretation of transformational analysis that we
have selected, one set of constituent boundaries for \( T(\mathbb{Z}, K) \)
is determined by \( T \), and \( T \) can be chosen so as to permit these
constituent breaks to be put in anywhere in a fixed \( T(\mathbb{Z}, K) \).

This freedom is important. E.g., one of the effects of
the passive transformation that converts \( Z = "\text{most people-like-}
summer" \) into "summer-is liked-by most people" is to introduce
the element \( \text{en by} \) between "like" and "summer", \(^{(16)}\) both of which are
terms of the proper analysis of \( Z \). But it is crucial that
the transformation be framed in such a way that "en" go with
"like", and "by" with "most people".

Such a construction of transformations tells us something
about the constituent boundaries in \( T(\mathbb{Z}, K) \), but nothing about
how \( C_0 \) holds for these constituents. But we need not require
that all of the information about \( C_0 \) be provided by the already
known algebra \( P \) (as suggested in the third paragraph of \( \text{78.1} \)).
Below, in \( \text{78.2} \), we will discuss certain general conditions
under which constituent structure can be automatically carried
over under transformation.

There is one asymmetry here that should be noted. The
product of a \( f_1 \)-derivation not only has a unique analysis,
given the derivation, but a consistent analysis. But a
transform \( T(\mathbb{Z}, K) \) does not necessarily have a consistent
analysis (though it has a non-overlapping proper analysis),
since the proper analysis may be 'completed' in various ways.
To require a consistent analysis it would be necessary either
to add some condition on \( P \), or to extend the sense in which
the constituent analysis is determined by \( T \) beyond the proper
analysis. I do not know whether

\[ \text{C6. For some instructions that the latter may be the case (though not strictly so).} \]

28.2. To give condition (C4) a precise content, we must
define the set of elementary transformations. The specific
motivation for this definition is much the same as that
behind the definition of grammatical transformations, so that
this definition is modelled closely on the statement of condition
(C6).

Suppose now that we have a system \( S \) which is a level or
a sum of levels. (17)

**Def.4.** \( J_{el} = \{ t_1 \mid 1 \geq 1 \} \) is a set of **elementary transformations**
defined by the following property:

for each pair of integers \( n \) and \( r \) such that \( n \leq r \),
there is a unique sequence of integers \( (a_0, a_1, \ldots, a_k) \)
and a unique sequence of strings in \( S(z_1, \ldots, z_{k+1}) \),
such that

(i) \( a_0 = 0; \ k \geq 0; \ 1 \leq a_i \leq r \) for \( 1 \leq i \leq k \)

(ii) for each \( Y_1, \ldots, Y_n \),

\[ t_i(Y_1, \ldots, Y_n; Y_n, \ldots, Y_r) = z_1^{a_1} z_2^{a_2} \ldots z_{k+1} \]

That is, the domain of \( t_i \) is the set of ordered pairs
\( (P_1, P_2) \), where \( P_1 \) is an \( n \)-ad of strings, \( P_2 \) is an \((k-n+1)\)-ad
of strings, and the last element in \( P_1 \) is the first element
in \( P_2 \). Thus \( t_i \) will be understood as converting the occurrence
of \( Y_n \) in the context
\[ \overline{X_1 \cdots \overline{X}_{n-1} \overline{X}_{n+1} \cdots \overline{X}_n} \]

into a certain string which is unique, given the sequence of terms into which \( \overline{X_1 \cdots \overline{X}_n} \) is divided, but which may vary for different divisions into \( r \) terms, or divisions into different numbers of terms.

We say that two elementary transformations are \( r \)-equivalent if they have the same effect, term by term, on all strings which are broken up into \( r \) terms.

**Def. 5.** Suppose that \( t_{\downarrow}, t'_{\downarrow} \in \langle \mathcal{E} \rangle \). Then \( t_{\downarrow} \) and \( t'_{\downarrow} \) are \( r \)-equivalent if and only if for all \( n \) and \( r \) such that \( n \leq r \),

\[
t_{\downarrow}(Y_1, \ldots, Y_n; Y_{n+1}, \ldots, Y_r) = t'_{\downarrow}(Y_1, \ldots, Y_n; Y_{n+1}, \ldots, Y_r)
\]

Given \( t_{\downarrow} \in \langle \mathcal{E} \rangle \), we define the derived transformation of \( t_{\downarrow} \).

**Def. 6.** \( t^*_{\downarrow} \) is the derived transformation of \( t_{\downarrow} \) if and only if for all \( Y_1, \ldots, Y_r \),

\[
t^*_{\downarrow}(Y_1, \ldots, Y_r) = t_{\downarrow}(Y_1; Y_1, \ldots, Y_r)^{t_{\downarrow}}(Y_1, Y_2; Y_2, \ldots, Y_r) \cdots \cdots^{t_{\downarrow}}(Y_1, \ldots, Y_r; Y_r)
\]

Thus the domain of a derived transformation \( t^*_{\downarrow} \) is an \( r \)-ad of strings, and this is carried into a unique string by \( t^*_{\downarrow} \).

We can give a simple example to illustrate the effect of (C9), given these definitions for 'underlying elementary transformations'. Suppose that we have an elementary transformation \( t \) such that

\[
(48) \quad t(Y_1; Y_1, Y_2) = Y_1 \wedge Y_2 \wedge Y_2 \wedge Y_1 \\
t(Y_1, Y_2; Y_2) = Y_1
\]
We can obviously carry over the terms constant part and term arrangement for elements by transformations, from the analogous use for grammatical transformations in §77.4. Thus from def. 4 we see that for each \( n \) and \( r \) such that \( n \leq r \), it has a unique constant part and a unique term arrangement.

In the case of (48), for \( n=1, r=2 \), the term arrangement of \( t \) is \((0, 2, 1)\), and the constant part is \((Z_1, Z_2, U)\). For \( n=2, r=2 \), the term arrangement is \((0, 1)\), and the constant part is \((U, U)\). The derived transformation \( t^* \) will have the property that

\[
(49) \quad t^*(Y_1, Y_2) = t(Y_1; Y_1, Y_2) \wedge t(Y_1, Y_2; Y_2) = Z_1^\wedge Y_2^\wedge Z_2^\wedge Y_1^\wedge Y_1
\]

Suppose that we have \( Q, Z, K \) as follows

\[
(50) \quad Q = \{ (X, a) \} \\
Z = a^\wedge b^\wedge c
\]

where \( f(X, a^\wedge b) \)

Then \( Q \) and \( t \) together define a transformation \( T \), which, as we know from (C9) has the following effect on \((Z, K)\).

\[
(51) \quad \text{The proper analysis of } Z \text{ wrt } K, T \text{ is } (a^\wedge b, c).
\]

\[
T(Z, K) = t^*(a^\wedge b, c) = Z_1^\wedge a^\wedge Z_2^\wedge a^\wedge b^\wedge a^\wedge b.
\]

The proper analysis of \( T(Z, K) \) wrt \( Z, K, T \) is the sequence

\[
(t(a^\wedge b; a^\wedge b, c), t(a^\wedge b, c; g)) = (Z_1^\wedge a^\wedge Z_2^\wedge a^\wedge b, a^\wedge b).
\]

Thus \( a^\wedge b \) plays the role of \( Y_1 \) and \( c \) the role of \( Y_2 \) from (48) and (49). The term arrangement of \( T \) is \((0, 2, 1, 1)\) and the constant part is \((Z_1, Z_2, U, U)\). In this case the term arrangement of \( T \) is formed by running together the term arrangements of \( t \) for \( n=1, r=2 \), and for \( n=2, r=2 \) (dropping the second \( 0 \)), and the same is true of the constant parts. But this is not necessarily the case as we
can see from the following example, formed from (50) by
taking $c = \mathcal{U}$.

(52) Suppose (48), (49). Suppose that:

$$Q' = \{ (X, U) \}$$
$$Z' = a^b$$
$$K' = \{ (Z, X) \}$$

where $f(X, a^b)$

Then $Q'$ and $t$ define a transformation $T'$, with $Q'$ as
associated restricting class and $t$ again as underlying
elementary transformation. $T'$ has the following effect
for $(Z', K')$.

(53) The proper analysis of $Z'$ wrt $K'$, $T$ is $(a^b, \mathcal{U})$.

$$T(Z, K) = t^*(a^b, \mathcal{U}) = Z_1^U Z_2^a b^a b^a b = Z_1^Z_2^a b^a b^a b.$$  

The proper analysis of $T(Z, K)$ wrt $Z, K, T$ is the sequence

$$(t(a^b; a^b, \mathcal{U}), t(a^b, \mathcal{U}; \mathcal{U})) = (Z_1^U Z_2^a b^a b, a^b) = (Z_1^Z_2^a b^a b, a^b).$$

Here $a^b$ again plays the role of $Y_1$ of (48) and (49), and $\mathcal{U}$ plays the role of $Y_2$. But now (cf. (C6ii), 122.4) the
term arrangement of $T'$ is $(0, 1, 1)$ and the constant part
is $(Z_1^Z_2^U, \mathcal{U})$.

28.3. We can now rephrase (C9) more precisely, in several steps.

(C10) Underlying each $T$ there is a $t \in J_{e_1}$ such that if

$$(Y_1, \ldots, Y_k)$$
is the proper analysis of $Z$ wrt $K, T$, then

$$T(Z, K) = t^*(Y_1, \ldots, Y_k),$$

where $t^*$ is the derived transformation of $t$.  

**Def. 8.** Suppose that $t$ underlies $T$ in the sense of (C10).

Suppose that $(y_1, \ldots, y_n)$ is the proper analysis of $Z$ wrt $K,T$. Then the proper analysis of $T(Z,K)$ wrt $Z,K,t$ is the sequence of terms $(t(y_1; y_1, \ldots, y_n), t(y_1, y_2; y_2, \ldots, y_n), \ldots, t(y_1, y_2, \ldots, y_n; y_n))$.

**Def. 9.** A sequence $S$ is a proper analysis of $T(Z,K)$ wrt $Z,K,T$ if and only if for some $t$ underlying $T$ in the sense of (C10), $S$ is the proper analysis of $T(Z,K)$ wrt $Z,K,t$.

As things now stand, although the proper analysis of $T(Z,K)$ wrt $Z,K,t$ is clearly unique, the proper analysis of $T(Z,K)$ wrt $Z,K,T$ is not, since various elementary transformations may underlie $T$. That the proper analysis of the transform is not necessarily unique can be seen from a simple example.

(54) Let $t_1$ be an elementary transformation such that

$$t_1(y_1; y_1, y_2) = c^d$$

$$t_1(y_1, y_2; y_2) = U$$

Let $t_2$ be an elementary transformation such that

$$t_2(y_1; y_1, y_2) = c$$

$$t_2(y_1, y_2; y_2) = d$$

Suppose that $c, d \in P$. But now let $T$ be any transformation with a two-termed associated restricting class, and suppose that $T$ maps every pair $(Z,K)$ in its domain into $c^d$. Then both $t_1$ and $t_2$ underlie $T$ in the sense of (C10). But the proper analysis of each $T(Z,K)$ wrt $Z,K,t_1$ is $(c^d, U)$, and the proper analysis of each $T(Z,K)$ wrt $Z,K,t_2$ is $(c, d)$.

But uniqueness of the proper analysis of the transform is called for by (C9). Thus we must add the further condition:

(C11) The proper analysis of $T(Z,K)$ wrt $Z,K,T$ is unique.
The conditions (C1)-(C11) are met by a set of elements which we call $\mathcal{J}_3$. Each $\mathcal{T}^3 \in \mathcal{J}_3$ is a set of ordered quintuples $\{(Z,K,Z',Pr^{(1)},Pr^{(2)})\}$, where $\{(Z,K,Z',Pr^{(1)})\} \in \mathcal{J}_2$ (cf. final paragraph of 4.2). $Pr^{(2)}$ is uniquely determined for each $(Z,K,Pr^{(1)})$. Furthermore, there is a $t \in \mathcal{J}_1$ associated with $\mathcal{T}^3$ such that if $(Z,K,Z',Pr^{(1)},Pr^{(2)}) \in \mathcal{T}^3$, and $Pr^{(1)} = (y_1,\ldots,y_T)$, then $Z' = t'(y_1,\ldots,y_T)$ and $Pr^{(2)} = (t(y_1; y_1,\ldots,y_T),\ldots,t(y_1,\ldots,y_T; y_T))$. Thus if we know the set of elements $\{(Z,K,Pr^{(1)})\}$ associated with a given $\mathcal{T}^3$, then knowledge of $t$ will complete the determination of $\mathcal{T}^3$. But the associated restricting class $Q$ determines the set of triples $\{(Z,K,Pr^{(1)})\}$. Hence a pair $(Q,t)$ of an associated restricting class and an underlying elementary transformation will suffice to determine an element of $\mathcal{T}^3$.

(C11) can be interpreted as a condition on the set of elementary transformations that can underlie a given $\mathcal{T} \in \mathcal{J}_3$. Let $PA_1(T)$ be the set of proper analyses of $Z$ wrt $K,T$. Is $PA_1(T)$ the set $\{Pr^{(1)}\}$ of elements that occur in the fourth place of the quintuples that belong to $T$. Suppose that $t_1$ and $t_2$ underlie $T$. Then for each $(y_1,\ldots,y_T) \in PA_1(T)$, and for each $n \leq T$, $t_1(y_1,\ldots,y_n; y_n,\ldots,y_T) = t_2(y_1,\ldots,y_n; y_n,\ldots,y_T)$. This is close to the definition of $r$-equivalence (cf. Def. 5, 178.2). In fact, for any sufficiently interesting $T$, it will always be the case that

(C12) If $t_1$ and $t_2$ underlie $T$, where $T$ is determined by an $r$-term associated restricting class, then $t_1$ and $t_2$ are $r$-equivalent.
If \( T \) meets (C12), we say that it is determinate. A necessary and sufficient condition for determinateness is that the term arrangement of \( T \) be unique, and uniqueness of the term arrangement can fail only when the membership of \( PA_1(T) \) is severely restricted. Below, in §62.2, we will see that there are good reasons for limiting ourselves to determinate transformations. But for the time being, we do not make this restriction.

Clearly \( J_3 \) is related to \( J_2 \) just as \( J_2 \) is related to \( J_1 \). That is, each \( T^2 \in J_2 \) corresponds to a subclass of \( J_3 \) consisting of elements which differ only in the proper analysis assigned to the transform. We have seen that each element of \( J_2 \) is determined (even over-determined) by a triple \((Q, C, A_\alpha)\), where \( Q \) is an associated restricting class, \( C \) is a constant part, and \( A_\alpha \) is a term arrangement. But such a triple does not give enough information to specify an element of \( J_3 \) as we can see from returning to example (54).

(54*) Suppose that \( t_1, t_2, c, \) and \( d \) are as in (54). Suppose that: \( a, b \in \mathbb{F} \),

\[
\begin{align*}
Z &= a \cdot b \\
K &= \{a \cdot b\} \\
Q &= \{(a, b)\}
\end{align*}
\]
Then \((Q, (c^\delta d))(0))\) is an ordered triple of a restricting class \(Q\), a constant part \((c^\delta d)\) and a term arrangement \((0)\) which defines an element of \(\mathcal{J}_2\), in fact the element \(T^2\) whose sole member is \((Z, K, c^\delta d, (a, b))\). But two elements of \(\mathcal{J}_3\) correspond to \((Q, (c^\delta d), (0))\), since either \((c^\delta d, U)\) or \((c, d)\) can be the proper analysis of the transform \(c^\delta d\), depending on whether \(t_1\) or \(t_2\) (which are not 2-equivalent) is chosen as the underlying elementary transformation.

We see, then, that an element of \(\mathcal{J}_3\) is determined not by an ordered triple \((Q, C, A_Z)\), but by an ordered pair \((Q, t)\), where \(Q\) is an associated restricting class, and \(t\) is an underlying elementary transformation. \(t\), in turn, is determined, for each \(n, z\), by a constant part and a term arrangement (cf. Def. 4). Each element of \(\mathcal{J}_3\) is uniquely determined by some such \((Q, t)\).

Not every \((Q, t)\) defines a transformation \(T \in \mathcal{J}_3\), however. We can arrive at a necessary and sufficient condition for \((Q, t)\) to define some \(T\) by noting that for each \(t \in \mathcal{J}_{e1}\), and for each \(r\) and \(n\) such that \(n \notin r\), there is a unique term arrangement \((s_0, s_1, \ldots, s_r)\). Since \(t^*\) is formed by concatenating elementary transforms of \(t\), for fixed \(r\) and for \(n=1, \ldots, m-r\) (in this order), we can define a unique term arrangement of \(t^*\), for fixed \(r\), in an obvious way.

**Def. 9.** Let \(t^*\) be the derived transformation of \(t \in \mathcal{J}_{e1}\).

Suppose that for each \(n \notin r\), the (unique) term arrangement of \(t\) for \(n, r\), is \(0, a_1(n), \ldots, a_r(n)\). Then the term arrangement of \(t^*\) for \(r\) is the sequence \((0, a_1(1), \ldots, a_k(1), a_1(2), \ldots, a_k(2), \ldots, a_1(r), \ldots, a_k(r))\).
Analogously, we can define the constant part of $t^*$ for $P$.

Note that the term arrangement of $t^*$ may not be the same as the term arrangement of the transformation $T$ defined in terms of $t$, although the latter term arrangement must be a subsequence of the former. Cf. (52)-(53), §72.2.

We can now characterize the set of pairs $(Q, t)$ that define some transformation.

(55) Suppose that $Q$ is the $P$-termed restricting class

$$\{ (w_1^{(1)}, \ldots, w_T^{(1)}) \mid 1 \leq i \leq N_Q \}.$$  

Suppose further that $t \in T_{el}$, $t^*$ is the derived transformation of $t$, and $(0, b_1, \ldots, b_m)$ is the term arrangement of $t^*$ for $P$, with $(Z_1, \ldots, Z_{m+1})$ being the constant part of $t^*$ for $P$.

Then a sufficient condition for $(Q, t)$ to define some transformation is that

(i) for each $k \leq m$, either for all $i$, $w_{b_k}(i) = U$, or for all $i$, $w_{b_k}(i) \neq U$.

(ii) $Z_1$ is a string in $P$.

This is also a necessary condition if we assume $Q$ to be minimal, in the sense that if for some $i$, $(w_1^{(1)}, \ldots, w_T^{(1)})$ is deleted from $Q$, then the resulting restricting class defines (in the sense of $(C^4)$, §72.2) a different domain than does $Q$.

We could in fact define restricting classes in such a way that they must be minimal, relative to a given system $P$. In practice we deal only with minimal $Q$'s, since the length of the description of $Q$ is a factor in the evaluation of the grammar, and a minimal $Q$ can be more simply described. Thus (55) will suffice for the practical purpose of deciding whether a given $(Q, t)$ legitimately determines a transformation, even if we do not define restricting classes as necessarily minimal.
We can now proceed to construct the set of grammatical transformations somewhat more systematically, summing up the remarks of 4.27-8.

Suppose we have a system $S$ which is a level or a sum of levels. $S$ must include $P$. $P$ was taken as the set of all elements of $P$ that bear $\emptyset$ to no element (cf. Def. 1, 4.27, chap. VI). We now extend $P$ to include all elements of $S$ that bear $\emptyset$ to no element. That is, we interpret the variable $I$ in the definition of $P$ as ranging over all strings. Since only elements of $P$ can bear $\emptyset$, this means that $P$ includes all strings of $S$ that belong to levels other than $P$, as well as $P$ in the old sense. In practice, this means that transformations can be defined on strings of words and morphemes (and into such strings). In other words, we regard $W$ and $\mathfrak{M}$ as embedded into $P$ as 'lowest level' elements of $P$.

**Def. 10.** Let $Q$ be an $r$-termed restricting class and $t$ an elementary transformation with the derived transformation $t^*$. Suppose that $(Q,t)$ meets (55).

Then $(Q,t)$ determines the set of ordered quintuples

$$T = \{(Z_{\alpha}, K_{\alpha}, Z_{\alpha}^i, \Pr_{\alpha}^{(1)}, \Pr_{\alpha}^{(2)}) \mid 1 \leq \alpha \leq N_Q\}$$

where (i) $Z_{\alpha}, Z_{\alpha}^i$ are strings in $P$

(ii) $\Pr_{\alpha}^{(1)}$ is the unique sequence $(Y_{l};...,Y_{T})$

such that (a) $Z_{\alpha} = Y_{l} \cdot \ldots \cdot Y_{T}$

(b) there is a $(W_{l}^{(1)},...,W_{l}^{(1)}) \in Q$ s.t.

for each $\mathfrak{N} \in \mathfrak{Q}, \mathfrak{E}_{\mathfrak{Q}}(Y_{l},\ldots,Y_{T},W_{l}^{(1)},Z_{l},K_{l})$

Then $Z_{i} = t^*(Y_{l};...,Y_{T})$,

and $\Pr_{\alpha}^{(2)} = (t(Y_{l};Y_{l};...,Y_{T});...,t(Y_{l};...,Y_{T};Y_{T}))$

(iii) if any quintuple $q$ meets (i) and (ii), then $q \in T$. 
Def. 11*. In this case, \( P_r^1 \) is the proper analysis of \( Z, T, Q \) wrt \( K, T \).

\[
\begin{align*}
\text{Pr}^1(1) & \quad \text{""} \quad \text{""} \quad \text{""} \\
\text{Pr}^1(2) & \quad \text{""} \quad \text{""} \\

Q & \text{ is associated with } T \\
t & \text{underlies } T \\
Z' & \text{ is denoted } "T(Z, K)"
\end{align*}
\]

We will continue to use the term 'domain' to cover the set \( \{(Z, K)\} \), and the term 'transform' for \( Z' = T(Z, K) \), just as if \( T \) were a simple mapping of \((Z', K')\) to \( Z' \), as in the case of \( J_1 \).

Def. 12*. \( T \) is a grammatical transformation if and only if there is a \( Q \) and a \( t \) meeting the premises of Def. 10* and such that \((Q, t)\) determines \( T \).

The set of grammatical transformations we denote by "\( J \)."

\( J \) is the set denoted "\( J_3 \)" in §78. 2.

Def. 13. \( J_2 \) is the set of quadruples \( \{(Z', K', Z, K) \} \) such that for some \( P_r^1(1), P_r^1(2), \ldots, P_r^N_Q \)

\[
\text{\( J_1 \) is the set of triples \( \{(Z, K, Z')\} \) such that for some \( P_r^1(1), \ldots, P_r^N_Q \), \( \{(Z', K', Z', K') \}\) is \( J_2 \).
\]

Th. 1. Determination, in the sense of Def. 10*, is unique, and the terms defined above meet the conditions laid down in §77-78.

29. 2. One emendation of this system of transformations will be quite useful below. We have limited the domain of each grammatical transformation \( T \) to a subset of \( \{(Z, K)\} \), but we will find it convenient to extend the domain of each \( T \) to cover
the whole set of pairs \((Z, K)\) such that \(Z\) is a string in \(\bar{F}\), and \(K\) a class of strings. This means that we must add to each \(T \ni \exists\) a quintuple \((Z, K, Z', Pr(1), Pr(2))\) for each \((Z, K)\) not already in the domain of \(T\), in the sense of \(\text{Def.}10\). The simplest and most convenient way to do this is by taking \(Z'\) as \(Z\), and both \(Pr(1)\) and \(Pr(2)\) as the one-termed sequence \((Z)\), in all such cases. Then \(T\) applies to all pairs \((Z, K)\) of a string in \(\bar{F}\) and a class of strings, but it applies only trivially to pairs which are not in the domain of \(T\) in the sense of \(\text{Def.}10\).

Note that the null transformation of \(\exists\), in the sense of \(\text{Def.}10\) (i.e., the null class of quintuples), is converted by this extension into an 'identity' transformation \(I\) having the property that \(I(Z, K) = Z\), and that the proper analysis of \(I(Z, K)\) wrt \(Z, K, I\) is identical with the proper analysis of \(Z\) wrt \(K, I\).

Definitions 10, 11, and 12 are now superseded by 10*, 11*, and 12*, as follows.

**Def.10**. Let \(Q\) and \(t\) be as in **Def.10**. Then \((Q, t)\) determines the set of ordered quintuples \(T = \{(Z_1, K_1, Z_1', Pr_1(1), Pr_1(2)) \mid 1 \leq i \leq N_{Q}\}\), where \(Z_1, Z_1'\) are strings in \(\bar{F}\) and \(K_1\) is a class of strings, and either: (I) there is exactly one sequence \((Y_1, \ldots, Y_{\bar{F}})\) s.t.

(a) \(Z_1 = Y_1 \ldots Y_{\bar{F}}\)

(b) there is a \((W_{1}, \ldots, W_{\bar{F}}) \in Q\) s.t. for \(n \leq F\),

\[ t(0, Y_1, \ldots, Y_{\bar{F}}, W_{1}, Z_1, K_1) \in P_{Q}(Y_1, \ldots, Y_{\bar{F}}, W_{1}, Z_1, K_1), \]

in which case: \(Z_1' = t^*(Y_1, \ldots, Y_{\bar{F}})\)

\[ Pr_1(1) = (Y_1, \ldots, Y_{\bar{F}}) \]

\[ Pr_1(2) = (t(Y_1; Y_1, \ldots, Y_{\bar{F}}), \ldots, t(Y_1, \ldots, Y_{\bar{F}}; Y_{\bar{F}})) \]

\[ Pr_1(3) = (t(Y_1, \ldots, Y_{\bar{F}}), \ldots, t(Y_1, \ldots, Y_{\bar{F}})) \]
or (II) it is not the case that there is exactly one sequence \((Y_1, ..., Y_n)\) meeting (a) and (b), in which case: 
\[
\begin{align*}
Z_i & = Z_i \\
Pr_i^{(1)} & = Pr_i^{(2)} = (Z_i)
\end{align*}
\]

**Def.11.** In this case, ... etc. as in **Def.11**.

**Def.12.** Exactly as **Def.12**, with determine now in the sense of **Def.10**, not **Def.10**.

Whenever we speak of "determination", transformation", "proper analysis", etc., below, we use these terms in the sense of Definitions 10-12, of this subsection.

This is by no means all that there is to say in this connection, but instead of pursuing this line of investigation further, we will turn to the further development of the theory. The major points to be kept in mind are that each transformation has fixed structural properties, in the sense of (C6) and **Def.4**; that each transformation imposes a proper analysis on the transform; and that each transformation can be characterized in a fixed way in terms of a pair \((Q, t)\) of an \(r\)-termed restricting class and an elementary transformation. An elementary transformation \(t\), in turn, can be characterized, for each \(m \leq r\), by giving the constant part and the term arrangement. Since we need never characterize \(t\) beyond the number of terms of the restricting class, we have a finite way to specify any grammatical transformation.
30.1. There are certain special kinds of transformations that seem to have a particular relevance to syntactic description. We will turn now to a discussion of these.

One kind of transformation that is of interest consists in the addition of certain elements to a string in a fixed syntactic position, or the deletion of certain elements in fixed position. Thus in converting such verb phrases as "eat lunch" into noun phrases such as "eating lunch" (\textit{ing} \textit{eat} \textit{lunch}) or "to eat lunch", we affix the element \textit{ing} or the element \textit{to} to the verb phrase in a fixed position. Or in forming the passive, we affix the element \textit{en} to the verb, and add the proper form of \textit{be} before the verb (along with other changes). And there seems to be no reason to exclude by definition the converse process of, e.g., forming an active from a passive by dropping these elements.\(^{(18)}\)

Addition and deletion of elements can be included in the same kind of transformation. Deletion can be effected by adding a 'minus' element (essentially, an element which is a left inverse to every element) before the element in question. The transformations which involve just addition and deletion of elements will be called \textit{deformations}. Combination of deletion of one element and addition of another in its place of course amounts to substitution. We can construct the set of deformations in the following series of steps.

We have assumed a system \(\mathcal{S}\) which is a level or a sum of levels. We now assume an enumeration \(X_1, X_2, \ldots\) of all strings in \(\mathcal{S}\). To avoid any confusion or misunderstanding, I will state
this condition more carefully. There is a slight extension
to include the 'minus' element.

Suppose we have a system $S$, where $S$ is a concatenation
algebra formed from

a set of levels containing in

particular the level $P$. Thus $S$ has a single unit element $U$,

and concatenation is permitted freely among the elements of

the various component levels. The primes of these component

levels are primes of $S$, and the algebraic properties of each

component level are carried over for the sub-algebra of $S$

consisting just of the elements of the given level.

Let $P = \{P_1, P_2, \ldots\}$ be the set of primes of $S$. Suppose

that an element $U$ is adjoined to $S$, and that $P$ is 'alphabetized',

i.e., that a total ordering $\prec$ is defined on it with

$P_1 \prec P_2 \prec \ldots$ Let $S(S)$ be the set of strings $X_i (i \geq 0)$ in $S$ such that

\[(56) \quad X_0 = U, \quad X_1 = U, \quad X_i = P_{i_1} \ldots P_{i_{\alpha_i}} \quad (i \geq 2; \quad i_1 \geq 1 \text{ for } i \leq \alpha_i)\]

Def. 14. $O_1(X_i, X_j)$ if and only if

(I) $i = 0$ or $1$, and $i < j$

or (II) $i > 1$, and $i \alpha_i < j \alpha_i$ (i.e., $X_i$ is shorter than $X_j$)

or (ii) $\alpha_i = j \alpha_i$ and there is a $W, P, Z, Z'$ s.t.

\[(a) \quad X_i = W P \prec Z\]

\[(b) \quad X_i = W^+ P \prec Z'\]

\[(c) \quad X < Z\]

In case (II), $P$ and $P$ are thus the earliest non-

identical primes of $X_I$ and $X_I$, perhaps the initial primes $P$ and $P'$,

in which case $W = U$. $O_1$ is a total ordering, and we
may assume that this ordering is \( X_0, X_1, X_2, X_3, \ldots \) finite.

We can easily order sequences of elements of \( S(S) \).

**Def. 15.** \( O_2((Y_1, \ldots, Y_m), (Z_1, \ldots, Z_n)) \) if and only if

1. \( m < n \)
   or \( m = n \) and there is a \( k^m \) such that
   1. \( O_1(X_k, Z_k) \)
   2. for all \( i < k, X_i = Z_i \)

\( O_2 \) is a total ordering on finite sequences of strings.

(57) Let \( A_{1k} \) be the \( 1 \)th sequence of \( 2k \) elements in terms of the ordering \( O_2 \) such that

1. \( A_{1k} = (X_{11}, \ldots, X_{12k}) \) \( \ (d_1 \geq 0) \)
2. \( d_1 \neq 0 \) for \( i = 2n \)

That is, the even elements \( X_{12}, X_{14}, \ldots, X_{12k} \) are not \( \sigma \).

**Def. 16.** Suppose that \( A_{1k} \) is as in (57). To \( A_{1k} \) we associate the elementary transformation \( \delta_{1k} \) such that

1. if \( k = n \), then \( \delta_{1k}(Y_1, \ldots, Y_n, Y_n, \ldots, Y_n) = W_n \)
   where (a) \( W_n = X_{12n} \), if \( X_{12n-1} = \sigma \)
   (b) \( W_n = X_{12n-1} Y_n X_{12n} \), if \( X_{12n-1} \neq \sigma \)
2. if \( k \neq n \), then \( \delta_{1k}(Y_1, \ldots, Y_n, Y_n, \ldots, Y_n) = Y_n \)

Inadmissibly, but suggestively, we might say that in case (i),
\[ w_n = (x_{12n-1}^n \ldots x_{12n}^n), \] and for any \( i, j \in \mathbb{U} \), assuming now that the operation inside the parentheses is carried out before the operation outside the parentheses, i.e., that associativity of concatenation is nullified.

\[ \delta_{jk} \] is an elementary transformation. I.e., \( \delta_{jk} \in \mathcal{X}_{el} \), for each \( i, k \). Where \( \delta^*_{jk} \) is the derived transformation of \( \delta_{jk} \), and \( A_{jk} \) is as in (57), we have

\[ \delta^*_{jk}(y_1, \ldots, y_k) = w_1 \ldots w_k, \]

where \( w_1 = z_1 \ldots x_{121}^1 \), and: \( z_1 = x_{121}^1 \ldots x_{121}^1 \) if \( x_{121}^1 \notin \sigma \)

\[ z_1 = \mathbb{U}, \text{ if } x_{121}^1 = \sigma. \]

**Def. 17.** Let \( \Delta \) be the set of grammatical transformations for which some \( \delta_{jk} \) is the underlying elementary transformation. The members of \( \Delta \) will be called deformations.

Thus given a transformation based on (57) and (58), \( \delta_{1i} \) is the \( i \)th term of the proper analysis of the transform,

\( Y_i \) is the \( i \)th term of the proper analysis of the transformed string \( Y_1 \ldots Y_k \).

Suppose for example that \( A_{12} \) (the \( 2 \)th sequence of 4 elements) is \((\mathcal{F}, a, b, c)\). Then \( \delta_{12} \) will be the elementary transformation such that

\[ \delta_{12}(y_1, y_2, y_3) = a \quad \left[ \text{as } (\mathcal{F}^\circ y_1)^\circ a, \text{ assuming non-associativity} \right] \]

\[ \delta_{12}(y_1, y_2, y_3) = b \cdot y_2^c \]

\[ \delta_{12}(y_1, y_2) = a^b \cdot y_2^c \quad \left[ \text{as } ((\mathcal{F}^\circ y_1)^\circ a)^\circ ((b^\circ y_2)^\circ c), \text{ assuming non-associativity} \right] \]
That is, if $T$ is based on $\sum_{k=2}^{n}$ and $(x_1,x_2)$ is the proper analysis of $Z$ wrt $K,T$, then $T(Z,K) = a^{b^{Y_2}}c$, and the proper analysis of $T(Z,K)$ wrt $K,T$ is the sequence $(a, b^{Y_2}c)$. Thus $T$ deletes the first term of the proper analysis of $Z$, replacing it by $a$, and $T$ replaces the second term $Y_2$ of the proper analysis of $Z$ by $b^{Y_2}c$; \[ T\text{~corr;} \quad Y_2 \rightarrow b^{Y_2}c. \]

Deformations have a certain property of exhaustiveness. For any $X_m,X_n \in S(S)$, there is some deformation which maps one into the other. In fact, there is a deformation based on $A_{yl} = (y_m,x_{yl})$ which maps every string into $x_m$. This of course does not imply that every $T \in \mathcal{I}$ is a deformation.

**30.2.** Deformations are transformations whose term arrangements are a subsequence (not necessarily a proper subsequence --- thus every index is included if the element $y$ does not occur in the sequence $A_{yl}$ which defines the deformation) of the sequence of indices of the terms of the proper analysis. That is, the order of these terms is never changed by a deformation. Another class of transformations that can be expected to play a major role in syntax is the class based on permutations, or, more generally, rearrangements of the indices of the terms of the proper analysis.

Suppose that we have an ordering $0_3$ of finite sequences of integers. Let $B_{jk}$ be the $j$th sequence, in terms of $0_3$, of $k$ integers $(a_1, \ldots, a_k)$, not necessarily distinct, with $a_i \leq k$ for $i \leq k$.

**Def. 18.** To each $B_{jk}$ we make correspond $\beta_{jk}$ such that

\[ \beta_{jk}(Y_1, \ldots, Y_n; Y_{n+1}, \ldots, Y_r) = Y_{q_n}, \quad \text{where: if } k=r, \text{ then } q_n = a_n \]

\[ \text{if } k \neq r, \text{ then } q_n = n \]
\( \beta_{1k} \) is an elementary transformation. Where \( \beta_{1k}^* \) is its derived transformation, we have

\[
(60) \beta_{1k}^*(Y_1, \ldots, Y_k) = Y_{a_1}^{-1} \cdots Y_{a_k}^{-1}
\]

**Def. 19.** Let \( B \) be the set of transformations based on some \( \beta_{1k}^* \).

The members of \( B \) will be called **rearrangements**.

Given \( T \in B \), where \( T \) is based on the underlying transformation \( \beta_{1k}^* \), we see that if the proper analysis of \( Z \) wrt \( K, T \) is \( (Y_1, \ldots, Y_k) \), then \( T(Z, K) = Y_{a_1}^{-1} \cdots Y_{a_n}^{-1} \), and \( Y_n \) is the \( n \)th term of the proper analysis of \( T(Z, K) \) wrt \( Z, K, T \).

Suppose, for example, that \( \beta_{13}^* = (1, 2, 1) \). Then \( \beta_{13}^* \) is the elementary transformation such that

\[
(61) \begin{aligned}
\beta_{13}^*(Y_1; Y_1, Y_2, Y_3) &= Y_1 \\
\beta_{13}^*(Y_1, Y_2; Y_2, Y_3) &= Y_2 \\
\beta_{13}^*(Y_1, Y_2, Y_3; Y_3) &= Y_1 \\
\beta_{13}^*(Y_1, Y_2, Y_3) &= Y_1^{-1} Y_2^{-1} Y_1 \\
\end{aligned}
\]

Suppose that \( (Y_1, Y_2, Y_3) \) is the proper analysis of \( Z \) wrt \( K, T \), where \( T \) is based on \( \beta_{13}^* \). Then \( T(Z, K) = Y_1^{-1} Y_2^{-1} Y_1 \), and the proper analysis of \( T(Z, K) \) wrt \( Z, K, T \) is \( (Y_1, Y_2, Y_1) \).

The sequences of integers \( (a_1, \ldots, a_k) \) which define a rearrangement can be enumerated outside of any particular grammar. So the specification within a grammar of which rearrangement appears will not add to the complexity of the particular grammar. But the alphabetization of strings must be given for each particular grammar. Thus the specification of deformations is a factor in the complexity.
We arrive at deformations by concentrating on the constant part of the transformation. Deformations do not affect the arrangement of terms of the proper analysis, except, perhaps by dropping terms. Rearrangements, on the other hand, are transformations which have a null constant part, but which do affect the order of terms of the proper analysis. That is, we arrive at rearrangements by considering the term arrangement of the transformation exclusively. There is one ('identity') transformation which is both a deformation and a rearrangement.

If $B_{4k}$ contains each $a_{1\leq k}$ exactly once, then we will call any transformation based on the corresponding $\beta_{4k}$ a permutation.

Def. 20. Let $\Pi$ be the set of transformations based on an elementary transformation $\tau_{4k}$ whose defining sequence $B_{4k}$ is a permutation of the integers $(1, \ldots, k)$. The members of $\Pi$ will be called permutations.

Deformations and permutations will be the basic elements in the transformation analysis of English.

We will use the terms 'deformation' and 'permutation' to denote the grammatical transformation as well as the elementary transformation that underlies it. We will use $\Delta_{el}$, $B_{el}$, and $\Pi_{el}$ to denote the sets of elementary deformations, rearrangements, and permutations, respectively.

81. If we had a technique for compounding grammatical transformations, we could investigate the possibility of generating all members of $\gamma$ from such subsets as $\Delta$ and $\Pi$ by compounding. A grammatical transformation $T$ applies to a pair $(Z,K)$, i.e., to a string with a given constituent analysis. Hence to apply a transformation $T_2$ to the product of a transformation $T_1$, it would be necessary to have associated with
the transform $T_1(Z,K)$ a fixed class $K$ which provides the string $T_1(Z,K)$ with a derived constituent structure. But we have not yet succeeded in developing the notion of 'derived analysis' in this sense, although we have developed transformations in such a way as to impose on the transform a partial analysis into terms.

We can, however, investigate the adequacy of such subsets as $\Delta$ and $B$ in an indirect manner, by developing a method for compounding elementary transformations. We can then determine whether some subset of the elementary transformations (e.g., $\Delta_1$, $B_1$, etc.) will provide, by compounding, a set $\mathcal{T}$ of elementary transformations which is extensive enough so that all of $\mathcal{J}$ (or all of some interesting subpart of $\mathcal{J}$) is determined by pairs $(Q,t)$, where $t \in \mathcal{T}$.

The interest of this investigation extends beyond the fact that it gives us more insight into the abstract structure of the system of grammatical transformations which we hope to apply in syntactic analysis. We will see below, in studying derived constituent structure, that the crucial notions of this study can be defined readily and naturally for deformations, rearrangements, and certain similar transformations, but not for transformations in general. Hence if we can show that these subsets of $\mathcal{J}$ are adequate to generate all of $\mathcal{J}$, or all that we need for some purpose, by compounding, then we can define such notions directly only for the cases where the definition is natural and motivated, and we can carry these notions over indirectly for the elements of $\mathcal{J}$ which are generated by compounding.

A further motivation for the study of compounding comes
from an analysis of the reasoning involved in the development of a system of transformations in the first place, in §27-8. We noted there that if grammatical transformations are to be available for the actual construction of grammars, and if the complexity of the characterization of transformations is to be a factor in the evaluation of these grammars, then there must be a fixed and finite way to characterize transformations (just as in chapter III and chapter VI we found it necessary to develop a fixed form for grammars on lower levels). In §79 we saw that any transformation can be determined by a pair \((Q,t)\), where \(Q\) is a finite set of finite sequences of strings. But not every elementary transformation can be represented in a fixed and finite manner. However, any deformation \(\delta_{\text{dk}}\) can be so characterized by stating \(A_{\text{dk}}\), and any rearrangement \(\beta_{\text{dk}}\), by stating \(B_{\text{dk}}\), both \(A_{\text{dk}}\) and \(B_{\text{dk}}\) being finite sequences. And the same will be true of the \(\gamma\)-transformations that we introduce below. Thus, in particular, any finite compound of these elementary transformations is available for grammatical analysis.

81.2. Given two elementary transformations \(t_1\) and \(t_2\) we can form a compound \(t_3=t_2(t_1)\) which is also an elementary transformation. \(t_3\) is defined (cf. Def. 4, §78.2) by stating its term arrangement and constant part for each \(n,\pi\) such that \(n \leq \pi\), and stating these in terms of the term arrangements and constant parts of \(t_1\) and \(t_2\). The obvious way to do this is as follows:
Def. 21. Given \( t_1, t_2 \in \mathcal{E}_1 \). Then \( t_2(t_1) \) is the elementary transformation \( t_3 \) such that for all \( \mathfrak{A} \), and all \( n \leq \mathfrak{A} \),
\[
t_3(Y_1, \ldots, Y_n; X_1, \ldots, X_n) = t_2(W_1, \ldots, W_n; W_1, \ldots, W_n),
\]
where \( W_n = t_1(Y_1, \ldots, Y_n; X_1, \ldots, X_n) \) (for \( n \leq \mathfrak{A} \)).

Th. 2. \( t_2(t_1) \) as thus defined, is in fact an elementary transformation of \( \mathcal{E}_1 \).

By virtue of Th. 2., we see that Def. 21 enables us to speak of all finite compounds \( t_n(t_{n-1}(\ldots(t_1)\ldots)) \), where \( t_i \in \mathcal{E}_1 \) for \( 1 \leq i \leq n \). Any such compound is itself an elementary transformation.

Suppose, for example, that \( t_1 \) and \( t_2 \) are elementary transformations such that

\[
(62) \quad t_1(Y_1; Y_1, X_2, Y_3) = Y_3 \quad t_2(Y_1; Y_1, X_2, Y_3) = Z_1
\]
\[
t_1(Y_1; X_2, Y_2, X_3) = Y_2 \quad t_2(Y_1; X_2, Y_2, X_3) = Y_2^\wedge Z_2
\]
\[
t_1(Y_1; X_2, Y_3, X_3) = Y_1 \quad t_2(Y_1; X_2, Y_3, X_3) = Z_3^\wedge Y_3^\wedge Z_4
\]

Thus \( t_1 \) is a permutation and \( t_2 \) is a deformation.

Where \( (t_i(t_i))^* \) is the derived transformation of \( t_i(t_i) \), we have, in the case of (62),

\[
(63) \quad (t_1(t_1))^*(Y_1, Y_2, Y_3) = t_1^*(Y_3, Y_2, Y_1) = Y_1^\wedge Y_2^\wedge Y_3
\]
\[
(t_1(t_2))^*(Y_1, Y_2, Y_3) = t_1^*(Z_1, Y_2^\wedge Z_2, Z_3^\wedge Y_3^\wedge Z_4) = Z_3^\wedge Y_3^\wedge Z_4^\wedge Y_2^\wedge Z_2^\wedge Z_1
\]
\[
(t_2(t_1))^*(Y_1, Y_2, Y_3) = t_2^*(Y_3, Y_2, Y_1) = Z_1^\wedge Y_2^\wedge Z_2^\wedge Z_3^\wedge Y_3^\wedge Z_4
\]
\[
(t_2(t_2))^*(Y_1, Y_2, Y_3) = t_2^*(Z_1, Y_2^\wedge Z_2, Z_3^\wedge Y_3^\wedge Z_4) = Z_1^\wedge Y_2^\wedge Z_2^\wedge Z_3^\wedge Y_3^\wedge Z_4^\wedge Z_4
\]

If it is furthermore the case that for \( \mathfrak{A} \neq 3, \mathfrak{A} \leq \mathfrak{A} \), then
\[
t_1(Y_1, \ldots, Y_n; X_1, \ldots, X_n) = X_n, \text{ then } t_1(t_1) \text{ is the identity elementary}
\]
transformation.

In 80 we developed two major sets of elementary transformations, the set $\Delta$ of deformations, and the set $B$ of rearrangements. We see at once that not every elementary transformation can be represented as a finite compound of deformations and rearrangements. Suppose that we call an elementary transformation finitary if there is an $a$ such that for all $r > a$, for all $n \leq r$, $t(x_1, \ldots, x_n; y_1, \ldots, y_n) = y_n$. Then clearly only a finitary elementary transformation can be represented as a finite compound of deformations and rearrangements. On the other hand, finitary elementary transformations are sufficient to enable us to reconstruct completely the set $\mathfrak{T}$ of grammatical transformations. That is, every $T \in \mathfrak{T}$ is determined by some $(Q, t)$, where $t$ is finitary. For if $Q$ is an $r$-termed restricting class, then the effect of $t$ on proper analyses of more than $r$ terms is irrelevant to the determination of $T$ by $(Q, t)$.

But $\Delta$ and $B$ do not even suffice to generate all of the finitary transformations needed to determine $\mathfrak{Y}$. The strongest statement that we can make, limiting ourselves to deformations and rearrangements is

**Th.3.** The set $\mathfrak{Y}_1$ is generated by compounding from $\Delta$ and $B$.

I.e., if $T^1 \in \mathfrak{Y}_1$, then there is a $Q$ and a $t$ such that $(Q, t)$ determines $T^1$ and $t = t_n(t_{n-1}(\ldots(t_1)\ldots))$, where each $t_i (1 \leq i \leq n)$ is a deformation or a rearrangement.

$\mathfrak{Y}_1$, it will be recalled, is the set whose members are sets of ordered triples $\{Z, K, T(Z, K)\}$. Each $T^1 \in \mathfrak{Y}_1$ becomes a grammatical transformation of $\mathfrak{Y}$ when a proper analysis of $Z$
and a proper analysis of \( T(\mathbb{Z}, K) \) are associated in the correct manner with each \( (\mathbb{Z}, K, T(\mathbb{Z}, K)) \in \tilde{T} \). Th. 2 follows directly from the fact that the unit \( U \) can appear as a term of the proper analysis and the restricting class.

**31.3.** To obtain a stronger result we consider a third class of elementary transformations that combine certain features of \( \Delta \) and \( B \) in that they have a null constant part, but that the rearrangement of terms is effected by 'attaching' terms to some fixed term of the proper analysis, as in a deformation.

We suppose once again that we have an ordering \( O_3 \) of finite sequences of integers. Let \( C_{1k} \) be the \( i \)th sequence, under the ordering \( O_3 \), of \( 2k \) integers \( (a_1, \ldots, a_{2k}) \) such that for each \( 1 \leq k \), \( 0 \leq a_i \leq k \).

**Def. 22.** To \( C_{1k} \) we associate the elementary transformation \( \gamma_{1k} \) such that

\[
\gamma_{1k}(Y_1, \ldots, Y_n; Y_{n+1}, \ldots, Y_{k}) = W_n, \quad \text{where}
\]

\[
W_n = \begin{cases} 
Y_n & \text{if } k = r; \text{ and } Y_0 = U \\
\frac{Y_n}{a_{2n-1}} \cdot \frac{Y_n}{a_{2n}} \cdot \frac{Y_n}{a_{2n}} & \text{if } k \neq r \\
W_n &= Y_n, & \text{if } k \neq r
\end{cases}
\]

\( \gamma_{1k} \) is an elementary transformation.

**Def. 23.** Let \( H \) be the set of transformations for which \( \gamma_{1k} \) is the underlying elementary transformation. The members of \( H \) will be called simply \( \gamma \)-transformations.
Suppose, for example, that \( c_{4,2} \) (the \( i \)th sequence of four elements) is \((2,0,1,1)\). Then \( h_{4,2} \) is the elementary transformation such that

\[
\begin{align*}
\hat{h}_{4,2}(x_1; x_1, x_2) &= x_2 x_1 x_0 = x_2 x_1 \\
\hat{h}_{4,2}(x_1, x_2; x_2) &= x_1 x_2 x_1 \\
\hat{h}_{4,2}^*(x_1, x_2) &= x_2 x_1 x_1 x_2 x_1
\end{align*}
\]

It is still not the case that \( \mathcal{J} \) can be recovered from \( \Delta \), \( B \), and \( H \). But we do have a result that appears to be just as effective for the purposes of syntactic description.

Actually, the only terms of interest in the proper analysis of a string are the non-units. If two proper analyses differ only by units, then they yield the same constituent breaks. Recognizing this, suppose that we define a relation \( E \) holding between \( A \) and \( B \) if \( B \) is a sequence of strings, and \( A \) is formed from \( B \) either by adding unit terms on to the end of \( B \) or deleting unit terms from the end of \( B \). Thus the string formed by concatenating the elements of \( A \) is identical with the string formed by concatenating the elements of \( B \), and one of the sequences \( A, B \) contains the other as an initial subsequence. \( E \) is an equivalence relation.

A transformation \( T \) is a set of quintuples \( \{(Z, K, Z', Pr_1^{(1)}, Pr_1^{(2)})\} \). We say that two transformations \( T_1 \) and \( T_2 \) are \( E \)-related if there is a 1-1 relation between \( T_1 \) and \( T_2 \) that carries each

\[(Z, K, Z', Pr_1^{(1)}, Pr_1^{(2)}) \in T_1 \quad \text{into} \quad (Z, K, Z', Pr_1^{(1)}, Pr_1^{(2)}) \in T_2,\]

where \( E \) holds between \( Pr_1^{(1)} \) and \( Pr_1^{(1)} \), and between \( Pr_1^{(2)} \) and \( Pr_1^{(2)} \). Then for each \( Z, K \), \( E \)-related transformations carry \((Z, K)\) into the same string \( Z' \), and they give the same constituent breaks in \( Z' \).

We can now state
Th. 4. For any $T_1 \in \mathcal{S}$, there is a $T_2 \equiv T_1$ such that $T_2$ is determined by a pair $(Q, t)$, where $t$ is a finite compound of deformations and $\mathcal{Q}$-transformations.

It seems reasonable to assume, in the light of this result, that the purposes of syntactic description will be adequately served by that part of $\mathcal{S}$ which is generated by compounding from $\Delta$, $B$, and $H$. All the transformations that we will actually require in the analysis of English which will be presented below, in chapter IX, are in fact determined directly by compounds of elementary transformations drawn from $\Delta_{e_1}$, $B_{e_1}$, and $H_{e_1}$.

82.1. We can now return to the important topic (introduced above in §78.1) of the effect of transformations on constituent structure. The central idea behind transformational analysis is that it will be profitable to select among grammatical sentences a certain kernel of basic sentences for which a simple phrase structure can be described, and in which all grammatical relations and selectional relations can be discovered. Only the kernel sentences are derived in the grammar by $\mathcal{S}_1$-derivations; hence only these sentences have $\mathcal{P}$-markers conferred on them in that part of the grammatical statement that corresponds to the level $\mathcal{P}$. But even though we need not attempt to find $\mathcal{P}$-markers for such sentences as

(65) whom have they nominated
(66) the game was won by John
(67) I know the man standing at the bar

which are deleted from the kernel, we must still assign some constituent analysis to them. The major systematic motivation for the development of the notion of "derived constituent
structure lies in the fact that transformations must be compoundable. Thus from the transform (66) we must be able to form the second transform "was the game won by John". But a transformation can apply only to a string with an assigned constituent analysis. Hence we must somehow assign constituent structure to such sentences as (66), and in general, to all transforms.

There are two sources from which information about this derived constituent analysis can be drawn. We can determine the place of constituent breaks, the selectional relations, and the content of $C_0$ for a derived sentence by (i) investigating the sentence (or sentences) from which it is derived and the manner of its derivation from these sentences; or, as we have noted in §28.1, by (ii) applying information already available in the statement of the level $P$. Our present concern is to investigate in some detail just how these sources can be exploited.

Until now the development of the theory of transformational analysis has been fairly straightforward. The presystematic demands imposed on the construction of this theory by the nature of the linguistic material and by the purposes of such analysis were fairly clear, and we were able to meet them in what seemed to be a natural manner. The constructions of §4.82-8 appear, by comparison, somewhat makeshift and arbitrary. In large measure this is due to the fact that the presystematic demands on this construction are not too clear. That is, it is difficult to determine just what is an intuitively adequate derived constituent structure for non-kernel sentences. But there is also little doubt that
a more careful study of the formal structure of the system of transformations could lead to a more satisfactory and natural conception of derived constituent structure. We will concentrate now on developing a conception which will be adequate for the needs of the transformational analysis of English to be undertaken in chapter IX, and we will make no systematic attempt to generalize much beyond this goal.

82.2. The proper analysis of $Z$ wrt $K, T$ is the sequence of terms into which $Z$ is divided for the purpose of applying $T$. We can begin the discussion of derived constituent structure by noting that a term of the proper analysis of $Z$ is either carried over unaltered by transformation (as in a rearrangement), dropped completely (as in deformations with the element $\gamma$), or else it has adjoined to it either other terms of the proper analysis (as in $\gamma$-transformations) or constant terms (as in deformations). But it can undergo no internal changes. If a term is carried over unaltered, it is natural to require that the constituent structure of its contained segments be invariant under the transformation. Thus in transforming

(68) your friend - will - bring the book tomorrow

into

(69) will - your friend - bring the book tomorrow,

the initial noun phrase "your friend" and the final verb phrase "bring the book tomorrow" (with its contained NP "the book") are carried over intact. We would certainly like to say that "your friend" and "the book" are still NP's in (69). To permit such statements in general, we must associate with each term of
the proper analysis of \( T(Z,K) \) a unique term of the proper analysis of \( Z \) which is in a certain sense its root under \( T \). This notion will play a central role in the discussion of derived structure.

If the order of terms is not changed by the transformation \( T \) (e.g., if \( T \) is a deformation), then the root of a term of the proper analysis of \( T(Z,K) \) will be just its pre-image under the elementary transformation \( t \) that underlies \( T \). I.e., if

\[
t(Y_1, \ldots, Y_n; Y_n \ldots, Y_x) = Y_1^x Y_n^x Y_2^x,
\]

then the root of \( Y_1^x Y_n^x Y_2^x \) is \( Y_n^x \). Similarly, if terms of the proper analysis of \( Z \) are adjoined to some \( Y_i^x \) of the proper analysis of \( Z \), as in an \( L \)-transformation, then it seems natural to take \( Y_i^x \) as the root of the resulting term of the proper analysis of \( T(Z,K) \).

If \( T \) is a rearrangement, then the root of the \( i \)th term of the proper analysis of \( T(Z,K) \) will naturally be the \( j \)th term of the proper analysis of the transformed string \( Z \), where \( i \) corresponds to \( j \) under the rearrangement. E.g., if \( Z = Y_1^x Y_2^x Y_3^x \) goes into \( T(Z,K) = Y_3^x Y_2^x Y_1^x \), then the occurrence (19) \( Y_1^x \) of \( Y_1^x \) in \( Z \) is the root of the occurrence \( Y_3^x Y_2^x Y_1^x \) of \( Y_1^x \) in \( T(Z,K) \).

In the case of a rearrangement, then, each term is identical with its root.

**Def. 24.** \((Y_i^x, Z_i^x)\) is the root of \((W_i^x, Z_i^x)\) wrt \((Y_1^x, \ldots, Y_x^x)\), if and only if (i) \( i, j \leq x \)

(ii) \( Z_i^x = Y_1^x \ldots Y_j^x \) [i.e., \( Z_i^x \) is an occurrence of \( Y_j^x \) in \( Y_1^x \ldots Y_x^x \)]

(iii) \( W_i^x = t(Y_1^x, \ldots, Y_j^x; Y_j^x, \ldots, Y_x^x) \)

(iv) \( Z_i^x = t(Y_1^x, Y_1^x, \ldots, Y_x^x) \ldots t(Y_1^x, \ldots, Y_j^x; Y_j^x, \ldots, Y_x^x) \) \[i.e., \( Z_i^x \) is an occurrence of \( W_i^x \) in \( t^*(Y_1^x, \ldots, Y_x^x) \)]
and either (v) or (vi)

(v) \( t \in A_1 \) or \( t \in H_1 \), and \( i = 1 \)

(vi) \( t \) is a \( B_1 \) based on \( B = (a_1, \ldots, a_n) \), and

\[
\begin{align*}
&\text{if } k = 2r \\
&\text{if } k \neq 2r
\end{align*}
\]

We can then define the root of \( (W_1, Z_1^w) \) wrt \( Z, K, T \) as

(70) the root of \( (W_1, Z_1^w) \) wrt \( (Y_1, \ldots, Y_r), t \), where \( (Y_1, \ldots, Y_r) \)

is the proper analysis of \( Z \) wrt \( K, T \), and \( t \) underlies \( T \).

There is thus a natural way to define "root" for \( \Delta \), \( B \),

and \( H \), though there is no obvious way to define it for

transformations in general. But we can extend the notion of

root to compound elementary transformations, which, as we

have seen, provide us with all the transformations needed for

grammatical analysis. Thus we can define a relation "root" related to "root" roughly as "ancestor" is related to "parent".

Ref. 25. \( (X_1, Z_1^y) \) is the root of \( (W_1, Z_1^w) \) wrt \( (Y_1, \ldots, Y_r), t \) if and

only if either (i) \( (X_1, Z_1^y) \) is the root of \( (W_1, Z_1^w) \) wrt

\( (Y_1, \ldots, Y_r), t \)

or (ii) there is a \( t_1, t_2, (X_1, Z_1^x) \) such that

(a) \( t = t_2(t_1) \)

(b) \( (X_1, Z_1^y) \) is the root of \( (X_1, Z_1^x) \) wrt

\( (Y_1, \ldots, Y_r), t_1 \)

(c) \( (X_1, Z_1^x) \) is the root of \( (W_1, Z_1^w) \) wrt

\( (t_1(Y_1, Y_2, \ldots, Y_r), t_1(Y_1, \ldots, Y_r; Y_r)), t_2 \)
We can now define the $\text{root}^*$ of $(W_i, Z_i)$ wrt $Z, K, T$, for any $T$ that we need for grammatical analysis, as (70), with "root*" replacing "root".

82.3. But now we face the new difficulty that in terms of this definition, the root* is not unique. The reason for this is that an elementary transformation can often be derived by compounding in several different ways from elements of $\Delta, B,$ and $H$, and as we have defined roots, these alternatives may assign different roots to a given occurrence of a term in the proper analysis of the final transform. In particular, it can be shown that any rearrangement $\beta$ which is not a permutation is identical with a compound of elementary transformations which assigns to some term of the proper analysis of the transform $\beta^*(Y_1, \ldots, Y_T)$, a different root than is assigned when $\beta$ is simply taken as a rearrangement. To demonstrate this, it is sufficient to prove Th.5. Any non-permutational elementary rearrangement is identical with a compound of elements of $\Delta_1$ and $H_1$.

A non-permutational rearrangement assigns $(Y_i, Z_i)$ as a root to $(W_i, Z_i)$ for some $i$ and some $i' \neq i$, where the proper analysis of the transformed string is $(Y_1, \ldots, Y_T)$ and the proper analysis of the transform is $(W_1, \ldots, W_T)$, and $Z_i$ and $Z_{i'}$ are as in Def.24. But deformations and $\gamma$-transformations can assign $(Y_i, Z_i)$ as a root to $(W_i, Z_i)$, under these circumstances, only if $i=i'$ (cf. (v), Def.24), so the non-uniqueness of the root follows from Th.2.
To see that \( \text{Th. 5} \) is true, consider a rearrangement based on

\[(71) \ B = (a_1, \ldots, a_k), \text{ where } l \leq a_1 \leq k, \text{ and for a certain } m \text{ which is } \leq k, \text{ } m \text{ does not appear as one of the } a_i \text{'s.} \]

This is a necessary and sufficient condition for \( \beta \) to be a non-permutation.

For each \( i \leq k \), let \( \delta_i \) be the elementary transformation such that

\[(72) \begin{cases} 
\delta_i(Y_1, \ldots, Y_k; Y_1, \ldots, Y_k) = U \\
\delta_i(Y_1, \ldots, Y_k; Y_1, \ldots, Y_k) = Y_n, \text{ for } n \neq i \text{ or } n = k.
\end{cases} \]

Thus \( \delta_i \) is a deformation, and \( \delta_i \) deletes the \( i \)th term of \( (Y_1, \ldots, Y_k) \). I.e., \( \delta_i(Y_1, \ldots, Y_k) = Y_1 \cdots Y_{i-1} U Y_{i+1} \cdots Y_k \).

For each \( i, j \leq k \), let \( \gamma_{ij} \) be the elementary transformation such that

\[(73) \begin{cases} 
\gamma_{ij}(Y_1, \ldots, Y_k; Y_1, \ldots, Y_k) = Y_i Y_j \\
\gamma_{ij}(Y_1, \ldots, Y_k; Y_1, \ldots, Y_k) = Y_n, \text{ for } n \neq i \text{ or } n = k.
\end{cases} \]

Thus each \( \gamma_{ij} \) is an \( \gamma \)-transformation, and \( \gamma_{ij}(Y_1, \ldots, Y_k) = Y_1 \cdots Y_{i-1} (Y_i Y_j) Y_{i+1} \cdots Y_k \). Notice that the compound \( \gamma_{ij}(\delta_i) \) essentially replaces the \( i \)th term by the \( j \)th term. I.e.,

\[(74) \text{ Let } \varphi_{ij} = \gamma_{ij}(\delta_i). \text{ Then } \varphi_{ij}(Y_1, \ldots, Y_k) = Y_1 \cdots Y_{i-1} Y_i Y_{i+1} \cdots Y_k. \]

Let \( \iota \) be the identity elementary transformation such that for all \( n, k \) such that \( n \leq k \),

\[(75) \iota(Y_1, \ldots, Y_n; Y_n, \ldots, Y_k) = Y_n \]
We can now easily construct a finite sequence of elementary transformations \( t_1, \ldots, t_k \) such that \( t_k(\ldots(t_1)\ldots)=\beta \) and each \( t_i \) is a compound of \( \gamma_i \)'s as in (74) or is \( \epsilon \), as in (75) — hence each \( t_i \) is a compound of deformations and \( \gamma \)-transformations. The construction is by induction.

We know that \( \beta^*(Y_1, \ldots, Y_k)=\gamma_{a_1}^{\gamma_1} \ldots \gamma_{a_k}^{\gamma_k} \). We construct \( t_1, \ldots, t_k \) such that, where \( t_1=t_1(\ldots(t_1)\ldots) \), then for all \( Y_1, \ldots, Y_k \),

\[
(76) \quad t_1^*(Y_1, \ldots, Y_k)=\gamma_{a_1}^{\gamma_1} \ldots \gamma_{b_1}^{\gamma_{b_1}} \ldots \gamma_{b_{k-1}}^{\gamma_{b_{k-1}}} \quad \text{where for each } i \leq k, \gamma_{a_i}^{\gamma_i} \text{ is one of } Y_{a_1}^{a_1}, Y_{b_1}^{b_1}, \ldots, Y_{b_{k-1}}^{b_{k-1}}.
\]

**Step 1**: Case I. Suppose \( m=1 \) (where \( m \) is as in (71)).

Then \( t_1=\gamma_{-1} \). Put \( \bar{t}_1=Y_{a_1}^{a_1} \gamma_2^{\gamma_2} \ldots Y_k^{\gamma_k} \), and for each \( i \leq k, Y_{a_i}^{\gamma_i} \)

one of \( Y_{a_1}^{a_1}, Y_2^{a_2}, \ldots, Y_k^{a_k} \), since only \( Y_m^{a_m} \) has been deleted, and \( m \), by assumption, is not one of the terms of \( \beta^*(a_1, \ldots, a_k) \).

**Case II**: Suppose \( m\neq1 \). (a) Suppose \( a_1=1 \). Then \( t_1=1 \).

(b) Suppose \( a_1 \neq 1 \). Then \( t_1^*=\gamma_{-1}^{a_1} \gamma_{a_1}^{a_1} \).

In case IIa, no term is deleted by \( t_1^* \). But in case IIb,

\[
t_1^*(Y_1, \ldots, Y_k)=\gamma_{-1}^{a_1} \gamma_{a_1}^{a_1} \gamma_2^{\gamma_2} \ldots \gamma_{m-1}^{\gamma_{m-1}} Y_1^{Y_{m-1}} Y_{m+1}^{Y_{m+1}} \ldots Y_k^{Y_k}.
\]

Hence in case IIb, only \( Y_m^{a_m-1} \) is deleted.

Hence in either case I or case II, \( t_1^*=t_1^* \) meets (76).
Step 2: Suppose that for some $i < k$ we have constructed $t_1, \ldots, t_i$ such that $t_i^*$ meets (76), where $t_i = t_1(\ldots(t_1)\ldots)$. We rename $Y_{a_1}, \ldots, Y_{a_i}, Y_{b_1}, \ldots, Y_{b_{k-1}}$ as $Z_1, \ldots, Z_k$, respectively.

By the inductive hypothesis, we know that $Y_{a_{i+1}}$ appears as one of the strings $Z_1, \ldots, Z_k$. And since, by hypothesis, (76) holds for all choices of $Y_{a_1}, \ldots, Y_{a_k}$, we know that $a_{i+1}$ actually appears as one of the indices $a_1, \ldots, a_i, b_1, \ldots, b_{k-1}$.

Suppose that $a_{i+1}$ corresponds to $q$ under the renaming which we have just carried out, so that $Y_{a_{i+1}}$ has been renamed $Z_q$. We know that $m$ does not appear in the sequence $B = (a_1, \ldots, a_k)$ and that $m, a_1, \ldots, a_k < k$. Hence there are at most $k-1$ distinct numbers among $a_1, \ldots, a_k$. But the sequence $B^* = (a_1, \ldots, a_i, b_1, \ldots, b_{k-1})$ is a sequence of $k$ terms containing each term of $B$. Hence there must be at least one term $\bar{x} \in B^*$ which $Q$ of $B^*$ for which either (i) or (ii) of (77) is the case:

(77)(i) $Q$ is not $m$ a term of $B$.

(ii) $Q$ is identical with an earlier term of $B^*$.

If a term of $B^*$ has this property, we say that it is redundant with respect to $B$. Suppose that the $n$th term of $B^*$ is the least term of $B^*$ which is redundant with respect to $B$. Then we know that no matter how $Y_{a_1}, \ldots, Y_{a_k}$ were originally chosen, either $Z_n$ is distinct from all of $Y_{a_1}, \ldots, Y_{a_k}$ or there is an $\bar{m}/n$ ($\bar{n} \leq k$) such that for some $i < k$, $Z_n = Z_{\bar{m}} = Y_{a_i}$ (i.e., to put it loosely, $Z_n$ is not the only instance of $Y_{a_i}$ in $Z_1, \ldots, Z_k$).

Clearly, then, even if the $n$th term of $B^*$ the sequence
$Z_1, \ldots, Z_k$ is deleted, each of $Y_{a_1}, \ldots, Y_{a_k}$ appears in the remaining sequence $Z_1, \ldots, Z_{n-1}, Z_n, \ldots, Z_k$. We can therefore proceed, analogously to step 1, with $n$ now taking the place of $m$ in step 1.

We have selected $n$ so that the $n$th term in the sequence $\{a_1, \ldots, a_i, b_1, \ldots, b_{k-i}\}$ is either not from $B$ or is identical with an earlier number in this sequence.

Case I: Suppose that the $n$th term in this sequence is not from $B$. Then $n \geq i + 1$ (i.e., the $n$th term is one of $b_1, \ldots, b_{k-i}$).

Ia: Suppose $n = i + 1$. Then $t_{i+1} = s_{i+1, a}$ (where $a$ corresponds to $a_{i+1}$, as above)

But $t_{i+1, a}^* (Z_1, \ldots, Z_k) = Z_1 \cdot \cdots \cdot Z_i \cdot a_i Z_i+2 \cdots Z_k$

$= Y_{a_1} \cdot \cdots \cdot Y_{a_i} \cdot Y_{a_{i+1}} \cdot Y_{b_2} \cdots \cdot Y_{b_{k-i}}$

and only $Z_n$ has been deleted, so that $t_{i+1}^*$ meets (75).

Ib: Suppose $n > i + 1$.

(a) Suppose $b_1 = a_{i+1}$. Then $t_{i+1} = 1$.

(b) Suppose $b_1 \neq a_{i+1}$. Then $t_{i+1} = S_{i+1, a} (J_{n, i+1})$.

But then $t_{i+1}^* (Z_1, \ldots, Z_k) = Z_1 \cdot \cdots \cdot Z_i \cdot a_i Z_i+2 \cdots Z_{n-1}$

$\cdots Z_{i+1} Z_n \cdots Z_k$

$= Y_{a_1} \cdot \cdots \cdot Y_{a_i} \cdot Y_{a_{i+1}} \cdot Z_i+2 \cdots$

$\cdots Z_{n-1} Z_i+1 Z_n+1 \cdots Z_k$.

But only $Z_n$ has been deleted without reappearing, so that $t_{i+1}^*$ meets (76).
Case II. Suppose that the $n$th term in the sequence $E^* = (a_1, a_2, a_3, \ldots, a_{k-1})$ is from $B$. But this term has been chosen so as to be redundant with respect to $E$. Hence (77ii) must hold for this term -- there must be an $\tilde{a} < \tilde{n}$ such that the $\tilde{a}$th term of $E^*$ is identical with the $n$th term and in consequence, $z_{\tilde{n}} = y_{\tilde{a}_1}$, for some $1 \leq k$.

IIa. Suppose $n > l+1$. Then proceed as in case I. (In the case analogous to (b) we must be careful to choose $a_{l+1}$. But this is always possible. $a$ must be chosen so that $\tilde{a}_{l+1}$ is the $a$th term of $E^*$. If $\tilde{a}_{l+1}$ is the $n$th term of $E^*$, it is also the $\tilde{a}$th term (where $\tilde{n} = n$), and $a$ can be taken as $n$. Since only $z_{\tilde{n}}$ is deleted by this procedure, and since $z_{\tilde{n}} = z_{\tilde{n}}$ remains, it follows that $z_{\tilde{n}} = t_{l+1}$ meets (76).
Case II: Suppose that the $n^{th}$ term in the sequence $\mathbf{B}=(a_1, \ldots, a_i, b_1, \ldots, b_{r-1})$ is from $B$. But this term has been chosen to be redundant with respect to $B$, so that (77ii) must hold for this term; there must be an $\tilde{r}/\in\mathbb{N}$ such that the $\tilde{r}^{th}$ term of $\mathbf{B}$ is identical with the $n^{th}$ term and in consequence, $Z_{\tilde{r}}=Z_n=\frac{Y_{\tilde{r}}}{Y_{\tilde{r}+i}}$, for some $i\neq k$.

IIa: Suppose $n \neq i+1$. Then proceed as in case I. Since only $Z_n$ is deleted by this procedure, and since $Z_{\tilde{r}}=Z_n$ remains, it follows that $t_{i+1}$ meets (76).

IIb: Suppose $b_1=\tilde{a}_{i+1}$. Then set $t_{i+1}=t_1$, as in case Ib(a).

IIc. Suppose $n \neq i$ and $b_1 \neq \tilde{a}_{i+1}$. Then $Z_{\tilde{r}}=\frac{Y_{\tilde{r}}}{Y_{\tilde{r}+i}}=Z_n=\frac{Y_n}{Y_n}$, and (77i).

(i) Suppose furthermore that $q>i+1$.

Then set $t_{i+1}=\frac{Y_{\tilde{r}}}{Y_{\tilde{r}}} \left( \frac{Y_{\tilde{r}}}{Y_{\tilde{r}+i}} \right)^{i+1} \left( \frac{Z_n}{Z_n} \right)^{i+1}$.

In this case, $t_{i+1}(Z_1, \ldots, Z_k)=Z_1 \cdots \frac{Y_{\tilde{r}}}{Y_{\tilde{r}+i}} \frac{Z_1}{Z_1} \frac{Z_2}{Z_2} \cdots \frac{Z_k}{Z_k}$.

In other words, the effect of $t_{i+1}$ is simply to interchange $Z_1(=\tilde{a}_{i+1})$ with $Z_{i+1}$. No terms are deleted, so $t_{i+1}$ meets (76).

(ii) Suppose that $q<i$. Suppose that there are $p$ distinct terms among $a_1, \ldots, a_i$. But since $\tilde{a}_{i+1}$ is not among these, it must be the case that $a_{i+1}=\tilde{a}_i$ for some $j \neq i$. Therefore there are $n_{\tilde{a}_{i+1}}$ distinct terms among $a_1, \ldots, a_{i+1}$. Therefore there are
at most $p+k-1$ distinct terms among $a_1, \ldots, a_k$, 
i.e., less than $p+k-1$ terms.

By assumption, $n > 1$ and $n$ is the last term of $E^*$ which is redundant with respect to $E$. Therefore all the terms $b_1, \ldots, b_{k-1}$ of $E^*$ are terms of $E$ and are distinct from one another and from all the terms $a_1, \ldots, a_k$ (which are the initial terms of $E^*$ as well as $E$). But since $a_1, \ldots, a_k$ are $p$ distinct terms, the sequence $E$ must contain $p+k-1$ distinct terms, since $a_1, \ldots, a_k, b_1, \ldots, b_{k-1}$ are all terms of $E$.

Thus the assumption that $n \leq 1$ leads to a contradiction, and this case is ruled out.

Hence in case I or case II, $\bar{t}_{i+1}^*$ meets (76) on the assumption that $\bar{t}_i^*$.

Hence in case I or case II, $\bar{t}_{i+1}^*$ meets (76). It follows that we can construct $\bar{t}_k^* = \delta$ in this manner, where $\bar{t}$

Hence in case I or case II, $\bar{t}_{i+1}^*$ meets (76). It follows that we can construct $\bar{t}_k^* = \delta$ in this manner out of elementary deformations and $\eta$-transformations, thus establishing Theorem 5.

The elementary transformations $\delta_i$ which figure so heavily in this demonstration will play an important role below.
Hence in case I or case II, \((t_{i+1}(t_i))\) meets (76) on the assumption that \(f_i\) does.

It follows that we can construct a finite sequence \(t_1, \ldots, t_k\) of elementary transformations such that \(t_k(\ldots(t_1)\ldots) = \beta\), and each \(t_i\) is a compound of \(\gamma\)-transformations and deformations, thus establishing theorem 5. The elements \(T_{ij}\), which figure so heavily in this demonstration will play an important role below.

Suppose that we call an elementary transformation equivocal wrt a set \(L\) of elementary transformations if it can be compounded in various ways from members of \(L\) giving various different root assignments to some transformation that it underlies. We have just seen that non-permutational rearrangements are equivocal wrt \(L' = \Delta_{el} + B_{el} + H_{el}\). But non-permutational rearrangements are not equivocal wrt \(L = \Delta_{el} + \Pi_{el} + H_{el}\). Hence a simple way to avoid this particular difficulty is to limit (vi), Def. 24, to permutations. Thus we now consider \(L\), instead of \(L'\), as our basic set of elementary transformations. Non-permutational rearrangements are still derivable from \(L\), but they are not equivocal wrt \(L\).

82.4. The decision to restrict the set of basic elementary transformations in terms of which "root" is defined to \(\Delta_{el}\), \(\Pi_{el}\), and \(H_{el}\), except the rest of \(B_{el}\), has consequences that require more explicit statement.

Suppose, for example, that we have an elementary transformation \(t\) such that

\[(78) \ t^*(x_1, x_2, x_3) = x_2 - x_1 - x_3,\]
with the proper analysis as indicated by the dashes.

If \( t \) were treated as a rearrangement, then the root of the second term of \( t^*(Y_1,Y_2,Y_3) \) would be the first term of \( Y_1^\wedge Y_2^\wedge Y_3 \), i.e., it would be \( (Y_1,Y_1) \). But if we treat \( t \) in the manner of the proof of Theorem 2, as a compound of deformations and \( \gamma \)-transformations, then the root of the second term of \( t^*(Y_1,Y_2,Y_3) \) is the second term of \( Y_1^\wedge Y_2^\wedge Y_3 \), i.e., it is \( (Y_2,Y_1^\wedge Y_2) \). If each term of the proper analysis of the transform is to be unique, then one of these analyses of \( t \) must be rejected. But there is enough independent motivation to each root assignment to suggest that it would be profitable to retain each analysis, with two distinct senses of the notion of "root".

We were originally led to the notion of "root" in \( \text{(82.2)} \) by noting that we clearly must be able to say that both "your friend" and "the book" are \( \text{NP} \)'s in

(79) will- your friend - bring the book tomorrow \((\text{=}(69))\)

which is derived by permutation from

(80) your friend - will - bring the book tomorrow \((\text{=}(68))\).

But there is an important difference between the case of "your friend" and that of "the book". The analysis of "the book" as an \( \text{NP} \) is a matter purely internal to one of the terms of the proper analysis. But the analysis of "your friend" as an \( \text{NP} \) is a matter that might affect the total constituent structure of the transform \((\text{82})\), since "your friend" is a complete term of the proper analysis, and to call it an \( \text{NP} \) determines the role that this term of the proper analysis may play in the constituent structure of the sentence as a whole. The fact that a given
term of the proper analysis of the transform is an NP may affect the analysis of other terms of the proper analysis, and the analysis of strings of these terms, as we will see below. Given a term of the proper analysis of the transform, then, we can distinguish between its internal structure, which is immaterial to the determination of the derived analysis of the rest of the transform, and its external structure, including its own constituent assignment, and the assignment of strings of which it is a part. In the case of (79)–(80), this distinction is of no particular importance, but in the case of such transformations as \( t \) in (78), it may become important.

The motivation for considering \( t \) of (78) to be a rearrangement comes from the consideration of internal constituent structure. The internal structure of the second term of the transform is the same as that of the first term of the transformed string. Thus from this point of view, we would like to consider \((\bar{Y}_1, Y_1)\) to be the root of \((\bar{Y}_1, \bar{Y}_1)\), as in the treatment of \( t \) as a rearrangement.

The motivation for considering \( t \) to be a compound \( \zeta (\zeta) \), where

\[
(81) \quad \zeta^*(\bar{Y}_1, \bar{Y}_2, \bar{Y}_3) = \bar{Y}_1 - \bar{Y}_2 - \bar{Y}_3
\]

\[
\zeta^*(\bar{Y}_1, \bar{Y}_2, \bar{Y}_3) = \bar{Y}_1 - (\bar{Y}_2 \bar{Y}_1) - \bar{Y}_3 = \bar{Y}_1 - \bar{Y}_2 - \bar{Y}_3
\]

comes from the consideration of the external constituent structure of the second term of the transform. When \( t \) is analyzed as in (81), it is the elementary transformation \( \zeta_{21}^* \), as this is defined in (73), (74), in \( \zeta_{21} \). That is, it amounts to a substitution of \( \bar{Y}_1 \) for \( \bar{Y}_2 \) in \( \bar{Y}_1 - \bar{Y}_2 - \bar{Y}_3 \). We would naturally expect the substituend \( \bar{Y}_1 \) to play the same role in the overall
structure of the sentence as did the term \( Y_2 \) for which it is substituted. From this point of view, then, we are led to consider \((Y_2, Y_1 \wedge Y_2)\) to be the root of \((Y_1, Y_1 \wedge Y_1)\).

In the last paragraph of \( \text{§}82.2 \), we chose the latter analysis. Thus the notion of 'root' as we are developing it here will have particular relevance to external constituent structure. In actually defining derived constituent structure, we will have to account for the purely internal structure of terms of the proper analysis of the transform in at least partially independent terms.

The real significance of the distinction that we have drawn between external and internal constituent structure will become clearer when we study generalized transformations, below, in \( \text{§}87 \). There we will see that (78) serves as an instance of a very important type of transformation. It will prove convenient to derive such sentences as

\[(82)\text{ that he was unhappy — was quite obvious}\]

from

\[(83)\text{ that he was unhappy — it — was quite obvious}(20)\]

taken as \(Y_1 \wedge Y_2 \wedge Y_3\). But now \(t\) as in (78) will convert this into

\[(84)\text{ that he was unhappy — that he was unhappy — was quite obvious}\]

and a further deformation which carries \(Y_1 \wedge Y_2 \wedge Y_3\) into \(U \wedge Y_2 \wedge Y_3\) \(\left(\delta_1\right)\text{ of (72), where }k=3\), will carry (84) into (82). But now the internal structure of the second term of (84) (= the first non-unit term of (82)) is the same as that of the first term of (83) (ultimately, it is that of the sentence "he was unhappy", from which this first term is derived). The external
constituent structure of the second term of (34), however, is that of the second term of (83). That is, "that he was unhappy" in (82) is the noun phrase subject of (82), just as "it" is the NP subject of "it was quite obvious".

We have already seen in §82.3 that by combining Λ-transformations and certain deformations, we can achieve the effect of a substitution of one term of the proper analysis for another, with the substituend having the same external structure as the term for which it is substituted. Such compound transformations as these appear often enough and significantly enough to receive a special name. Generalizing somewhat over (73), (74), we define the class of substitutions as follows.

Def. 26. t is a substitution if and only if there is a \( t_1, \ldots, t_n \) such that \( t = t_n(t_1, \ldots, t_1, \ldots, t_1) \), and for each \( i \leq n \),

(i) \( t_i \) is either a deformation or an Λ-transformation, and \( t_i \in \Delta_{\lambda i} \).

(ii) if \( t_i \in \Delta_{\lambda} \), and \( \Delta \) is its defining sequence, then the terms of \( \Delta \) are either Ω or \( U \) (cf. §80.1).

(iii) if \( t_i \) is an Λ-transformation, and its defining sequence is \( \underline{\Sigma} = (a_1, \ldots, a_k) \), then for each \( i \leq k \), (a) \( \underline{a}_{2_i-1} = 0 \),

(b) \( \underline{a}_{2_i} = 0 \) or \( \underline{t}_{i-1}(y_1, \ldots, y_i; x_1, \ldots, x_i) = U \),

where \( \underline{t}_{i-1} \) is a transformation such that \( \underline{t}_{i-1}(\ldots(t_1)\ldots) \).

(iv) If \( \underline{t} \in \Delta_{\lambda i} \).

The class of elementary substitutions will be denoted \( \Sigma_{el} \), and the class of grammatical transformations based on them, \( \Sigma \).
Th. 6. For each substitution $t_3$ there is a sequence $D = (a_1, \ldots, a_{2k})$

such that

(i) $a_{2i-1} = 0$ or $\tau$.

(ii) if $a_{2i-1} = 0$, then $a_{2i} = 0$.

(iii) if $a_{2i-1} = \tau$, then $a_{2i}$ is an integer $m$ s.t. $0 \leq m < k$.

(iv) there is an $m$ s.t. $1 \leq m < k$, $a_{2m-1} = \tau$, and for all $i \leq 2k$, $a_i \neq m$.

(v) there is a $1 \leq k$ such that $a_{2i} = 0$, $a_{2i} \neq 1$.

(vi) $t_3(Y_1, \ldots, Y_n; Y'_1, \ldots, Y'_n)\begin{cases} Y_{a_{2n}} & \text{if } r = k \text{ and } a_{2n-1} = \tau, \text{ where } Y_0 = Y \\
Y_n & \text{if } r \neq k, \text{ or } a_{2n-1} = 0. \end{cases}$

and each sequence $D$ such that (i)-(v) determines some substitution $t_3$ such that (vi).

Thus $D$ as in Th. 6 determines the substitution $t_3$ that carries $Y_n$ of $Y_1, \ldots, Y_n$ into $(Y_{a_{2n-1}} Y_n) \cap Y_{a_{2n}}$, where $Y_0 = U$, $Y_0 = T$, and associativity is assumed inoperative. Substitutions can thus be characterized in much the same way as deformations and $\lambda$-transformations.

Suppose, for instance, that $D = (\tau, 0, \tau, 1, 0, 0)$. Then the associated $t_3$ is the elementary transformation such that

\[(85)\] $t_3(Y_1; Y_1, Y_2, Y_3) = U$

$t_3(Y_1, Y_2; Y_2, Y_3) = Y_1$

$t_3(Y_1, Y_2, Y_3; Y_3) = Y_3$

$t_3^*((Y_1, Y_2, Y_3) = U Y_1 Y_3 = Y_1 Y_3$

i.e., $t_3$ is the elementary transformation that carries (84) into (82).

82.5. With this clarification, we can return to the discussion of \(82.2-3\). We found in \(82.3\) that the requirement of uniqueness...
Thus $D$ as in Th.6 determines the substitution $t_s$ that carries $Y_{\underline{n}}$ of $Y_{1,\underline{n}}$ into a string $\underline{w}_{\underline{n}}$ which is identical with $Y_{\underline{n}}$ if $s_{2n-1}=0$ and is identical with $Y_{\underline{2n}}$ (where $Y_{0}=\emptyset$) if $s_{2n-1}=1$. Condition (iv) ensures the deletion of some term $Y_{\underline{m}}$ which is not reintroduced elsewhere by $t_s$ (hence $t_s$ is certainly not a permutation). Condition (v) guarantees that there is at least one term $Y_4$ which displaces some other term. By means of the sequence $D$, substitutions can be characterized in much the same way as deformations and $\gamma$-transformations.

Suppose, for instance, that $D=(\emptyset, 0, 1, 1, 0, 0)$. Then the associated $t_s$ is the elementary transformation such that

\[(65)\quad t_s(Y_{1,\underline{2}}, Y_{2,\underline{2}}, Y_{3,\underline{2}}) = \emptyset\]

\[t_s(Y_{1,\underline{2}}, Y_{2,\underline{2}}, Y_{3,\underline{2}}) = Y_1\]

\[t_s(Y_{1,\underline{2}}, Y_{2,\underline{2}}, Y_{3,\underline{2}}) = Y_2\]

\[t_s(Y_{1,\underline{2}}, Y_{2,\underline{2}}, Y_{3,\underline{2}}) = Y_3\]

\[t_s(Y_{1,\underline{2}}, Y_{2,\underline{2}}, Y_{3,\underline{2}}) = U_{Y_1} Y_3 = Y_1 \colon Y_3\]

I.e., $t_s$ is the elementary transformation that carries (34) into (32).

82.5. With this clarification, we can return to the discussion of 82.2-3. We found in 82.3 that the requirement of uniqueness
for the root led us to exclude non-permutational rearrangements from the class of basic elementary transformations for which the root is directly defined. But even when we define the basic set of elementary transformations as \( L = \Delta + H \), there remain elementary transformations that are equivocal wrt \( L \), even in the set \( L \) itself. In fact, we can show that a deformation whose defining sequence contains a \( \triangledown \) at any point is equivocal wrt \( L \). That is, there is an analysis of this deformation which contains exactly one permutation (distinct from the identity \( \iota \)). We have seen in §2.4 that in the case of non-permutational rearrangements, the equivocation revealed a real equivocation in the notion of root, but in the case of these deformations, and in certain other cases, the alternative analysis in which a permutation figures seems to have no independent motivation.

One simple way to eliminate all further equivocation and to guarantee uniqueness of the root, is to restrict the occurrence of permutations in compounds. We can define a **compoundable** sequence of elementary transformations as a sequence which contains permutations only if there is no way to achieve the effect of the compound formed from this sequence without using permutations.

**Def. 27.** \( t_1, \ldots, t_n \) are **compoundable** if and only if for all \( 1 \leq n \), \( t_1, \Delta + H \), and either (i) or (ii)

(i) for all \( 1 \leq n \), \( t_1, \Delta + H \).

(ii) there is no \( t_1, \ldots, t_m \) such that

(a) \( t_m((t_1)\ldots) = t_n((t_1)\ldots) \)

(b) for all \( 1 \leq m \), \( t, \Delta + H \).
If in the general definition of "root" for compounds, we limit ourselves to compoundable sequences, then there can be no equivocation, and, at the same time, there is no restriction on the generative power of compounding. That is, every elementary transformation \( t \) that can be derived by compounding from rearrangements, deformations and \( \zeta \)-transformations, will assign a unique term of the proper analysis as a root to each term \( t(Y_1, \ldots, Y_n; y_{n}, \ldots, y_{n}) \) of the transform. A less stringent and more flexible requirement than compoundability in this sense might also be devised.

We thus arrive, finally, at the definition of "root", combining the definitions of "root 1" and "root 2" (Definitions 24, 25), with the emendations of the intervening discussion.

**Def. 28.** \( (Y_1, Z_1^Y) \) is the root of \( (W_1, Z_1^W) \) wrt \( (Y_1, \ldots, Y_n) \) if and only if either (I) or (II)

(I)(i) \( i, j \leq n \)

(ii) \( Y_i \cap Y_j = \emptyset \)

(iii) \( W_i = t(Y_1, \ldots, Y_n; y_{n}, \ldots, y_{n}) \)

(iv) \( Z_i^W = t(Y_1; Y_{n}, \ldots, Y_n) \cap t(Y_1, \ldots, Y_n; y_{n}, \ldots, y_{n}) \)

and either (v) or (vi)

(v) \( t \in A_{e_1} + H_{e_1} \), and \( i = j \)

(vi) \( t \in H_{e_1} \), and \( t \) is based on \( B(a_1, \ldots, a_k) \);

and \( \left\{ \begin{array}{ll}
\text{if } k = 2, \text{ then } i = a_1 \\
\text{if } k = 1, \text{ then } i = 1
\end{array} \right. \)

(II) there is a compoundable sequence \( t_1, \ldots, t_m \) and a pair \( (X_1, Z_1^X) \) such that

(i) \( \bar{t} = t_{m-1} \ldots (t_1) \ldots \)

(ii) \( (Y_1, Z_1^Y) \) is the root of \( (X_1, Z_1^X) \) wrt \( (Y_1, \ldots, Y_n), \bar{t} \)

(iii) \( (X_m, Z_m^X) \) is the root of \( (W_1, Z_1^W) \) wrt \( (W_1, \ldots, W_n), \bar{t} \)
Th. 2. Suppose (i) $t_1, \ldots, t_n$ are elementary transformations of

$L = l + i + j$

(ii) $t = t_n \ldots (t_1)$

(iii) for $y_1, \ldots, y_n$, and all $m \leq n$,

$$t(y_1, \ldots, y_m; y_{n+1}, \ldots, y_n) = w_n$$

Then there is a unique pair $(y_1, y_1', \ldots, y_1')$ such that

$(y_1, y_1', \ldots, y_1')$ is the root of $(w_n, w_1', \ldots, w_n')$ wrt $(y_1, \ldots, y_n)$, s

where $j, n \leq n$.

Def. 29. $(y, z^y)$ is the root of $(w, z^w)$ wrt $z, k, t$ if and only if

there is a $y_1, \ldots, y_n, t$ such that

(i) $(y_1, \ldots, y_n)$ is the proper analysis of $z$ wrt $k, t$

(ii) $t$ underlies $T$

(iii) $(y, z^y)$ is the root of $(w, z^w)$ wrt $(y_1, \ldots, y_n)$, s.

Th. 8. Suppose that some elementary transformation underlying $T$

is a finite compound of permutations, deformations, and $\eta$-transformations.

Then for each $z, k$, every term of the proper analysis of

$T(z, k)$ wrt $z, k, t$ (and only such terms) has a term of

the proper analysis of $z$ wrt $k, t$ as its root.

We require further that this root must be unique. Th. 7.

assures us that for each underlying $t$, the root of each term

is unique (since the proper analysis of $z$ wrt $k, t$ is unique).

But there may be various underlying transformations. In $\beta 28.3$,

we proposed a condition $(C12)$ which defined a set of determinate

transformations. We noted that determinate transformations

are those with a unique term arrangement. It is in fact the

case that
Th. 2. If \( T \) meets the condition of Th. 8, and \( T \) is determinate, then every term of the proper analysis of the transform has a unique root with respect to \( T \).

Any transformation that is general enough in its application to be of interest in syntactic analysis is determinate. Henceforth, we will always assume that the grammatical transformations under discussion are determinate. That is, we restrict ourselves to that part of \( \mathcal{Y} \) which meets (Cl2), \( \mathfrak{b} (2.2) \), as well as (Cl)-(Cl1), and we can thus assume uniqueness of root assignment.

Th. 4, \( \mathfrak{b} (8.2) \), and Th. 5, \( \mathfrak{b} (8.2) \), \( \mathfrak{b} (8.2) \), that the condition of Th. 8 is not too restrictive for syntactic analysis, since we can construct a transformation equivalent to any grammatical transformation, within the limits of this condition. The condition of compoundability does however impose a restriction on root assignment which may turn out not to be desirable, in view of the role of the notion of "root" in the determination of derived constituent structure. There is good reason to be somewhat suspicious about the maneuvers that have been carried through here to ensure the existence and uniqueness of the root. But they do leave intact the \( \delta \) that we will find useful in the transformational analysis of English, and, as we have seen, they have systematic motivation as well as some linguistic motivation.

Note in particular that a pair \( \Pi, \delta \) or \( \delta, \Pi \) (where \( \Pi \in \Pi \), \( \delta \in \Delta \), \( \delta \in \Delta_e \)) are compoundable only if \( (\Pi) \delta \) does not delete any terms of the proper analysis, i.e., if its defining sequence does not contain \( \sigma \) as a term. We have already seen the
utility of defining substitutional elementary transformations composed of \( \delta, \iota \) where the defining sequence of \( \delta \) contains only \( \iota \) (and \( \upsilon \)), i.e., where \( \delta \) only deletes terms, adding nothing. In fact, outside of the elements of \( \Delta_{el}, \Pi_{el}, \) and \( H_{el} \), the only elementary transformations that appear in the analysis of English that we will present below are substitutions and \( \Pi-\delta \) transformations of the type just described. Thus the machinery that we have developed will be adequate for the analysis to be presented in chapter IX, in this respect. (22)

The fact that the only deformations that actually appear in compounds are those with no deletions or those with only deletions (i.e., with null constant part) suggests an alternative approach from which it might be useful to study the basic structure of the system \( \gamma_{el} \).

83.1. We began the discussion of the constituent structure of transforms by proposing that to each term \( W_i \) of the proper analysis of the transform, there be assigned a term \( Y_i \) of the proper analysis of the transformed string, as the unique root of \( W_i \). Having provided the means for this assignment, we can go on to investigate derived constituent structure.

Suppose, throughout this discussion, that \( Z \) is a string, and that \( K \) is a class of strings which provides the analysis of \( Z \) (thus a \( P \)-marker, if \( Z \) is directly derived by a \( \beta_1 \)-derivation on the level of phrase structure). Suppose further that we have a transformation \( T \), with \( (Y_1, \ldots, Y_n) \) being the proper analysis of \( Z \) wrt \( K, T \), and \( (W_1, \ldots, W_n) \) being the proper analysis of \( T(Z, K) \) wrt \( Z, K, T \). We know that for each \( i \), there is a unique \( j \) such that the occurrence \( Y_i \) of \( Y_i \) is the root of the occurrence \( W_i \) of \( W_i \).
The problem that we now face is analogous to a problem to which much of chapter VI was devoted (cf. §48.1, (1), and the following sections). We were interested there in developing the notion "X is a Y of Z" which was to hold whenever X is a 'significant' segment of Z (i.e., a constituent), and when the given occurrence of X in fact functions as a Y. Thus we wanted to be able to say that the man is an NP in the man was here; that called up is a Verb in John called up his friends, though not in John called up the stairs, etc. This problem was solved in a general and apparently effective manner by the development of P-markers (which always give a consistent analysis, cf. Th. 15, §49.2) and the relation $\mathcal{E}_0$ (cf. §49.1, Def. 4). But now we are assuming that the algebra $\mathcal{P}$ and the relation $\mathcal{J}$ are limited so that only certain sentences have P-markers, namely, the kernel sentences. And we are interested in constructing notions for non-kernel sentences (i.e., transforms) that will fulfill the function served for kernel sentences by P-markers and $\mathcal{E}_0$.

The optimal solution to this problem would be to construct a notion of derived interpretation in such a way that the derived interpretation $K$ of $T(Z,K)$ is a set of strings uniquely determined by $Z,K,T$ and giving an consistent analysis of $T(Z,K)$, just as $K$ gives an analysis of $Z$. Then $\mathcal{E}_0$ could hold without alteration for $T(Z,K),K'$.

There are many difficulties in this course, and we have not managed to achieve such a satisfactory solution. Instead, we will approach the construction of "derived interpretation" indirectly, by means of a relation $\mathcal{E}$ more general than $\mathcal{E}_0$. 
This construction will not ensure that the derived interpretation
will provide a consistent analysis, and in its details, it is
rather too closely tied down to the specific requirements of
the transformational analysis of English. Hence this construction
should at most be regarded as providing a partial set of conditions
that will have to be met by some more general and revealing
construction of the concepts of derived constituent structure.
It is possible to generalize these conditions in various
ways. My own impression is that we are again at a point where
reasonably theoretical construction must be deferred until the
actual empirical consequences of various approaches are more
thoroughly explored.

83.2. In §78.1 we noted that our previous knowledge about the
level \( P \) can automatically provide us with a certain amount
of information about derived constituent structure. It may
be the case that a term \( W_1 \) of the proper analysis of \( T(Z,K) \),
or a string of terms \( W_1 \ldots W_{i+1} \) is represented by some prime
\( P_k \) or \( P_k \) (i.e., \( P_k \) bears \( f \) to this term or string of terms).
E.g., we know from \( P \) that "by John" is a Prepositional Phrase (PP).
That is, from the proper fragment of the grammar associated with \( P \) in
English we can reconstruct the following partial specification as \( f \):
(86) \( f(P_P, P'_N P) \), \( f(P, by) \), \( f(N_P, John) \).
Hence \( f(P_P, by\,John) \). Thus when we form the passive
(87) Bill was accused by John
from "John accused Bill", with the proper analysis as
indicated by the dashes, then we know automatically that
by John is a PP in (87).

It is very important to notice that it is not necessary
here that "by John" actually occur in some kernel sentence as
a PP. We know (cf. §50) that the algebra \( P \) is non-restricted.
Hence even if a certain phrase (e.g., "by John") never occurs as a $P_L$ in the restricted part of $P$ (i.e., in a restricted $\mathcal{F}$-derivation), it may still be a $P_L$ for reasons inherent in the construction of the phrase structure of kernel sentences and having nothing to do with transformational analysis. Thus as long as (86) is required by considerations of simplicity internal to the system of phrase structure developed for the kernel grammar, then the analysis of "by John" in (87) as a PP will be forthcoming.

Note in particular the significance of Ax. 10, chap. VI (§52), in this connection. This axiom requires that the classes of the absolute analysis (cf. §32) be carried over into $P$, thus increasing greatly the degree to which $P$ is non-restricted. Let us consider for a moment the effect of this requirement on the derived analysis of the passive. The absolute categories, it will be recalled, are intended to be those broad categories such as Noun, Verb, Adjective, etc., that mark the major distributional distinctions and that establish an overall dichotomy between grammatical and ungrammatical.

It will certainly be the case that "by" and "from", for instance, belong to the same absolute category. Similarly, it would seem reasonable to assume that the differences in distribution between two subclasses of adjectives $A_1$ and $A_2$ will not be marked on the absolute level, where $A_1$ is the class containing "tired", etc., adjectives that can occur after "very", and $A_2$ contains those elements such as "accused" that cannot. Hence there will be a prime Prep in $P$ such that $\mathcal{F}(\text{Prep, by})$ and $\mathcal{F}(\text{Prep, from})$, and a prime $A$ such that $\mathcal{F}(A, \text{entire})$ and $\mathcal{F}(A, \text{accuse})$. 
Suppose further that $f(\text{NP}, \text{John})$ and $f(\text{NP}, \text{the\ trip})$. It would seem that all of these assumptions must turn out to be the case in any adequate statement of the level $\mathbb{P}$, if the notions of phrase structure are to have their intended significance.

Suppose now that (87) is deleted from the kernel, to be reintroduced by transformation, but that (88) Bill was tired from the trip

is retained in the kernel. We will see in chapter IX that there are good reasons why this should be the case. (24) But in the statement of phrase structure for kernel sentences, (88) is analyzed as an instance of (89) NP-was-A-PP.

Hence, under the assumptions we have just made, it follows that (87) also receives this analysis, automatically. But this was just the analysis of passives to which we were led in $\S 62.3$. This is important, because it means that even if passives are dropped from the kernel, their derived constituent structure is that of a kernel sentence. This is not in general true of transforms, and it can account for the intuitively intermediate status of passives commented on in $\S 73.2$. In discussing this analysis of passives as (89) in $\S 62.3$, we noted that it was intuitively inadequate in that the 'verbal force' of, e.g., "accused" in (87) (as opposed to the really 'adjectival' status of "tired" in (88)) was unexplained. But this inadequacy is now eliminated (once we show that passives must be deleted from the kernel and derived transformationally, cf. $\S 94$), since the verbal force of "accused" is explained by the fact that (87) is derived from a sentence where "accuse" is actually a transitive verb. It is thus explained on a higher level than the
level $P$ of phrase structure.

We can state the condition on derived constituent structure that has been implicit in this discussion as the definition of a notion $E_1$, related to $E_0$, but including $T$ as an extra "parameter".

**Def. 30.** $E_1(x, y, z, k, l, t)$ if and only if there is a $w_1, \ldots, w_l$ such that (i) $(w_1, \ldots, w_l)$ is the proper analysis of $T(z, k)$ wrt $z, k, t$

(1) $1 \leq i \leq j \leq k$

(2) $x = w_1 \ldots w_j$

(3) $y = w_1 \ldots w_j$

(4) $v = w_1 \ldots w_j$ [i.e., $v$ is an occurrence of $x$ in $T(z, k)$]

(5) Suppose: $(r_k, s_k)$ is the root of $(w_k, w_1 \ldots w_j)$. Suppose: for each $k$ such that $1 \leq k' \leq j$,

\[ s_{k+1} = s_k \cup r_{k+1} \]

Then for some $k$ such that $1 \leq k' \leq j$, $w_k \cup r_k$

(6) $s$ is a prime

(7) $f(s, x)$

Thus the occurrence $v$ of $x$ is an $s$ of $T(z, k)$ if $s$ is a prime that represents $x$ in the kernel grammar, where $x$ is a string of one or more terms of the proper analysis of $T(z, k)$, and $x$ has been produced by $f$ of $s$, not simply carried over intact from $z$. (5) is necessary in Def. 30 (and Def. 31) to preclude the assignment of a new and irrelevant analysis to a constituent that does not change at all under the transformation.

Note that since (87) has the analysis (89), it also has the analysis NP-VP, since $f(VP, was A PP)$ must be the case on the level $P$ to account for (88). The fact that
was accused by John is a VP of (87) thus follows from Def. 30, taking as X a string of three terms of the proper analysis of (87) (namely, was, accused, by John). This demonstration then follows on the same assumptions about $\mathfrak{T}$ that were made above to show that the transform (87) has the analysis (89). Similarly we can show that (87) is a Sentence. Thus the passive (87) has the full constituent structure of a kernel sentence, by virtue of Def. 30. This is so far true only of the external structure of terms of the proper analysis (cf. 482, 4). In definitions 33 and 35 below we add further conditions on internal structure from which it will follow finally that passives actually have $\mathfrak{f}$-derivations and $\mathfrak{T}$-markers, even when deleted from the kernel, although not, of course, restricted $\mathfrak{f}$-derivations.

We can weaken the assumption about absolute categories upon which these comments about the status of passives have been based. We assumed that the distinction between $A_1$ (containing "tired"-en'tire) and $A_2$ (containing "accused"-en'acuse) would not be marked on the absolute level of analysis. But even this reasonable assumption can be dropped if we generalize $\mathcal{E}_1$ to a relation $\mathcal{E}_2$ which will assert that $X$ is an $S$ if there is an $X'$ with some representing string in common with $X$ and such that $\mathfrak{f}(S,X')$.

**Def. 31.** $\mathcal{E}_2(X,Y,S,Z,K,T)$ if and only if

1. $X$ is a term of the proper analysis of $T(Z,K)$ wrt $Z,K,T$
2. $Y$ is an occurrence of $X$ in $T(Z,K)$
3. There is an $X', W$ such that (a) $\mathfrak{f}(S,X')$
   (b) $\mathfrak{f}(W,X')$
   (c) $\mathfrak{f}(W,X)$
(iv) $S$ is a prime

(v) $(v)$, Def. 30, where $x = w_1 \cdots w_k$

But we have seen in Chapter VII that there is a prime $V_T$

such that $\mathcal{F}(V_T, \text{tire})$ and $\mathcal{F}(V_T, \text{accuse})$. That is, both "tire" and "accuse" are transitive verbs, though they belong to different subclasses of $V_T$. The decision to set up such an element $V_T$ is unaffected by the deletion of passives from the kernel. But now, whether or not $A_1$ and $A_2$ are distinguished on the absolute level, it follows that $\text{accused}$ is an $A$, if $\mathcal{F}(A, \text{tired})$. For "accused" within the level $P$ appears as $\text{en}^{\text{accuse}}$, and "tired" as $\text{en}^{\text{tire}}$. (25) Hence both $\text{en}^{\text{accuse}}$ and $\text{en}^{\text{tire}}$ are represented by $\text{en}^{V_T}$. Taking $\text{en}^{\text{accuse}}$ as $X$, $\text{en}^{\text{tire}}$ as $X'$, $\text{en}^{V_T}$ as $W$, and $A$ as $S$, we have the conclusion that $\text{en}^{\text{accuse}}$ is an $A$ if $\mathcal{F}(A, \text{en}^{\text{tire}})$, by Def. 31, where $T$ is the passive transformation, and $Z = "\text{John accused Bill}"$. I.e., in this case, $\epsilon_2(\text{en}^{\text{accuse}}, \text{Bill}^{\text{was}^{\text{en}^{\text{accuse}}}, \text{A}, \text{Z}, \text{K}, \text{T})$.

It may, however, turn out to be the case that Def. 31 is too strong in certain other cases. (cf. 410, 1.)

83.2. We see, then, that a good deal of useful information about derived constituent structure can be drawn from the system of phrase structure that is constructed for kernel sentences. But naturally, this source of information will not suffice to determine derived structure completely. It will be recalled that the grammar constructed for kernel sentences was based on a restricted relation $\mathcal{F}_R$ from which $\mathcal{F}$ could be mechanically reconstructed (cf. 50). But as sentences are withdrawn from the kernel, the relation $\mathcal{F}_R$ will become more limited in application. In fact, the primary systematic goal
of transformational analysis is to delete from the kernel a sufficient number of sentences so that a simple characterization of \( f^T \) (i.e., an 'optimal' grammar) can be presented. Hence the relation \( f^T \) as it is reconstructed from the kernel grammar will not be sufficiently rich and diversified to account for the total constituent structure of transforms.

But even though transforms may not have \( F \)-markers, they do have constituent structure, and this must be stateable in terms of the level \( P \). Thus we must construct a relation \( f^T \) which will supplement \( f^R \), and we must revise the requirement of \( \mathfrak{A} \), requiring now that \( f^T \) be mechanically recoverable from \( f^R \) and \( f^T \).

We will have to characterize \( f^T \) somehow on the level of transformational analysis. If \( f^T \) were known, then the problem of defining "\( X \) is an \( S \) in \( T(Z,K) \)" would be considerably simplified. If \( f(S,X) \) in the kernel grammar, then \( f^T \) or an elaboration of \( \mathfrak{A} \) may prove sufficient. If it is not the case, in terms of the kernel grammar, that \( f(S,X) \), then we can use the fact that \( f^T(S,X) \) in a definition perfectly analogous to Def. 3D.

On the other hand, if the relation "\( X \) is an \( S \) in \( T(Z,K) \)" is known completely, then the characterization of \( f^T \) will pose no problem. We can simply define \( f^T \) as the relation holding between \( S \) and \( X \) where for some transform \( T(Z,K) \), \( X \) is an \( S \) of \( T(Z,K) \). Thus the dual problem of determining \( f^T \) and the relation "is a" for transforms can be approached in various ways.

Suppose that we choose the first approach of characterizing \( f^T \) independently, then defining "\( X \) is an \( S \) of \( T(Z,K) \)" in terms of it. The grammar for \( f^T \) can be constructed
in the same manner as the grammatical statement based on $f^T$. We would develop this grammar so as to give the simplest characterization of a relation $f^T$ that will suffice to give all the derived constituent structure necessary for compounding of transformations, wherever this is called for.

This is a possible course, but one which it would be much preferable to avoid. The systematic motivation for transformational analysis is the desire to eliminate the immense complexity of a full statement of constituent structure. There is no point in simply transplanting a vast segment of this complexity to a new level. Furthermore, this procedure has an unpleasantly ad hoc flavor. We will find that by a further study of the roots of the terms of transforms, and the manner in which a transform is derived, we can eliminate the necessity for characterizing derived structure in this ad hoc and cumbersome fashion, by stating general conditions under which a derived constituent structure is assigned. Each step that we take in defining derived constituent structure is designed to eliminate the need for a part of the listing $f^T$ in the actual grammar of English. It will appear to be the case that $f^T$ can be eliminated without residue, at least to the extent that we have experimented with transformational analysis in English in chapter IX. But it should be kept in mind that if a certain transformational analysis requires an assignment of constituent structure to a transform in a way that cannot be derived from knowledge of $P$ and knowledge of the transformation $T$, then it is possible to save this analysis by listing the requisite $f^T$, if this results in an overall simplification of the grammar.

We now proceed with the second course of defining derived
constituent structure in a general way, leading to a partial or (we hope) complete specification of $f^T$ in terms of the notion "$X$ is a $Y$ of $T(Z, k)$".

83.4 Suppose that

(90) I-called my friend up

is dropped from the kernel to be reintroduced by transformation from

(91) I called up my friend.

In (91), "called up my friend" is a VP with the subject "I", and in (90) we should like to say that "called my friend up" is a VP as well. In fact, the conjunction criterion that we found so useful in Chapter VII necessitates this analysis, since we have

(92) I called my friend up and invited him

and "invited him" is clearly a VP.

The motive for dropping such sentences as (90) from the kernel is obvious, when we consider the general inadequacy of the system of phrase structure developed in Chapter VI for the case of discontinuous elements. We return to the transformational analysis of such sentences in ..., chapter IX.

If we follow the first approach suggested in §83.2, we might supply the required information by adding to the grammar statements of the form

(93) $f^T(VP, \text{"called my friend up"})$

But it is clearly both possible and preferable to state in linguistic theory general conditions under which such results as (93) will be derivable, so that no special statement at all
need be added to the grammar of English to provide the constituent analysis.

(90) is formed from (91) by a permutation that carries $Y_1-Y_2-Y_3-Y_4$ into $Y_1-Y_2-Y_4-Y_3$. The rule that we would like to construct should tell us that in such cases, if $Y_2-Y_3-Y_4$ is an $\mathcal{E}$, then $Y_2-Y_4-Y_3$ is an $\mathcal{S}$ as well.

The general requirement can be given as a definition of $\mathcal{E}_3$.

**Def. 32.** $\mathcal{E}_3(X,Y,Z,K,T)$ if and only if there is a $Y_1, \ldots, Y_i, W_1, \ldots, W_i$, $i \in I$ such that

(i) $(Y_1, \ldots, Y_i)$ is the proper analysis of $Z$ wrt $K,T$

(ii) $(W_1, \ldots, W_i)$ is the proper analysis of $T(Z,K)$ wrt $Z,K,T$

(iii) $X = \bigcap_{k \leq i} W_i$ \quad $0 \leq i \leq i-1$

(iv) $X = W_i \cap \cdots \cap W_{i-1}$

(v) $V = W_i \cap \cdots \cap W_1$

(vi) for each $k$, $0 \leq k \leq i$, there is an $m$, $0 \leq m \leq i$, such that

(a) $(Y_{i+m}, Y_{i+m})$ is the root of $(W_{i+k}, W_{i+k})$ wrt $Z,K,T$.

(b) $E_0(Y_{i+m}, Y_{i+m}, Y_{i+m}, Y_{i+m}, Z,K)$

(vii) $Z$ is a prime of $P$.

If $T$ is the permutation that converts $Y_1-Y_2-Y_3-Y_4$ into $Y_1-Y_2-Y_4-Y_3$, then Def. 32 will account for the fact that "called my friend up" is a VP in (90) if "called up my friend" is a VP in (91).

But Def. 32 does not require that $T$ be taken as a permutation. If $T$ is not a permutation, then $k=m$ in (vi), Def. 32. There are at least three interesting cases that are correctly dealt with in terms of $\mathcal{E}_3$ as defined here, where $T$ is not a permutation.
(i) Suppose that \( T \) is a substitution based on an elementary transformation \( t_{se} \Sigma_{el} \) (cf. Def. 26, §82.4). In §82.4 we noted that the substituend must have the same external constituent structure as the term for which it is substituted. Thus "it" is the NP subject of "it was quite obvious". But we form (94) that he was unhappy was quite obvious (= (82)) by substituting "that he was unhappy" for this occurrence of "it". Hence "that he was unhappy" must be the NP subject of (94).

Def. 32 tells us that if the effect of \( T \) is to replace \( y_{1}^1 \) in \( z = ... y_{1}^1 ... \) by \( y_{1}^4 \), then this occurrence of \( y_{1}^1 \) is an \( S \) if the given occurrence of \( y_{1}^4 \) was an \( S \) (and any string of terms of the proper analysis of \( T(z, k) \) containing this occurrence of \( y_{1}^1 \) is an \( S \) only if the corresponding string of roots containing \( y_{1}^4 \) is an \( S \)). Hence (94) will be correctly analyzed, where (94) (= (82)) is derived from (84) in the manner of §82.4, and (94) will be a substitute for the form \( \kappa \). 

(ii) Suppose that \( T \) is a deformation having the effect of (95), where hinges much more on the proper analysis as throughout this section.

(95) \( T: y_{1}^1 y_{2}^1 y_{3}^1 \rightarrow y_{1}^2 w_{1}^1 w_{2}^1 y_{2}^3 y_{3}^3 \)

In this case, \( T \) can be regarded as substituting \( w_{1}^1, w_{2}^1 \) for \( y_{2}^1 \) in \( z = y_{1}^1 y_{2}^1 y_{3}^1 \), and it seems reasonable to require that \( w_{1}^1, w_{2}^1 \) play the same role in \( T(z, k) \) that \( y_{2}^1 \) played in \( z \), for reasons similar to those that appeared in the case of substitutions.

As an instance, suppose that we were to introduce "very" transformationally, modifying adjective phrases (AP). Thus,

where \( w_{1} = \text{very}, y_{2} = \text{old}, w_{2} = \text{u} \), \( T \) converts

(96) \( y_{1}^1 \text{ old } y_{3}^3 \)
into $Y_1$, very"old"$Y_2$. But "very old" is an AP in the transform, where "old" is an AP in (96). And if "old" in (96) is part of an NP "old men", then "very old" in the transform is part of an NP "very old men". This is provided for by Def. 32.

We will see that there are reasons for not introducing "very" transformationally. But there are many instances similar to the one just sketched, where transformational treatment is in order. Suppose, for example, that in (95), $Y_2$ is one of the $M$'s, i.e., $Y_2$ is one of "can", "will", . . . (cf. § 61.2). Suppose that $W_1 = Y$, and $W_2$ is the element $M$'t which, suffixed to an $M$, gives "can't", "won't", etc. But these negatives must be analyzed as $M$'s, because they are subject to the same further transformations as "can", "will", etc. E.g., from "he can't come" we can form "can't he come?", just as we can form "can he come?" from "he can come", etc. But again, that "can't" is an $M$ if "can" is an $M$, follows from Def. 32, where the negative element is introduced by transformation as here outlined. Similarly, if "can come" is a VP, then "can't come" is a VP in the transform.

(iii) If $W_1$ or $W_2$ in (95) is one of $Y_1, Y_2$, or $Y_3$, then $T$ is an $\eta$-transformation. As an instance, consider the mapping $\overrightarrow{\Phi_1}^{P}$ (cf. § 63.1, and (35), § 60.2), which, in particular converts (97) into (98).

(97) . . . ing\textasciitilde eat\textasciitilde lunch . . .

(98) . . . eat\textasciitilde ing\textasciitilde lunch . . .

We might treat this as a permutational mapping, but it is also possible, with much superior results, to treat it as based on an \eta\textasciitilde-transformation \eta such that
(99) \( \mathcal{L} : \ldots \mathcal{X}_1 \mathcal{Y}_2 \mathcal{Y}_3 \ldots \rightarrow \ldots \mathcal{X}_1 \mathcal{Y}_2 \mathcal{X}_1 \mathcal{Y}_3 \ldots \)

followed by a deformation \( \mathcal{S} \) such that

(100) \( \mathcal{S} : \ldots \mathcal{Z}_1 \mathcal{Z}_2 \mathcal{Z}_3 \ldots \rightarrow \ldots \mathcal{U} \mathcal{Z}_2 \mathcal{Z}_3 \ldots \)

Neglecting now the surrounding context indicated by the dots, \( \mathcal{S}(\mathcal{L}) \) will be the elementary transformation such that

(101) \( \mathcal{S}(\mathcal{L}) : \mathcal{X}_1 \mathcal{Y}_2 \mathcal{X}_3 \rightarrow \mathcal{X}_1 \mathcal{Y}_2 \mathcal{X}_1 \mathcal{X}_3 \rightarrow \mathcal{U} \mathcal{Y}_2 \mathcal{X}_1 \mathcal{X}_3 = \mathcal{Y}_2 \mathcal{X}_1 \mathcal{Y}_3 \)

or, in particular,

(102) \( \text{ing} \rightarrow \text{eat, lunch} \rightarrow \text{ing-eat'ing-lunch} \rightarrow \text{U-eat'ing-lunch} \rightarrow \text{eat'ing-lunch} \)

The advantage of this analysis with a \( \mathcal{S}(\mathcal{L}) \)-transformation is that \( \mathcal{Y}_2 \mathcal{Y}_1 \) is now an \( \mathcal{S} \) in the transform in (99) and (101), if \( \mathcal{Y}_2 \) is an \( \mathcal{S} \) in \( \ldots \mathcal{X}_1 \mathcal{Y}_2 \mathcal{Y}_3 \ldots \); and \( \mathcal{Y}_2 \mathcal{Y}_1 \mathcal{X}_3 \) is an \( \mathcal{S} \) in the transform if \( \mathcal{Y}_2 \mathcal{Y}_3 \) is an \( \mathcal{S} \) in \( \ldots \mathcal{X}_1 \mathcal{Y}_2 \mathcal{Y}_3 \ldots \). In particular, it follows from Def. 32 that "eating" is a transitive verb \( \mathcal{X}_1 \mathcal{Y}_3 \), and "eating lunch" is a \( \mathcal{V} \mathcal{P}_1 \). This shows us the way out of one of the recurrent difficulties in chapter VII. We noted (cf., for instance, last paragraph of \S 61.2) that the conjunction criterion requires that

(103) John is eating lunch

be analyzed with the constituent breaks given by the \( \text{is'-ing', hyphens} \) as \( \text{NP - auxiliary phrase - VP}_1 \). But at the same time, we found that the most effective analysis of the verb phrase is achieved by considering "is'-ing" to be an auxiliary, with "eat lunch" the \( \mathcal{V} \mathcal{P}_1 \), and a mapping \( \Phi \) converting is'-ing'-eat'-lunch into is'-eat'-ing'-lunch. In chapter VII, these analyses did not seem to be compatible. But we see now that if the mapping in
question is considered to be a \( \lambda(x) \) transformation (as in \((101)) \), then these two analyses are in fact perfectly compatible, by virtue of Def.\,32. \( (\text{Cf. } \lambda^4_3, \text{ chap. VIII, and } \lambda^4_8, \text{ chap. IX, below}) \)

**83.6.** In §82.4 we noted a distinction between external and internal constituent structure. We have discussed the former in §83.4-2, and we can now turn to a discussion of the internal structure of terms of the proper analysis.

Suppose that \( Y \) is a \( \text{prt} \) (or the whole) of \( Y_1 \), and \( Y_1 \) is the root of \( W_1 \). Suppose further that \( Y_1 \) is not deleted in forming \( W_1 \). Thus \( W_1 \equiv Z_1 \uparrow Y_1 \uparrow Z_2 \) (where \( Z_1 = Z_2 = \mathcal{U} \), if the transformation is a permutation). Then, since \( Y \) is unaffected by the transformation, the matched occurrence of \( Y \) in \( W_1 \) is an S \( \mathcal{E}_{\text{prt}} \) if the given occurrence of \( Y \) in \( Y_1 \) is an S. \( \text{E.g., } "\text{the other team}" \) is an NP in "the game was won by the other team", as is "the game" (in the latter case, \( Y = Y_1 = W_1 \)). Similarly, both "your friend" and "the book" are NPs in (69) as in (68) (§82.2).

We give this requirement as a definition of \( \mathcal{E}_4 \).

**Def.\,33.** \( \mathcal{E}_4(Y, \forall, S, Z, K, T) \) if and only if there is a \( Y_1, \ldots, Y_n, W_1, \ldots, W_\ell \), \( a, b, c, d, i, i \) such that

(i) \( (Y_1, \ldots, Y_n) \) is the proper analysis of \( Z \) wrt \( K, T \).

(ii) \( (W_1, \ldots, W_\ell) \) " " " " " " " " \( T(Z, K) \) wrt \( Z, K, T \)

(iii) \( 1 \leq i, j \leq \ell \)

(iv) \( \forall = \forall^\uparrow \forall^\uparrow \forall^\uparrow X \), where \( \forall = \begin{cases} \mathcal{U}, & \text{if } i = 1 \\ W_1 \uparrow \ldots \uparrow W_1, & \text{if } i > 1 \end{cases} \)

(v) \( W_1 = a^\uparrow X^\uparrow b^\uparrow d \)

(vi) \( Y_1 = a^\uparrow X^\uparrow b \)

(vii) \( (Y_1, \ldots, Y_1) \) is the root of \( (W_1, W_1 \ldots W_1) \) wrt \( Z, K, T \).
(viii) \( \mathcal{E}_0(\mathcal{X}, \mathcal{X}^\prime, \mathcal{X}, \mathcal{Z}, \mathcal{K}) \), where \( \mathcal{X} = \begin{cases} \{y_1, \ldots, y_d \} & \text{if } d > 1 \\ \{y_1 \ldots y_{d-1} \} & \text{if } d = 1 \end{cases} \)

The model behind this is the following.

\[
Z = x_1 \ldots x_d \ldots x_n
\]

\[
\text{root: } W_1 \ldots W_l \ldots W_r
\]

\[
\mathcal{T}(Z, K) = c \cdot a \cdot x \cdot b \cdot d
\]

If \( T \in \mathcal{T} \), then \( c, d = \mathcal{U} \). Def \( \mathcal{S} = \{ x \} \). \( \mathcal{U} \) is \( \mathcal{S} \).

This definition serves the required purpose for \( T \) is based on permutations, deformations, and elementary compounds of these, but it clearly is not adequate for the affixed term of an \( L \) transformation. E.g., consider the \( L \)-transformation \( T \) based on \( L \) with the defining sequence \( (0, 0, 1, 0) \). Thus

\[
\begin{align*}
\mathcal{L}(Y_1; Y_1, Y_2) &= Y_1 \\
\mathcal{L}(Y_1, Y_2; Y_2) &= Y_1 \cdot Y_2 \\
\mathcal{L}(Y_1, Y_2) &= Y_1 ^ \prime Y_2
\end{align*}
\]

Thus \( T \) carries \( Y_1 - Y_2 \) into \( Y_1 \cdot Y_1 \cdot Y_2 \). But the root of the second term \( Y_1 \cdot Y_2 \) of the proper analysis of the transform is \( Y_2 \). For occurrences of \( \mathcal{X} \) in \( Y_2 \), \( \mathcal{E}_4 \) will be adequate, but not for occurrences of \( \mathcal{Y} \) in the first part \( Y_1 \) of \( Y_2 \). Instead of defining \( \mathcal{E}_5 \) so as to remedy this deficiency in general, we will define it only for the special case of substitutions (cf. \( \S 82.4 \)). Since the definition is simplest for these cases, and these are the only cases needed for our analysis of English.
Ref. 32. \( E_5(X, Y, S, Z, K, T) \) if and only if there is a \( Y_1, \ldots, Y_n, W_1, \ldots, W_n \),\( a, b, i, j \) such that

(i) \( (Y_1, \ldots, Y_n) \) is the proper analysis of \( Z \) wrt \( K, T \)

(ii) \( (W_1, \ldots, W_n) \) \( \in \) \( T(Z, K) \) wrt \( Z, K, T \)

(iii) \( 1 \leq i, j \leq n \)

(iv) \( Y = \bar{Y}^a \bar{X}^b \), where \( \bar{Y} = \begin{cases} U, & \text{if } i = 1 \\ Y_1 \ldots Y_{i-1}, & \text{if } i > 1 \end{cases} \)

(v) \( W_1 = \bar{W}_1 = a \bar{X}^b \)

(vi) \( T \) is based on the elementary transformation \( t_s \in \Sigma_{el} \), and the defining sequence of \( t_s \) is

\[
D = (a_1, \ldots, a_{2m-2}, \ldots, a_{2n}) \text{ \quad [cf. Th. 6, \( b82.4 \)]}
\]

(vii) \( E_0(X, Y, a \bar{X}^b, S, Z, K) \), where \( \bar{Y} = \begin{cases} U, & \text{if } i = 1 \\ Y_1 \ldots Y_{j-1}, & \text{if } j > 1 \end{cases} \)

The model behind this is the following (the order of \( i \) and \( j \) is irrelevant):

(306) \[
Z = Y_1 \ldots Y_i \ldots Y_n \quad \frac{\bar{Y}^a \bar{X}^b}{\text{root}} \quad \text{root} \quad \frac{W_1 \ldots W_i \ldots W_n}{Y_1 \ldots Y_{i-1}} \quad \frac{(a \bar{X}^b)^c}{\text{associativity}} \quad \frac{Y_1}{Y_1}
\]

In this case the root of \( W_1 \) is not \( Y_1 \), but \( Y_1 \). But the internal constituent structure of \( W_1 \) is that of \( Y_1 \). The fact that we cannot use the root in giving this definition is connected with the distinction between the two senses of "root" discussed in \( b82.4 \).

83.2. One further requirement for derived constituent structure is that if \( T(Z, K) = \ldots X \ldots \), then \( X \) is an \( X \) in \( T(Z, K) \). This is needed to complete the parallel between the derived notion of "is a" and
\( \mathcal{E}_0 (\text{cf. Def. 4, §8.1}). \)

**Def. 35.** \( \mathcal{E}_0 (X, Y, S, Z, K, T) \) if and only if (i) \( Y \) is an occurrence of \( X \) in \( T(Z, K) \)

(ii) \( X=S \)

In §3.1 we set ourselves the task of constructing a notion for transforms analogous to \( \mathcal{E}_0 \) for kernel sentences. We can now define a relation \( \mathcal{E}_0 \) which meets this requirement (subject to the qualifications suggested in §3.1).

**Def. 36.** \( \mathcal{E}(X, Y, S, Z, K, T) \) if and only if (I)(i) there is an \( i \) such that \( \mathcal{E}_i (X, Y, S, Z, K, T) \)

(ii) \( X=S \)

of (II) there is an \( X_1, X_2, Y_1, S_1, S_2 \) such that

(I) \( \mathcal{E}(X_1, Y_1, X_1, S_1, Z, K, T) \)

(ii) \( \mathcal{E}(X_2, Y, S_2, Z, K, T) \)

(iii) \( Y=Y_1 \)(X_1=X_2=Y_1 \( X \)

(iv) \( S=S_1 \)(S_2)

The inductive condition, (II), states that if \( X_1 \) is an \( S_1 \) and \( X_2 \) is an \( S_2 \) in \( Z \), where \( Z=X_1 \)(X_2), then \( X=X_1 \)(X_2) is an \( S_1 \)(S_2) in \( Z \). The analogous property holds for \( \mathcal{E}_0 \). (Iii) is added to preclude the assignment of constituent structure to unit terms of the proper analysis of \( T(Z, K) \). This restriction is necessary to ensure that the axiom system for \( \mathcal{E}_0 \) will be met, where \( \mathcal{E}_0 \) is recovered partially from \( \mathcal{T} \) which is defined in terms of \( \mathcal{E}_0 \), as suggested in §83.3.

Now that we have at least partially developed the notion of derived structure, in the form of a definition of \( \mathcal{E}_0 \), we can proceed to state the condition on \( \mathcal{T} \) as proposed in §83.3.

(107) If \( \mathcal{E}(X, Y, S, Z, K, T) \) and \( X=Y \), then \( \mathcal{T}(S, Y) \)

This can be reconstructed as a necessary and sufficient condition (i.e., a definition) for \( \mathcal{T} \) if we are prepared to accept the conclusion that \( \mathcal{T} \) never need be independently stated, as even in part (cf. §83.3). If we are willing to allow a certain amount of independent
specification of \( \mathcal{S}^T \) under the condition of overall simplification of the grammar, then (107) will be an axiom to be met on the level of transformational analysis by the independent notions \( E_{1-6} \) and \( S^T \).

The form of Def. 36 is intended to suggest that we have not completed the characterization of \( E_6 \), and that further conditions can be added as additional relations \( E_{1-6} \). But its incompleteness is only one respect in which the characterization of \( E_6 \) is inadequate. As we have developed \( E_6 \) here, it is not sufficiently restrictive in its assignment of derived constituent structure. That is, it assigns constituent structure in certain ways that we would not like to accept. In particular, it does not in general provide a consistent analysis for transforms, a fact which may indicate a formal inadequacy.

Suppose that \( I \) is an identity transformation that carries \( Z \) with the proper analysis \( (Z) \) into \( I(Z,K)=Z \) with the proper analysis \( (Z) \) (cf. case II, Def. 10, \( \mathfrak{B}2.2 \)). Then

\[ \text{Th.10.} \quad E_0(X,Y,S,Z,K) \text{ if and only if } E(X,Y,S,Z,K,I) \]

Thus \( E_0 \) can be regarded as one of the special cases of \( E_6 \), with \( T \) in this case the identity transformation \( I \).

\[ \mathfrak{B}3.8 \] The second goal proposed in \( \mathfrak{B}3.1 \) was the construction for each \( T(Z,K) \) of a set \( K' \) of strings to be assigned to \( T(Z,K) \) as its derived interpretation, just as a \( P \)-marker is a set of strings giving the constituent interpretation of a kernel string. \( K' \) must clearly be taken as the set of strings that provides exactly the constituent structure carried over from \( (Z,K) \) under \( T \). Thus we have
Def. 27. \( K' \) is the derived interpretation of \( T(Z,K) \) wrt \( Z,K,T \) if and only if \( K' \) is a minimal set of strings such that for all \( X,Y,S \),
\[
E_0(X,Y,S,T(Z,K),K') \text{ if and only if } E(X,Y,S,Z,K,T)
\]

Th. 11. Suppose that \( Z \) is a string in \( P \), \( K \) a set of strings containing \( Z \), and \( T \) a grammatical transformation based on an elementary transformation \( t \) which is a finite compound of deformations, permutations, and \( Z \)-transformations.

Then there exists a unique set of strings \( K' \) such that \( K' \) is the derived interpretation of \( T(Z,K) \) wrt \( Z,K,T \).

If \( T=I \) (as in Th. 10), then (i) if \( K \) is a \( P \)-marker of \( Z \), then \( K=K' \). (ii) in any event, \( K' \subseteq K \).

We cannot in general state that \( K'=K \) if \( T=I \), because \( K \) may contain irrelevant material, while \( K' \) is minimal. But in any case that actually deal with, \( K \) is minimal as well, since all strings in \( T \) and in another are strings such that \( Z'K \) and \( Z'K' \) are \( T \) and \( T' \).

Remark: and derived interpretations contain no strings irrelevant to the constituent analysis. Thus, \( T' \) and a string \( Z' \) with the interpretation \( K' \) which is a more string \( Z' \) is not the derived interpretation of \( K' \). This is the fundamental to any constituent analysis.

832. Now that we have supplied a precise (though incomplete) sense for the notions of derived constituent structure and derived interpretation, we can proceed to develop a technique for compounding transformations. The problem in defining \( T_2(T_1(Z,K)) \) was that \( T_1(Z,K) \) is a string \( Z \), and \( T_2 \) must operate on a pair \( (Z',K') \). But now we do have a unique set \( K' \) associated with \( T(Z,K) \) and providing its constituent structure. We can therefore define a compound transformation \( T_2(T_1) \) by the following condition:
We cannot in general state that \( K' = K \) if \( T = 1 \), because \( K \) may contain irrelevant material, while \( K' \) is minimal.

Th. 11 permits us to characterize a great many transformations (including, apparently, all those in which we are interested) as sets \( T = \{(z, \xi, \zeta, \xi', \zeta')\} \), where \( z, \zeta' \) are strings in \( \mathcal{F} \) belonging to the sets \( K, K' \) respectively, and \( \zeta' = T(z, \xi) \) in some structural feature constant for \( T \). Thus \( T \) maps a string \( z \) with the interpretation \( K \) into a new string \( \zeta' \) with the derived interpretation \( K' \). This is the fundamental feature of transformational analysis.

§1.2. Now that we have supplied a precise (though in no doubt incomplete) sense for the notions of derived constituent structure and derived interpretation, we can proceed to develop a technique for compounding transformations. The problem in defining \( "T_2(T_1(z, \xi))" \) was that \( T_1(z, \xi) \) is a string \( z \), while \( T_2 \) must operate on a pair \( (\zeta', \xi') \). But now we do have a unique set \( K' \) associated with \( (z, \xi) \) and providing its constituent structure. We can therefore define a compound transformation \( [T_2(T_1)] \) by the following condition:
(108) Suppose that $T_1, T_2 \in \mathcal{J}$.

Then $\left[ T_2(T_1) \right](Z,K) = T_2(T_1(Z,K),K')$, where $K'$ is the derived interpretation of $T_1(Z,K)$ wrt $Z,K,T$.

$\left[ T_2(T_1) \right](Z,K)$ is denoted "$T_2(T_1(Z,K))"$

The derived interpretation of $T_2(T_1(Z,K))$ wrt $T_1(Z,K), K', T_2$ is called "the derived interpretation of $T_2(T_1(Z,K))$ wrt $Z,K,\left[ T_2, T_1 \right]$".

By assigning to $\left[ T_2(T_1) \right]$ proper analyses of the transformed string and of the transform, we could regard $\left[ T_2(T_1) \right]$ as a transformation, i.e., as a set of ordered quintuples $\{(Z,K,Z',Pr^{(1)},Pr^{(2)})\}$ (cf. 478, 3). We could then ask whether $\mathcal{J}$ is closed under compounding, i.e., whether $\left[ T_2(T_1) \right]$ is necessarily in $\mathcal{J}$, if $T_1$ and $T_2$ are. The answer to this is clearly in the negative. Thus we have to define a compound of more than two elements by an inductive definition more general than (108).

Def. 38. Suppose $T_1, \ldots, T_n \in \mathcal{J}$.

Suppose $n \leq 1$. Then:

(1) $\left[ T_1 \right](Z,K) = T_1(Z,K)$

(II) "The derived interpretation of $T_1(Z,K)$ wrt $Z,K,\left[ T_1 \right]$" is defined as "the derived interpretation of $T_1(Z,K)$ wrt $Z,K,T_1$".

Suppose $n > 1$. Then:

(1) $\left[ T_n \ldots, T_1 \right](Z,K) = T_n(T_n^{(2)} \ldots, T_1)(Z,K)$

where $T_n^{(2)} = \left[ T_n^{(1)} \ldots, T_1 \right](Z,K)$.

(II) "the derived interpretation of $T_n^{(2)}$ wrt $Z,K,\left[ T_n^{(1)} \ldots, T_1 \right]$" is defined as "the derived interpretation of $T_n^{(2)}$ wrt $Z,K,\left[ T_n^{(1)} \ldots, T_1 \right]$".
Def. 29. \([T_1^n, \ldots, T_1^n](Z, K)\) is denoted "\(T_1^n(\ldots (T_1(Z, K) \ldots)\)"

We know from §22.5 and Def. 13, §22.1, that each grammatical transformation \(T\) is associated with a set of ordered triples \(T^n = \{(Z, K, Z')\}_1^n\), where \(Z' = T(Z, K)\). Similarly each compound transformation \([T_1^n, \ldots, T_1^n]\) is associated with a set of ordered triples \(T_1^n = \{(Z, K, Z^n)\}_1^n\), where \(Z^n = T_1^n(\ldots (T_1(Z, K) \ldots)\). \(T^n\) meets conditions (C1)-(C7), §22 (with the emendation of §22.2). But \(T_1^n\) fails condition (C6). That is, the undesirable transformations of the type described in §22.4, and excluded by (C6), are readmitted by compounding. But the difficulty in §22.4 was not the availability of these transformations, but the fact that they were no more complex than the transformations of the type admitted by (C6), so that the latter set of intuitively much simpler transformations were in no way preferred. Now, however, the situation is quite different. The transformations excluded by (C6) can only be reintroduced by compounding; that is, they must be described by a sequence of transformations of the type admitted by (C6). Hence their characterization is considerably more complex, the complexity depending on how flagrantly they violate (C6), i.e., how long a sequence must be given to describe them. This result seems satisfactory. The descriptive potential of the theory is not reduced by (C6), but we can rank the transformations available for syntactic analysis by complexity (i.e., length of description) in what seems to be an intuitively acceptable way, with those that meet (C6) ranking above those that do not. Thus the transformations that meet (C6) will be preferred in grammars which are evaluated in terms of the standard of simplicity.
Before leaving the topic of compound transformations we will extend the
notation \("[T_n, \ldots, T_1]\)" to cover the case where the \(T_i\)'s themselves may be
compound transformations.

Def. 40. Suppose that for each \(i\) such that \(1 \leq i \leq n\), \(T_i\) is the compound
transformation \([T_{m_i}, \ldots, T_{1_i}]\), where the \(T_i\)'s are each based on
a finite compound of elementary deformations, permutations and
\(\gamma\)-transformations.

Then \([T_n, \ldots, T_1]\) is the compound transformation

\[
[T_{m_n}, \ldots, T_{m_1}, T_{n-1}, \ldots, T_{n-1}, \ldots, T_{m_1}, \ldots, T_{1_1}]
\]

It remains to define a notion of equivalence among transformations, so that
such expressions as \("T_=\[T_n, \ldots, T_1]\)" or \("T_{a_m}, \ldots, T_{a_1}\) = \([T_{b_n}, \ldots, T_{b_1}]\)"
are provided with a clear sense.
Actually, we have given no clear sense to such expressions as
"T_1 = [T_2, \ldots, T_l]\) or \)[T_{m_2}, \ldots, T_{l_1}] = [T_{m_2}, \ldots, T_{l_1}]\),
which appear in Ax. 16 and in § 4-2. We can remedy this inadequacy by developing a notion of equivalence of transformations.

Def. 44. \(T_1\) and \(T_2\) are equivalent if and only if (i), (ii), (iii), (iv), or (v):

(i) \(T_1\) and \(T_2\) are grammatical transformations of \(\mathcal{J}\), and they are identical.

(ii) \(T_1\) is the compound transformation \([T_2]\), where \(T_2 \in \mathcal{J}\), or \(T_2 = [T_1]\), where \(T_1 \in \mathcal{J}\).

(iii) \(T_1\) is the compound transformation \([T_{m_1}, \ldots, T_{l_1}]\), and \(T_2\) is the compound transformation \([T_{n_1}, \ldots, T_{l_1}]\), where \(T_{m_1}, T_{n_1} \in \mathcal{J}\), and for all \(Z, K\):

(a) \([T_{m_1}, \ldots, T_{l_1}](Z, K) = [T_{n_1}, \ldots, T_{l_1}](Z, K)\).

(b) the derived interpretations are identical in case (iii).

(iv) \(T_1\) and \(T_2\) are compound transformations in the extended sense of Def. 40, and their expanded forms meet (a), (b) of (iii).

(v) there is a \(T_3\) equivalent to both \(T_1\) and \(T_2\).

In all cases we write: \(T_1 = T_2\) or \(T_2 = T_1\) (interchangeably).

Thus in all cases, if \(T_1 = T_2\), then \(T_1\) and \(T_2\) give the same transforms and derived analyses. This is a notion that deserves a good deal more study.
§4.1. We can now return to the consideration of the mappings of \( P \) into \( W \) which was broken off at the conclusion of chapter VI. Incidentally to the construction of transformations, we have developed machinery which will enable us to complete the characterization of the level of phrase structure. It will be recalled that the development of \( P \) was broken off at the point (§58.2) where it became necessary to state just how the mappings \( \Phi^P_1, \Phi^P_2 \) of \( P \)-markers into strings in \( \text{Gr}(W) \) take constituent structure into account, what sense should be attached to a compound mapping, how a mapping can be actually described in the grammar, and how \( \epsilon_0 \) holds of the strings of words which are the images of the mappings. But now it is clear that each of these problems is solved in terms of the development that we have just outlined if we formulate each mapping as a transformation. Although the construction of transformations was motivated by quite different considerations (cf. §72.4), we see that these constructions would have been necessary anyway, if only to complete the discussion of phrase structure.

In Ax. 1c, §58.3, we analyzed the mapping \( \Phi^P_1 \) into \( \Phi^P_2(\Phi^P_1) \), and in §58.2 and the sketch of English syntax in §67.2-2, we considered \( \Phi^P_1 \) to be itself a compound \( \Phi^P_1(\Phi^P_1) \). These analyses now have meaning, in terms of Definitions 28 and 32 if we regard mappings as transformations. We thus adjoin as a further Axiom to the Axiom system for \( P \) (cf. §58):

\[
\text{Ax. 1c. } \Phi^P = [\Phi^P_2, \Phi^P_1, \ldots, \Phi^P_1], \text{ and } \Phi^P_2, \Phi^P_1 \text{ are grammatical transformations.}
\]

Axiom 1b is now superfluous, except for the limitation that \( \Phi^P(\lambda, K) \) is a string in \( W \). Axioms 1c, 1d, further limit the class of transformations that can appear as part of the mapping \( \Phi^P \).
84. 2. It will be useful for later discussions to elaborate slightly the notations for describing mappings.

Def. 40. Suppose that $\mathbf{\varphi}_1^P = \begin{bmatrix} \mathbf{\varphi}_1^P \ldots \mathbf{\varphi}_1^P \end{bmatrix}$

Then $\mathbf{\varphi}_{m+1}^P = \mathbf{\varphi}_2^P$

$\mathbf{\psi}_k^P = \begin{bmatrix} \mathbf{\varphi}_1^P \ldots \mathbf{\varphi}_1^P \end{bmatrix}$, for $1 \leq k \leq m+1$

It follows from Ax. 12, §58. 1, that $\mathbf{\psi}_m^P(Z, K)$ is a string in $\mathbf{W}$, if $K$ is a $P-$marker of $Z$, and in this case, $\mathbf{\psi}_{m+1}^P(Z, K)$ is a string in $\mathbf{W}$. Furthermore, $\mathbf{\psi}_{m+1}^P(Z, K) \in \mathbf{Gr}(\mathbf{W})$, in this case. All of these statements may also be true if $K$ is not a $P-$marker of $Z$. In fact, we will formulate the level of transformations in such a way that they will be true when $K$ is the derived interpretation of $Z$.

We extend the definition of compound transformations $\begin{bmatrix} \mathbf{T}_1 \ldots \mathbf{T}_1 \end{bmatrix}$ so that the $\mathbf{T}_1$ themselves may be compounds.

Def. 41. Suppose that for $1 \leq i \leq n$, $\mathbf{T}_i$ is the compound transformation

$\mathbf{T}_i = \begin{bmatrix} \mathbf{T}_{i1} \ldots \mathbf{T}_{i1} \end{bmatrix}$, where $\mathbf{T}_{i1} \in \mathbf{G}$ for $1 \leq j \leq m_1$

Then $\begin{bmatrix} \mathbf{T}_1 \ldots \mathbf{T}_1 \end{bmatrix}$ is the compound transformation

$\begin{bmatrix} \mathbf{T}_{11} \ldots \mathbf{T}_{11} \mathbf{T}_{1n-1} \ldots \mathbf{T}_{1n-1} \ldots \mathbf{T}_{11} \ldots \mathbf{T}_{11} \end{bmatrix}$

Suppose that we define the complement of $\mathbf{\psi}_k^P$ as

Def. 42. $\mathbf{\psi}_k^P$ is the complement of $\mathbf{\psi}_k^P$, for $1 \leq k \leq m+1$, under the assumption of Def. 40, where

$\begin{bmatrix} \mathbf{\psi}_k^P \ldots \mathbf{\psi}_k^P \end{bmatrix}$

Th. 12. $[\mathbf{\psi}_k^P, \mathbf{\psi}_k^P] = \mathbf{\varphi}_2^P$
This treatment of mappings as transformations enables us to clarify a very important feature of transformational analysis. A given transformation may apply to \((Z,K)\), where \(Z\) is a string in \(F\) in the narrow sense (cf. b-2,1 -- i.e., where \(Z\) is composed of lowest level elements of \(F\), e.g., it is the product of a \(f\)-derivation), or it may apply only when \(Z\) has already been partially developed by mappings, or when \(Z\) has been totally developed into a string in \(W\). The fact that a transformation \(T\) may apply only when \(Z\) is partially developed by mappings is a major reason for analyzing \(F_2^P\) as a compound of many transformations. We can now make sense of this condition by requiring that \(T\) be compounded with just \(\Psi_k^0\), which is required for \(T\) to apply properly.

This of course implies that a compound transformation can in general be formed from two transformations only if these apply to strings which are developed by mappings to the same point, or if the second applies to a string developed beyond the point of application of the first member of the compound. This is a limitation which it might be possible and worthwhile to avoid. In certain cases it can be avoided by special devices, but it cannot be avoided in general unless each \(F_2^P\) is a one-to-one transformation. This suggests a possible extension of our general framework to include somehow the notion of an inverse of a transformation. Exactly how such an extension might be carried out (or whether the one-one assumption \(\psi\) is a correct one, in fact) is not clear. In the transformational system for English there seems to be no particular need for such an extension. But this point must be kept in mind as a possible defect in the system being developed here.
H Having constructed the set of grammatical transformations which are to be available for syntactic description, we can proceed with the program outlined in §76, that is, we can turn to the construction of the level $T$ of transformational analysis as a linguistic level, and to the determination of the criteria that are to apply in the evaluation of a grammar containing a transformational level. We can construe transformational analysis as a new level, having the same status and the same formal structure as other levels, and introduced for the same general systematic purpose, to simplify the generation of grammatical sentences. Since this is a syntactic level, our purpose is to simplify the description of the set $\text{Gr}(W)$ of grammatical strings of words. We must then construct a set of $\overline{P}$-markers which are mapped into a certain subset of $\text{Gr}(W)$ by a mapping $\Phi_T$, just as on the level $P$ we constructed $\overline{P}$-markers which are mapped into a certain subset of $\text{Gr}(W)$. Between them, $\overline{P}$-markers and $T$-markers must exhaust (under the respective mappings) the set $\text{Gr}(W)$. More generally, the combined range of $\overline{P}$ and $T$ must be exactly the set $\mu^W$ of $\overline{W}$-markers.\(^{(26)}\)

Consider the level $T$ defined as follows:

\[(109) \quad T = \left[ T, \wedge, \cdot, s^T, \mu^T, \Phi^T \right],\]

where $T$ is the set of primes of $T$, $s^T$ is the relation discussed in §83.2, $\mu^T$ is the set of $T$-markers, and $\Phi^T$ is the mapping that carries these into strings of words.

If the definition of $\Phi^T$ of (107), §83.2, is as suggested there, then $s^T$ can be dropped as an independent element, as we have seen.
It has been evident throughout our development of transformations that the level $T$ is intimately related to the level $P$, and as we axiomatize $T$, this relationship will become explicit in that we will have a set of conditions of compatibility holding for $P$ and $T$. To begin with, strings in $Gr(P)$ (the set of products of restricted $g'$-derivations) and $P$-markers of these strings must be taken as primes of $T$.

More generally, we proceed with the assumption of §79.1 and §80.1 that: we have a system $S$ including $P$, $M$, and $W$ and containing a single unit $U$ (which serves for all subalgebras of $S$) and a single concatenation operation, so that elements of the various subsystems of $S$ can be freely concatenated (in other words, $M$ and $W$ can be regarded as embedded in $P$; $P$ is extended to include all elements not bearing $g'$, i.e., $P$ in the old sense of Def.1, §47.2, as well as all strings in $M$ and $W$, the formal properties of $P$, $M$, and $W$ are carried over into $S$, for these subsystems. We can thus analyze the set $T$ of primes of $T$ as follows:

**Axiom 1:** $T = T_1 + T_2 + T_3 + T_4$,

where $T_1 = \{Z \mid Z$ is a string in $P\} + \{\sigma\}$ (cf. §80.1)

$T_2 = \{K \mid K$ is a set of strings in $P\}$

$T_3 = \{T_3 \mid T_3$ is a grammatical transformation \}$

$T_4 = \{T \mid T$ is a compound transformation \}$ (cf. §83.1)

Note that $T_2$ contains $P$-markers, and $T_3$ contains the mappings $P^P_{T_1}$, as we have seen in §84. Other special instances of $T_3$ are the identity transformations which carry each $(Z, K)$ into $Z$. We will select a single one of these, name it "I", and refer to it henceforth as the identity transformation. I can be taken as the transformation with the identity permutation $\langle$ as its underlying elementary transformation, and with $Q=\{U\}$ as its
restricting class. For any \( Z \in U \), \( I(Z,K) \) is thus an instance of case II, Def. 10, §79.2. Thus it is always the case that \( I(Z,K) = Z \), and that the proper analysis of \( Z \) wrt \( K, I = \) the proper analysis of \( I(Z,K) \) wrt \( Z, K, I = (Z) \). I must not be confused with the unit element of \( T \). It is a real transformation, which simply has no effect on any pair \( (Z,K) \).

In developing the level of Phrase Structure we were able to begin at once with its algebraic formulation, but before constructing \( T \) as a level, we found ourselves involved in a long discussion of transformations. The reason for this is obvious when we compare the primes of \( T \) with the primes of \( S \). It was pointed out repeatedly in chapter VI (e.g., §45) that the primes of \( S \) have no real content, whereas \( T \). (Although we used the notation "Noun Phrase" (NP), we had not really defined "Noun Phrase" but only "constituent"; and we decided to label one of the constituents NP, though there was no systematic significance to this labelling. But in the preceding discussion we have actually given the content of the set \( T_3 \) of primes of \( T \). In fact, given an ordering of the strings in \( S \) for a particular language, we can, in constructing the transformational level for this language, actually choose from an already given stock of transformations. Thus we know a good deal more about the transformational level than about the phrase level. Here we can, for instance, compare different languages with respect to the actual content of their transformational levels, not only with respect to their formal structure on this level, just as on the phonemic level we can compare phonemic structures in substantial terms (e.g., languages A and B both have voiced stops, etc.). On the phrase level, we must be content with formal comparisons only (e.g., languages A and B have \( P \)-markers with similar diagrams, in the
sense of $G_{\overline{4g.23}}$, etc.).

85.2. Strings in $T$ are formed by concatenating the primes of $T_i$.
The question is: how are we to interpret such concatenations?
We will interpret strings in $T$ in terms of the following correspondence:

$$(110) \quad Z^K T \text{ corresponds to } T(Z, K)$$

$$T_1 \ldots T_n \quad \overrightarrow{\left[ T_n, \ldots, T_1 \right]} \quad (\text{cf. Def. 39, Def. 41})$$

We can now define:

**Def. 44.** A is a normal string of $T$ if and only if $A = Z^K T_{i_1} \ldots T_{i_n}$,
where $Z \in T_1$, $K \in T_2$, $T_i \in T_3$ or $T_4$. (27)

(110) explains how we may understand any normal string,
and we may take normal strings to be the only significant strings
in $T$ (for the moment -- we generalize this below, in 482). In
particular, then, the $T$-markers will be normal strings. $T^T$
is the mapping which assigns $T$-markers to grammatical strings
in $W$. Hence $T^T$ is defined by (110). Thus we have

Ax. 2. Suppose that $A$ is the normal string $Z^K T_{i_1} \ldots T_{i_n}$
Then $T^T(A) = T_n(\ldots T_{i_1}(Z, K) \ldots) \quad (\text{cf. Def. 39})$

It is clear that concatenation of transformations is the
same as compounding, and that $T^T(A)$ does not depend on how
the ultimate grammatical transformations into which the terms
of the transformational part of $A$ are analyzed, are actually
grouped into compounds in $A$. That is,

**Th. 13.** Suppose that $A = Z^K T_{i_1} \ldots T_{i_n}$ and $B = Z^K T_{i_1} \ldots T_{i_m}$, where
$A$ and $B$ are normal, and $\overrightarrow{T_{i_1}, \ldots, T_{i_n}} = \overrightarrow{T_{i_1}, \ldots, T_{i_m}}$ (cf. Def. 41).
Then $T^T(A) = T^T(B)$. 
Thus we can set up equivalence classes of $T$-markers differing inessentially, in that the transformational part of one corresponds to the same compound transformation as does the transformational part of the other.

**Def. 45.** Suppose that $A$ and $B$ are normal strings in $T$ such that

$$A = Z^K T_{i_1}^{T_{i_1}} \ldots T_{i_n}^{T_{i_n}}$$

$$B = Z^K T_{j_1}^{T_{j_1}} \ldots T_{j_m}^{T_{j_m}}$$

Suppose further that $[T_{i_n}^{T_{i_n}}, \ldots, T_{i_1}^{T_{i_1}}] = [T_{j_m}^{T_{j_m}}, \ldots, T_{j_1}^{T_{j_1}}]$.

Then $A$ and $B$ are equivalent.

Equivalent normal strings will then be mapped into the same string in $W$ by $\underline{\phi} T$. However, certain non-equivalent $T$-markers may be mapped into the same string in $W$. It is important to distinguish these cases, since the second, and only the second will define constructional homonymy on the level $T$. Only in the case of identically-mapped non-equivalent $T$-markers is there a real case of structural ambiguity. It is not necessary to distinguish between equivalent $T$-markers in syntactic analysis.

**85. 2.** The level $P$ is characterized in part by a set $\text{Gr}(P)$ of strings in $W$. This is exactly the set of products of restricted $J$-derivations (cf. Th. 3, §42. 2), that is, the set of strings which are generated in the grammar reduced (in the sense of §51) from the level $P$. (28) We can define the kernel of a language as the subset of grammatical strings of words which is the image of $\text{Gr}(P)$ under $\underline{\phi} P$. 
Def. 46. The kernel is the set \( \Omega \) of strings such that
\[
Y \in \Omega \text{ if and only if there is a } Z \text{ and a } K \text{ such that }
\]
(i) \( Z \in \text{Gr}(P) \)
(ii) \( K \) is a \( P \)-marker of \( Z \)
(iii) \( Y = \Phi^P(Z, K) \)

Thus the kernel is included in \( \Omega^W \), but does not in general exhaust \( \Omega^W \). Our intention in constructing transformational analysis was to limit the kernel to a subset of \( \Omega^W \), in fact to just that subset for which a simple, systematic, and 'optimal' syntactic description can be provided, deriving the other members of \( \Omega^W \) from the kernel by transformation. Thus for every string in \( \Omega^W \) but not in the kernel, we must provide a \( T \)-marker based on a kernel string. We thus place the following condition on the set \( \Omega^T \) of \( T \)-markers:

Axiom 3. \( \Omega^T \) is a set of strings \( \{A_1, \ldots, A_n\} \) such that
\[
\text{(i) } A_1 \text{ is normal sim. + } Y
\]
(ii) \( A_1 = Z^K \ldots \), where \( Z \in \text{Gr}(P) \), and \( K \) is a \( P \)-marker of \( Z \).
(iii) If \( Z \) is not in the kernel, then
\[
Z \in \Omega^W \text{ if and only if there is an } A_1 \in \Omega^T \text{ such that }
\]
\[
\Phi^T(A_1) = Z
\]

For every \( Z \) in the kernel, there is in fact a string \( A_1 \) meeting (i) and (ii) and such that \( \Phi^T(A_1) = Z \), namely,
\[
\text{(iii) } A_1 = Z^KQ^P, \text{ for some } Z \text{ and } K.
\]

Hence we might alternatively drop the antecedent of (iii) in Axiom 3 and add all strings (iii) (with a proper choice of \( Z, K \)) to \( \Omega^T \).

In order for Axiom 3 to be met, it will be necessary to give an extremely complicated characterization of \( \Omega^T \) as matters now
stand. Below, in §87, we will generalize the algebra of $\mathcal{F}$ in such a way as to permit $Ax.3$ to be met significantly.

85.4. In §84.2 we noted that each transformation $T$ applies to the $\psi_k^*$-transform, for some $k$, of a string $Z$. And the transformation operation is completed by continuing to apply the mappings $\psi_1^P, \psi_2^P, \ldots$ until the result is a string in $\mathbb{F}$. Thus the typical $T$-marker will be of the form

$$Z^k \psi_k^* T^\psi \psi_k$$

if $T$ is a transformation applying to $\psi_k^*$-transforms. Suppose now that $T_1$ applies to $\psi_k^*$-transforms, and $T_2$ applies to $\psi_{k+1}^*$-transforms, and that we wish to apply $T_2$ after $T_1$ to form a compound transformation. The corresponding $T$ marker will be

$$Z^k \psi_k^* T_1^\psi \psi_k^* T_2^\psi \psi_{k+1}^*$$

Continuing to more general cases, it certainly seems reasonable to impose further conditions on normal strings in terms of the distribution of mappings $\psi_1^P$ in the string. Because of the potential trouble spot mentioned in §84.2, any condition we set is somewhat suspect. But it will be inconvenient to carry out an actual description of transformational structure without setting some condition. We will impose, tentatively, the strong condition that the segments of $\psi_1^P$ must be found as a unique substring of a normal string.

The notations $\psi_k^*$ and $\psi_k^*$ introduced in §84.2 are significant only in terms of a fixed decomposition of $\psi_1^P$ into $[\psi_1^P, \ldots, \psi_1^P]$. Hence any use of these notations presupposes a fixed decomposition. In practice, there will for any language be only a single
decomposition of $\Phi^P$ to which reference will ever be made in constructing the grammar, since if alternative decompositions are utilized, they must be independently stated, thus increasing the complexity of the grammar. Though this is not absolutely necessary, it would be quite possible to propose the uniqueness of this decomposition as a condition on that part of the grammar of Phrase Structure that deals with mappings. (It will be recalled that there was a gap in our formulation of the requirements for grammars just at this point — cf. \ref{422.2}.)

In any event, we assume here a fixed decomposition of $\Phi^P$ into $[\Phi^P_{m+1}, \ldots, \Phi^P_1]$, where $\Phi^P_{m+1} = \Phi^P_2$ (cf. \ref{428.1}, Ax. 1c, and \ref{84.1}), and the discussion is relative to this assumption.

**Def. 42.** $T$ is a component of $\Phi^P$ if and only if for some $i, j$ such that $1 \leq i \leq i+1 \leq m+1$, $T = [\Phi^P_{i+1}, \ldots, \Phi^P_j]$

In particular, given an analysis of $\Phi^P$ as assumed, $\Psi_k^P$ and $\Psi_k$ are components of $\Phi^P$. We see that components of $\Phi^P$ play a special role in $\mathfrak{T}$ in that they provide, in a sense, a certain skeletal structure for $\mathfrak{T}$ markers.

**Def. 43.** Suppose that $A$ is a string in $\mathfrak{T}$.

Then $(T_1, \ldots, T_k)$ is a $\Phi^P$-skeleton of $A$ if and only if

(i) $\Phi^P = [T_k, \ldots, T_1]$

(ii) each $T_i$ is a component of $\Phi^P$ (relative to the assumption of a fixed decomposition)

(iii) There are strings $x_1, \ldots, x_{k+1}$ in $\mathfrak{T}$ such that

$A = x_1^T T_1^T x_2^T \cdots x_k^T T_k^T x_{k+1}$

**Ax. 4.** A string of $\mathcal{A}^T$ has a unique $\Phi^P$-skeleton.
The definitions and axioms of $\hat{B}_5$ will be generalized and elaborated below in $\hat{B}_7$.

55.5 The level $\underline{\mathbf{T}}$ is established primarily in order to enable us to avoid the vast complexity and inelegance which is the necessary consequence of any attempt to state $\underline{\mathbf{W}}$ directly and exhaustively in terms of Phrase structure. The grammatical statement of $\underline{\mathbf{T}}$ for a given language, will contain a specification of the strings in $\underline{\mathbf{T}}$ which are $\underline{\mathbf{T}}$-markers of strings of $\underline{\mathbf{W}}$, and the characterization of those transformations that appear in these strings. The latter will be effected by stating, for each transformation, its underlying elementary transformation $\mathbf{t}$ and its restricting class $\mathbf{Q}$. We have discussed the form of this characterization above. If the transformation in question is compound, we describe it in terms of a sequence of its non-compound components of $\mathbf{J}$. A grammar containing a level $\underline{\mathbf{P}}$ and a level $\underline{\mathbf{T}}$ is thus evaluated by considering its total simplicity. In practice this means that for each class of strings of $\underline{\mathbf{W}}$, we must decide whether or not the strings in question belong to the kernel by determining the relative complexity of deriving them by $\underline{\mathbf{f}}$-derivation in $\underline{\mathbf{P}}$, or by a transformational analysis. Again, we have at best an evaluation procedure, since simplicity is a measure that applies to a complete grammar. For this evaluation procedure to be significant, it is necessary, actually, to prescribe a fixed form of grammatical statement for the level $\underline{\mathbf{T}}$, just as we did for $\underline{\mathbf{P}}$. We will not go into this in detail, though certain reductions will be described in the next two sections. To do so would duplicate a large portion of the discussion already given, and at this early stage of the study of transformational
analysis, there are many tasks that seem more immediately important. In actually applying transformational analysis to English, as a test case, we will drop sentences from the kernel only when it is quite clear that with any reasonable grammatical definition of the form of a statement for $T$, the transformational analysis is simpler. Cf. also §84, below, in this connection.

85-6. We have interpreted transformational analysis as adding a new level of linguistic structure, having the same status as other levels. There are arguments for and against such interpretation. As levels are characterized abstractly in Chapter II, transformational analysis can be put into the form of a linguistic level, as we have just seen. The level $T$ is clearly quite different, however, in many respects, from the lower levels that we have described. For one thing, there is no relation between left-to-right order in strings in $T$ and temporal order of sounds in the represented utterance. On the other hand, as we have seen in §14, this is too strong a requirement even for other levels. But it is true that on no other level is there so little relation between left-to-right and temporal order.

There are several considerations in favor of considering transformational structure to constitute a linguistic level, beyond the trivial and purely formal fact that it can be considered to have the basic form of a concatenation algebra. For one thing, this treatment adds considerably to the unity of linguistic theory. Just as we can represent each utterance by a sequence of phonemes, morphemes, words, and phrases, we can represent each utterance by a sequence of operations by which it is derived from a kernel sentence (more correctly, from a string in $\text{Gr}(P)$ which is mapped by $\Phi^P$ into a kernel sentence). The most
convincing support for the treatment of transformational analysis in this manner lies in the extension of the notion of constructural homonymity that results automatically from this treatment. Just as we have the possibility of multiple representation on every other level, we have, on the level $T$, the possibility of assigning several $T$-markers to a given sentence. We will see below that assignment of multiple analyses on the level $T$ has the same effect as constructural homonymity on any other level, namely, ambiguity of interpretation of the sentences which fall in the overlap of the distinct structural patterns provided by distinct markers. Finally, we will see below, in §87, that the interpretation of $T$ as a concatenation algebra leads to a quite natural and effective generalization of transformations, with little extra machinery needed.

§86.4. This theory of transformations has been based on a conception of transformations as single-valued mappings. There are however, many cases in which any one of a certain set of elements in a sentence is subject to what is essentially the same transformation. Thus any constituent $C$ can be replaced by $C'$ and $C''$ (with certain qualifications). Or, more narrowly, any Noun Phrase in a sentence (again with certain qualifications) can be replaced by the element "who" (or "what", depending on what sort of Noun Phrase it is) which is placed at the beginning of the sentence to form a question. E.g., given "I saw John reading a book", we have "whom did I see reading a book", "what did I see John reading", etc. We will find it quite useful to have at hand a method for indicating briefly that a certain transformation applies, in some sense, to all elements of a certain kind $g$ in a sentence. As matters now stand, the uniqueness condition on transformations requires that to
convey this we must construct a whole set of transformations separately, one for each occurrence of the element in question (constituents in the first example, Noun Phrases in the second). The inconvenience of this (and the complexity to which it leads in the statement of the grammatical strings in $\mathbf{T}$) leads us to the notion of a family of transformations. By a "family of transformations" we will mean a set of grammatical transformations, all with the same underlying transformation, and all having restricting classes meeting some fixed condition.

**Def. 49.** A family $F$ of transformations is defined by an ordered pair $(C, t)$, where $C$ is a condition on $\mathbf{T}$ elements (i.e., a schema $C(\alpha_1, \ldots, \alpha_k)$ with $\alpha_1, \ldots, \alpha_k$ as its only free variables), and $t$ is an elementary transformation. $T$ is a member of the family $F$ if and only if

(i) $T$ is a grammatical transformation

(ii) $t$ is the elementary transformation underlying $T$

(iii) if $Q = \{W(i)_{1}, \ldots, W(i)_{m}\}$ is the restricting class for $T$, then $n \in \mathbf{r}$ and for each $i,$

$$C(W(i)_{1}, \ldots, W(i)_{m})$$ (i.e., the terms of the restricting class, in the given order, meet the condition $C$)

Thus $C$ is a condition that the restricting classes of a transformation must meet to belong to $F$. Since a transformation is determined by an ordered pair $(Q, t)$, it is thus also determined by a properly chosen $(C, t)$ (though in this case, of course, determination is not in general unique).

Since it will in general be much simpler to give the condition $C$ than to spell out in detail what the restricting class must be so as to both to meet $C$ and at the same time to
define a grammatical transformation, this notion can lead to an essential simplification in the statement of the grammar of $\mathfrak{T}$, whenever it is the case that there is a set of $\mathfrak{T}$-markers that differ from one another only by members of a family $F$. In such a case, we can simply state the pair $(C,t)$ corresponding to $F$, and assert that any string containing a $T_i \in F$ in the proper position is a $\mathfrak{T}$-marker, without having to actually state these transformations $T_i$ separately and explicitly. We have not developed a rigid form of grammatical statement for the transformational level. If we were to go on to do so, we would introduce a notational convention permitting us to list "...$F$...", where $F$ is a family of transformations, in the list of $\mathfrak{T}$-markers in place of the set of statements "...$T_i$...", where $T_i \in F$. Naturally, this simplification permits many more transformations to appear in this position than would be permitted by an exhaustive list, since many transformations that it would have been unnecessary to define will meet the condition $C$. But this is a harmless duplication. No new sentences will be generated if $C$ is properly chosen.

The introduction of the concept of 'family' plays the same role on the level $\mathfrak{T}$ that the reductions of $\mathfrak{I}$ played on the level $\mathfrak{P}$. By permitting us merely to give the conditions that transformations must meet to appear in $\mathfrak{T}$-markers, this development leads to a considerable simplification of the statement of the grammar of $\mathfrak{T}$.

86.2. **Axiom 3** requires that each $\mathfrak{T}$-marker be a string of the form $Z^\ast \mathfrak{K}^\ast ...$, where $Z \in \mathfrak{I}(P)$, i.e., where $Z$ is actually derived on the level $\mathfrak{P}$. It is possible to develop another reduction of the grammar of $\mathfrak{T}$ which will enable us to
include in the list of $\mathcal{T}$-markers strings in $\mathcal{T}$ that begin with a string $Z \in \mathcal{Sr}(P)$, it being understood that $Z$ must be derived by one of the other transformational statements. This could always be avoided by giving a compound transformation instead, but only at the cost of some complexity. We will give this reduction in a more detailed and satisfactory form as a part of a more generalized characterization of $\mathcal{A}^T$ below (as condition 4, §87.4).

87.1. A grammatical transformation, as we have defined it, converts a string interpreted in a certain way into a second string with a derived interpretation. An investigation of the problems that motivated transformational analysis reveals certain inadequacies in this conception. Such sentences as

(114) (a) That John was unhappy was quite obvious
(b) The men who lost their jobs were bitter
(c) Since John liked the book, I decided to give it to him
(d) John read the book and enjoyed it thoroughly

are evidently instances of sentences constructed transformationally from a pair of kernel sentences. This suggests the need for a generalization of the notion of transformation to include transformations defined on $n$-ads of sentences (more correctly, $n$-ads of pairs of a string $Z$ and an interpretation $K$).

It may be that the best approach to this problem would be a direct and independent definition of generalized transformations; we will approach the problem, however, in a different way, extending the notions developed above. We will do this by stating a set of conditions which can be regarded as an extension of the axiom system for $P$ and $T$. This extension
actually necessitates some minor readjustments in the axioms that have already been given. These are fairly obvious, and we will not trouble to indicate them.

One way to generate a transform from two given sentences, within our present framework, is to run the two sentences together into a single long sentence and apply a transformation to this new compounded sentence. Given \((Z_1, K_1)\) and \((Z_2, K_2)\), we can form \((Z_3, K_3)\), where \(Z_3\) is \(Z_1 \land Z_2\), and \(K_3\) is essentially the Cartesian product of \(K_1\) and \(K_2\) (i.e., it is the set of strings that contains \(X_1 \land X_2\), wherever \(X_1 \in K_1, X_2 \in K_2\)). To avoid ambiguity, however, we must indicate in \(Z_3\) and in the strings that make up \(K_3\) exactly where the break between the two sentences lies. Thus we add to the set of primes of the system \(S\) (cf.§82.1) an element \(#\), indicating sentence boundary, and we extend the set \(\text{Gr}(P)\) to include complex strings containing \#.

Condition 1. There is an element \(# \in P\). There are no strings \(X_1, X_2, X_3\) such that \(X_1\) is a prime of \(P\) and \(\delta(X_1, X_2 \land # \land X_3)\).

Condition 2. If \(Z_1, Z_2 \in \text{Gr}(P)\), then \(Z_1 \land # \land Z_2 \in \text{Gr}(P)\).

Def.49. \(Z_1 \land # \land Z_2\) is a complex string.

Since Condition 2 is recursive, \(\text{Gr}(P)\) can include strings with any number of occurrences of \#. Thus \(\text{Gr}(P)\) consists of strings with restricted \(\delta_1\)-derivations, and complex strings whose ultimate components are the products of such derivations.

Given this extension, we can find transformations which will give (114) when applied to complex strings in \(\text{Gr}(P)\), and we can construct normal strings as \(\pi\)-markers of these sentences. The strings of \(\text{Gr}(P)\) from which (114) are derived might be:
(115) (a) John was unhappy # it was quite obvious
(b) The men were bitter # the men lost their jobs
(c) John liked the book # I decided to give it to him
(d) John read the book # John enjoyed it thoroughly

In all cases but (c), the transformation will presumably drop the element #. In case (c) this can be retained as an intonation marker. We have pointed out several times that a major deficiency of our constructions is their failure to cover suprasegmental features such as pitch, stress, and juncture. If these constructions were somehow extended to cover this aspect of grammar, then # would be one of a set of juncture elements of a considerably wider distribution and playing a much more central role in the grammatical description, and the element # would thus lose its special and unique character. But the extension to include suprasegmentals appears to be no trivial matter.

Since generalized transformations are, in terms of this construction, simply special cases of grammatical transformations, the notions of derived constituent structure, family of transformations, etc., carry over without special comment.

87.2. **Axiom 3**. §85.3 expresses the fundamental requirement for transformational analysis. We can test the adequacy of our construction by determining how neatly it enables us to meet **Axiom 3**, that is, how simple a characterization can be given of $\mu^T$ while still meeting this requirement.

- We know that **Axiom 3** could be met even without the generalization of §87.1 if we were willing to give sufficiently complex and sufficiently many transformations. For example,
in the case of the set of sentences of the form that Sentence was quite obvious (the form of which (114a) is an instance), it would be necessary to give a distinct deformational transformation for each distinct that-phrase, with "it - was quite obvious" as the string of Gr(P) from which all these sentences are derived. Thus (114a) could be derived by a deformation $\delta$ such that

$$(116) \delta^*(Y_1,Y_2) = \text{that he was unhappy} - Y_2.$$ 

Thus the defining sequence for $\delta$ is $(\delta, \text{that he was unhappy}, U, U)$ (cf. Def.16, §80.1). And "that he was nervous - was quite obvious" would be derived, again from "it - was quite obvious" (as $Y_1 - Y_2$) by a deformation $\delta^*$ such that

$$(117) \delta^*(Y_1,Y_2) = \text{that he was nervous} - Y_2.$$ 

Thus the defining sequence for $\delta^*$ is $(\delta, \text{that he was nervous}, U, U)$. Similarly with other instances of this type. Such treatment would not only have been intolerably complex, but it would have missed the point that in each case the term which is introduced by the deformation has itself a transformational origin. Now we can accomplish this same result with a single transformation based on the complex strings (115a), etc. Thus instead of the complicated analysis in terms of deformations of the type of (116), (117), etc. we have a single transformation $T_{th}$ which carries any instance of (118) into the corresponding case of (119).

$$(118) \text{Sentence - # - it - was quite obvious}.$$ 

$$(119) \text{that Sentence - was quite obvious}.$$ 

We then have a single statement to the effect that any string of the form (120) is a $T$-marker.

$$(120) Z^k T_{th}, \text{ where the } Z^k \text{-skeleton is properly inserted.}$$
This is oversimplified, and a good deal more generalization is possible. But the improvement brought about by the development of generalized transformations should be clear.

Note that $T_{th}$ here has the restricting class (118) (which can obviously be generalized beyond "was quite obvious") and

at the present stage of development of the theory, it appears to be based on an elementary transformation which is the compound of a substitution $t_s$ and a deformation $\delta$ where

\[
\begin{align*}
\delta &: \ Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4 \rightarrow \text{that}^\wedge Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4 \\
\begin{align*}
t_s &: Z_1 \rightarrow Z_2 \rightarrow Z_3 \rightarrow Z_4 \rightarrow U \rightarrow U \rightarrow Z_1 \rightarrow Z_4 = Z_1 \rightarrow Z_4 \\
t_s(\delta) &: Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4 \rightarrow U \rightarrow U \rightarrow \text{that}^\wedge Y_1 \rightarrow Y_4 = \text{that}^\wedge Y_1 \rightarrow Y_4
\end{align*}
\]

In the case of (115a), $Y_1$ = "John was unhappy", $Y_2$ = $\#$, $Y_3$ = "it", and "$Y_4$ = "was quite obvious".

We have discussed the derived constituent structure of such sentences above in \S 83.5, (i), pointing out that "that John was unhappy" is the NP-subject of (114a) since it substitutes for "it" of "it was quite obvious". We return to such sentences below, in \S 87.6.

87.2. Even though generalized transformations offer a considerable improvement, certain artificialities in the treatment of more complicated cases still remain and indicate the need for a further development.

Before going on to discuss these inadequacies, we define more
carefully what we mean by the interpretation of a complex string.

**Def. 50.** If \( K_1 \) and \( K_2 \) are classes of strings, then

\[
\text{\( K_1 \ast K_2 \) is the class of strings \( Y \pi Y' \pi Z \), where \( Y \in K_1 \) and \( Z \in K_2 \).
}
\]

I.e., \( K_1 \ast K_2 = \{ X \mid \text{there is a} \ Y, Z \ \text{s.t.} \ Y \in K_1, \ Z \in K_2, \ \text{and} \ X = Y \pi Y' \pi Z \}\).

Thus \( K_1 \ast K_2 \) is the interpretation of the complex string \( Z_1 \pi Y \pi Z_2 \), where \( K_1 \) is the interpretation of \( Z_1 \), and \( K_2 \) is the interpretation of \( Z_2 \).

The remaining deficiencies in our conception of generalized transformations become most clear when compounding of transformations is required. We illustrate this by an example.

(122) Suppose that: \( Z_1 = \) "they had found John"; \( K_1 \) is the \( \pi \)-marker of \( Z_1 \)

\( Z_2 = \) "it was quite obvious"; \( K_2 \) is the \( \pi \)-marker of \( Z_2 \)

\( T_{th} \) is the transformation described above with

(118) as restricting class, and \( t_s(\delta) \) of

(121) as underlying transformation.

Suppose further that we have a transformation \( T \) such that

\( Z_3 = T(Z_1, K_1) = \) "John had been found"; \( K_3 \) is the derived interpretation of \( Z_3 \) wrt \( Z_1, K_1, T \).

It should clearly be the case that (123) and (124)

(123) that they had found John was obvious

(124) that John had been found was obvious

are derived in essentially the same way, by \( \pi \)-markers of

the form (120). That is, the \( \pi \)-marker of (123) should be

(125) \( (Z_1 \pi Z_2)^\pi (K_1 \ast K_2)^\pi T_{th} \) (30) (omitting the \( \pi \)-skeleton, as in (120))

and the \( \pi \)-marker of (124), analogously, should be

(126) \( (Z_3 \pi Z_2)^\pi (K_3 \ast K_2)^\pi T_{th} \).
But $Z_3 \#^{#} Z_2 \not\in \text{Gr}(F)$, since $Z_3$ is derived by the transformation $T$. Thus (126) does not qualify as a $\frac{T}{T}$-marker (cf. $A'k.3$, $\frac{\text{485}.3}$). This means that we must construct a new and distinct transformation $T'$ which combines features of $T_{th}$ and $T$ of (122), and then we must derive (124) by a $\frac{T}{T}$-marker:

(127) $Z_1 ^{#} Z_2 \sim (K_1 K_2)^{T'}$

But this is obviously unsatisfactory. This treatment fails to indicate the intuitively obvious fact that (123) is derived from $Z_1$ in the same way that (124) is derived from $Z_3$. It should be possible to give a uniform treatment of (123) and (124) in terms of the transformations $T$ and $T_{th}$ which are independently necessary, and dispensing with the new and ad hoc transformation $T'$.

This difficulty might be avoided in the manner of $\frac{\text{485}.2}$ by defining an appropriate reduction in the grammar which would allow us to state (126) in the list of $\frac{T}{T}$-markers, it being understood that $Z_3$ is derived by another transformation. Alternatively, the difficulty can be avoided by extending the class of normal strings to include strings of the form

(128) $Z_1 ^{K_1} T - Z_2 ^{K_2} T_{th}$

which in the case of (122) will be interpreted as applying $T_{th}$ to the string $T(Z_1 K_1)^{#} Z_2$ with the interpretation $K_3 K_2$. We will first exploit this approach, returning $\frac{n}{9}$ to the suggestion of $\frac{\text{485}.2}$ below.

We now proceed to extend Def. $\frac{44}{44}$, $\frac{\text{485}.2}$, permitting "$T_1$" in this definition to be replaced by "$Z_1 ^{K_1} T_1$". Def. $\frac{44}{44}$ is now superseded by
Def. 51. A is a normal string in $T$ if and only if $A = S_1 \cdots S_n$ ($n \geq 1$)
where

(i) $S_1 = Z_1^i K_1^i T_1$

(ii) $S_1 = Z_1^i K_1^i T_1$ or $S_1 = T_1$

where $Z_1 \in T_1$, $K_1 \in T_2$, $T_1 \in T_3 \cdots T_n$ (cf. Ax. 1, 82.1, 11, and Th. 27.)

Thus (128) must be rewritten with the identity transformation $T$ as

(129) $Z_1^i K_1^i T^* Z_2^i K_2^i T^* T_{th}$

and interpreted as applying $T_{th}$ to $T(Z_1, K_1)^\# I(Z_2, K_2)^\# T(Z_1, K_1)^\# Z_2$, just as before. The normal strings of Def. 54 are special cases of the normal strings of Def. 51, with each $S_i = T_i$, for $i \geq 1$.

We must now replace Ax. 2, 82.2, by a more general statement covering normal strings as defined in Def. 51, and supplying the interpretation for (128) and (129) that we have informally described. Before rephrasing Ax. 2 for the more general case, we define the value of a normal string.

Def. 52. Let $A = S_1 \cdots S_n$ be a normal string, where $S_1, \ldots, S_n$ are as in (i), (ii), Def. 51.

For $m \leq n$, we define the value of $S_1 \cdots S_m$ as

$Val(S_1 \cdots S_m)$

as the ordered pair $(Z_m^i, K_m)$, where

Case I: $m = 1$. \(\{Z_m^i, T_1(Z_1, K_1)\}

$K_m^i$ is the derived interpretation of $Z_m^i$ wrt $Z_1, K_1, T_1$ (cf. Def. 27, Th. 11, 83.8)

Case II: $m > 1$, $S_m = T_m$. \(\{Z_m^i = T_m(Z_m^i - 1, K_m^i - 1)\}

$K_m^i$ is the derived interpretation of $Z_m^i$ wrt $Z_m^i - 1, K_m^i - 1, T_m$
Case III. \(m > 1\), \(S_m = Z_m^{K_m} T_m\).

\[
Z_m^i = Z_m^{i-1} T_m (Z_m^i, K_m)
\]

\(K_m = K_m^{i-1} * K_m\), where \(K_m\) is the derived interpretation of \(T_m (Z_m, K_m)\) wrt \(Z_m, K_m, T_m\).

In all cases, we denote \(Z_m^i\) by "Val_1 (S_1 \cdots S_m)" and \(K_m\) by "Val_2 (S_1 \cdots S_m)".

Replacing Ax.2, \(\eta 85.2\), we have

Condition 2: If \(A_m\) is a normal string, then \(\Phi (A_m) = \text{Val}_1 (A_m)\).

We regard \(\Phi (A)\) as undefined for non-normal strings in \(T_n\).

Since a normal string can be analyzed in one way as \(S_1 \cdots S_n\) such that (1), (ii), Def. 51, it follows that \(\Phi (A_m)\) is unambiguous.

As an example of Def. 52 and Condition 2, let \(A_n = (129)\).

Then \(n = 3\), and \(A_n = S_1^3 S_2^3 S_3^3\), where

\[
\begin{align*}
S_1 &= Z_1^3 K_1^3 T \\
S_2 &= Z_2^3 K_2^3 I \\
S_3 &= T_3h
\end{align*}
\]

Then we have the following set of values:

(131) \(\text{Val}_1 (S_1) = T(Z_1, K_1) = Z_3\) (cf. (122))

\(\text{Val}_2 (S_1) = K_3\) (cf. (122))

Thus \(\text{Val}(S_1) = (Z_3, K_3)\)

\(\text{Val}_1 (S_1 S_2) = Z_3^3 \# I(Z_2, K_2) = Z_3^3 \# Z_2\)

\(\text{Val}_2 (S_1 S_2) = K_3 * K_4\) (cf. Th. 11, \(\eta 83.8\), last line)

Thus \(\text{Val}(S_1 S_2) = (Z_3^3 \# Z_2, K_3 * K_4)\)

\(\text{Val}_1 (A_n) = \text{Val}_1 (S_1 S_2 S_3) = T_3h (Z_3^3 \# Z_2, K_3 * K_4)\)

Therefore \(\Phi (A_n) = T_3h (Z_3^3 \# Z_2, K_3 * K_4)\), just as in the informal account given above.
Condition 2 contains $Ax.2$ as a special case.

In §85.2, after defining $\mathfrak{T}$ implicitly by $Ax.2$, we went on (Def. 45) to define equivalence classes of $\mathfrak{T}$-markers with equivalent transformational parts. This was important, since it is necessary to state that we have constructional homonymy on the level $\mathfrak{T}$ only when we have non-equivalent $\mathfrak{T}$-markers $A$ and $B$ such that $\mathfrak{T}(A)=\mathfrak{T}(B)$. Definition 52 permits new cases of normal strings that are mapped into the same string in $\mathfrak{P}$ by $\mathfrak{T}$ for reasons inherent in the construction of these strings in $\mathfrak{P}$ and having nothing to do with the language under analysis. The discussion of equivalence of $\mathfrak{T}$ is facilitated when we note that each normal string $A$ has a substring $Z_1^*K_1^*Z_2^*K_2^*\ldots Z_n^*K_n^*$ consisting of the strings in $\mathfrak{P}$ on which it is based and their interpretations, and a substring $Y_1^*Y_2^*\ldots Y_n^*$ consisting of the transformations that appear in $A$.

Def. 53. Let $A$ be a normal string in $\mathfrak{T}$.

Then the $\mathfrak{P}$-basis of $A$ is the maximal sequence $((Z_1^*,K_1^*),(Z_n^*,K_n^*))$ such that

(i) $Z_1^*\in \mathfrak{P}$ (i.e., $Z_1^*$ is a string in $\mathfrak{P}$)

(ii) $K_1^*\in \mathfrak{P}$ (i.e., $K_1^*$ is a set of strings)

(iii) there is a $Y_1^*,\ldots,Y_n^*$ such that

$$A=Z_1^*K_1^*Y_1^*Z_2^*K_2^*Y_2^*\ldots Z_n^*K_n^*Y_n^*$$

In this case, $(Y_1^*,\ldots,Y_n^*)$ is called the transformational part of $A$.

Th. 14. The $\mathfrak{P}$-basis and the transformational part of a normal string exist and are unique.

In a direct generalization of Def. 45, §85.2, we might say that two normal strings are equivalent if, term by term, their transformational parts correspond to equivalent transformations.
(in the sense of Def. 43, 184.2). But this characterization of equivalence will still permit cases of non-equivalent multiple $\Xi$-markers which because of their universality, do not mark genuine homonymy.

Suppose that

\[(132) \quad Z = Z_1 \neq Z_2 \ ; \ Z_1, Z_2 \in \text{Gr}(P)\]

$K_1$ is the interpretation of $Z_1$, and $K_2$ of $Z_2$

$K = K_1 \cdot K_2$

$T(Z, K)$ is a string of words.

As $\Xi$-markers of $T(Z, K)$ we may have, among others

\[(133) \quad (i) \quad Z^\wedge K^\wedge T \]

\[ (ii) \quad Z_1^\wedge K_1^\wedge T \cdot Z_2^\wedge K_2^\wedge T \]

(i) is an instance of the normal string $A_1 = S_1$; (ii) is an instance of $A_2 = S_1^\wedge S_2^\wedge S_3^\wedge$, where $S_1 = Z_1^\wedge K_1^\wedge I$, $S_2 = Z_2^\wedge K_2^\wedge I$, and $S_3^\wedge = T$. In general then, two $\Xi$-markers present a legitimate case of constructional homonymy only if under all substitutions of strings and interpretations for the non-complex components of these $\Xi$-markers, the same string in $\Xi$ results by $\Theta^T$.

We could easily go on to formalize this condition. Though obvious, it is rather involved, and we can bypass this construction, merely noting that in practice, considerations of simplicity of grammar will generally militate against the admission of equivalent $\Xi$-markers into $\Theta^T$. 

87.4. The extension of the class of normal strings by Def. 51 enables us to give a more elegant characterization of $\Theta^T$ in particular cases, by extending the possibilities of transformational treatment in what seems a natural way. We can, in terms of
it, give a more adequate condition of on \( \pi \). \textbf{Ax. 2}, \( \frac{85.3}{85.3} \), will be replaced by \textbf{Condition 4}, which incorporates generalized transformations, and certain other simplifications (e.g., \( \frac{86.2}{86.2} \)).

The most direct way to generalize \textbf{Ax. 2} would be to require that each term \((Z_{1_i}, K_{1_i})\) of the \( P \)-basis of a \( T \)-marker must meet (ii), \textbf{Ax. 2}, i.e., that each \( Z_{1_i} \) be in \( \text{Gr}(P) \) with \( K_{1_i} \) as its \( P \)-marker. But this condition (call it "condition C") would lead to certain artificialities and complications that are not resolved by the generalization carried through by \textbf{Def. 51}.

Suppose that we have a normal string \( A=S_1^{\downarrow} \cdots \downarrow S_n \), with \( S_i = Z_{1_i}^{\downarrow} \downarrow T_i \downarrow \) and \( S_{i+1} = T_i \downarrow T_{i+1} \) for a certain \( i \) such that \( 1 \leq i \leq n \). Thus \( T_{i+1} \) is a generalized transformation operating on the string

\[ \text{Val}_i \left( S_1^{\downarrow} \cdots \downarrow S_{i-1} \downarrow \right)^{\#} T_i \left( Z_{1_i}, K_{1_i} \right) \quad (\text{cf. Case III, Def. 52}) \]

The suggested generalization of \textbf{Ax. 2} would require that \( Z_{1_i} \), in this case, must be in \( \text{Gr}(P) \), thus that \( T_{1_i} \) be a compound transformation incorporating the changes that \((Z_{1_i}, K_{1_i})\) undergoes before \( T_{i+1} \) applies to it. For each of the possible transformational developments of \((Z_{1_i}, K_{1_i})\), a different \( T_{1_i} \) must be given.
Suppose further that there are transformations $V_1, \ldots, V_\tau$ such that all combinations $V_1(Z_1^{1,1}, V_2(Z_2^{1,1}, \ldots, V_\tau(Z_\tau^{1,1}, V_2(V_1(Z_1^{1,1}, Z_1^{1,1}), V_3(V_1(Z_1^{1,1}), \ldots, V_\tau(V_1(Z_1^{1,1})), \ldots$, etc., can occur significantly on the level $\tau$. In other words, in stating the grammar of $\tau$, we will provide that any string of the form

$$Z_1^{1,1} < V_1 \gamma(V_2) \ldots < V_\tau >$$

is in $\mathcal{A}_\tau^\mathcal{M}$, where the angle notation is carried over from earlier chapters (cf. $4\text{II.}3$), with the emendation that at least one of $V_1, \ldots, V_\tau$ must occur.

Suppose now that $T_{i+1}$ can apply not only to (134), but to any string

$$\text{Val}_1(S_1^{1,1} \ldots S_{i-1}^{1,1})^\# X_i$$

where $X_i$ is given by one of the strings generated by (135). Then we will be forced to state, alongside of $A_1$, a large number (in this case, $2^\tau - 1$) of $\tau$ markers differing only in that in place of $T_i$, they have some transformation corresponding to one of the cases of (135). This is not only overly complicated, but also repetitious in a way we would like to avoid, since all of (135) will essentially be repeated, case by case, in describing this set of $\tau$-markers.

But this case is actually the rule. Very often a generalized transformation will apply when various transforms appear as components of the complex string to which it applies. This suggests that we relax condition C, and require merely that each $Z_i$ in the $P$-basis be either a string in $\text{Gr}(P)$ or be $\mathcal{P}_\tau(X)$, for some $\tau$-marker $X$. This relaxation of condition C gives 486.2 as a special case.
There are reasons for relaxing condition C still further. Typically, a T-marker $A=S_1 \ldots S_n$ will have $S_n = \psi_k$. That is, its final term will be the remainder of the mapping $\Phi^T$ (see Definition 4). We have not found it necessary to state this as a special case, since $\psi_k$ is itself a transformation. But it is clear from investigation of the motives for transformational analysis that this is typical, and that $S_n$ will fail to be $\psi_k$ (for some $k$) only if the mapping $\Phi^P$ has been completed before $S_n$ is applied, i.e., if the transformation $\alpha$ applies to strings of words. (31)

We might say then that the correct $\psi_k$ is automatically added on to each T-marker, to convert the transform into a string in $\tilde{W}$. When a term $(Z, K)$ appears in the $P$-basis of a T-marker, we expect $Z$ to be not $\Phi^T(X)$, for some T-marker $X$, but rather $\Phi^P(X)$, where $X=Y \psi_k$. In other words, the components of a complex string will not ordinarily be mapped completely into words. Thus we define

Definition 4. If $A=S_1 \ldots S_n$ is a normal string, and for some $k$, $S_n = \psi_k$, then we define the reduced correlate of $A$ as

$A_{\text{red}} = S_1 \ldots S_{n-1}$

Replacing $A_{\text{red}}$, we now have

Condition 4. $\mu^T$ is a set of strings $X_1, \ldots, X_m$ in $\tilde{T}$ such that

(i) $X_i$ is normal

(ii) For each $1 \leq i \leq n$, if $((Z_1, K_1), \ldots, (Z_m, K_m))$ is the $P$-basis of $X_i$, then for each $1 \leq m$,

\[ \text{either (a) } Z_i \in \text{Gr}(P) \text{, and } K_i \text{ is a } \tilde{P} \text{-marker of } \tilde{Z}_i \]

\[ \text{or (b) } (Z_i, K_i) = \text{Val}(Z_{\text{red}}), \]

where $Z_{\text{red}}$ is the reduced correlate of some $Z \in \mu^T$.

(iii) $X_i \in \mu^W$ if and only if there is an $X_i \in \mu^T$ such that $\Phi^T(X_i) = \tilde{Y}$.
Note that with this construction, a non-kernel string of \( \mu^W \) need not have a \( T \)-marker based on kernel string \( \mu^g \), but it will always have a \( T \)-marker with "branches" each of which originates in a kernel string. Note also that every string in \( \mu^W \) has a \( T \)-marker, kernel strings having the \( T \)-marker \( \mathcal{K}^T \mathcal{F}^P \), where \( \mathcal{K} \mathcal{F}^P \).

87.2. The structural characterization of normal strings begun in §85.4 must be generalized to include the new cases of normal strings permitted by Def. 51, with Ax. 4 retained as a special case in this extension. Partly because of the difficulties pointed out in §84.2, I am not quite sure how this extension should be carried out. Unless some such condition is laid down, however, the definition of \( \mu^T \) given above will be far too broad -- far too many strings will be permissible in \( \mu^T \) and an important feature of abstract transformational structure will be omitted.

There are two ways of proceeding at this point. We may simply apply this system as given above to language without laying down further restricting conditions on normal strings, using ad hoc measures in each particular case to make sure that no complication arises. Since \( \mu^T \) as characterized above is too broad, we can be sure that nothing will be left out by this procedure. Or we may attempt to lay down a restricting condition on normal strings in the manner of §85.4, but generalized, and we may then apply the system within the limits of this restriction. We can thus determine, for any particular suggested restriction on normal strings, just what is the cost, for a particular grammar, of setting up this restriction in the general theory. Since
ultimately some such restriction will be necessary, I think that the latter course will be more interesting. We will thus go on to develop what seems to be the most natural generalization of \(Ax.4\).

Suppose that \(A = S_1 \cdots S_n\) is a normal string. There is a subsequence \(S_{\alpha_1}, \ldots, S_{\alpha_m}\) containing each \(S_i\) which is \(T_i\) rather than \(Z_i K_i T_i\). We can place the following condition on normal strings: For each \(i\),

(i) \(S_{\alpha_i}, S_{\alpha_{i+1}}, \ldots, S_{\alpha_m}\) contains the equivalent of \(\Psi_k^i\), for some \(k\). I.e., there is a subsequence \(S_{\beta_1}, \ldots, S_{\beta_{\tau}}\) of \(S_{\alpha_1}, \ldots, S_{\alpha_m}\) such that for some \(k\),

\[\Psi_k^i = [S_{\beta_1}, \ldots, S_{\beta_{\tau}}]\]

(ii) If \(S_{\alpha_i} = Z_i K_i T_i\) directly precedes \(S_{\alpha_{i+1}}\) in \(A\) (i.e., if \(\alpha_i = i+1\)), then one of \(S_{\alpha_1}, \ldots, S_{\alpha_m}\) contains the equivalent of \(\Psi_k^i\).

(a) if \(Z_i \in \text{Gr}(P)\), then \(T_i\) must contain an equivalent of \(\Psi_k^i\). (case Iia, condition 4)

(b) if \(Z_i \in \text{Val}_1(Z_{\text{red}})\), for some \(Z\), then \(Z_{\text{red}}\) must contain an equivalent of \(\Psi_k^i\). (case iib, condition 4)

We can give this formally as condition 5 on strings in \(\mu^T\).
Condition 2. Suppose that

(i) \( A = S_1 \wedge \ldots \wedge S_n \) is a normal string in \( T_n \), where the \( S_i \)'s are such that (i), (ii), Def. 51, \#87.3.

(ii) \( A \in \mu^T \)

(iii) \( S_{n+1} = U \)

(iv) \( Y_1, \ldots, Y_{n+1} \) is a subsequence of \( S_1, \ldots, S_n, S_{n+1} \) such that

(a) \( S_i \) is one of \( Y_1, \ldots, Y_n \) if and only if \( S_i \) is a component of \( \Phi^P \).

(b) \( Y_{n+1} = s_{n+1} = U \)

(v) \( S_{k} \wedge S_{k+1} \wedge \ldots \wedge S_{t+1} \) is a segment of \( A \), where for some \( k < t+1 \)

(a) \( S_{k+1} = Y_k \)

(b) \( 1 \leq t < t+1 \leq n+1 \)

(c) If \( t > 1 \), then \( S_{t-1} = Y_{k-1} \)

(d) \( S_{t+1} = T_{t+1} \) or \( Z_{t+1} = Y_{k+1} \) for \( 0 \leq k < i < j \).

Then for each \( i \) such that \( S_{t+i} \wedge T_{t+i} \) either (I) or (II).

(I) \( Z_{t+i} \in \text{Gr}(P) \), and \( T_{t+i} \wedge Y_k \wedge Y_{k+1} \ldots \wedge Y_{t+1} \) is a \( \Phi^P \)-skeleton

(II) \( Z_{t+i} = \text{Val}(Z_{t+i}^\text{red}) \), where \( Z_{t+i}^\text{red} \) is the reduced correlate of some \( Z_{t+i}^\mu \), and \( Z_{t+i}^\text{red} \wedge Y_k \wedge Y_{k+1} \ldots \wedge Y_{t+1} \) contains a unique \( \Phi^P \)-skeleton.

Thus \( S_{t} \wedge \ldots \wedge S_{t+1} \) is a segment of \( A \) enclosed between \( Y_k \) and \( Y_{k+1} \), if \( t > 1 \); between \( U \) and \( Y_1 \), if \( t = 1 \). In particular, \( S_{t} \wedge Y_1 \wedge \ldots \wedge Y_{t+1} \) contains a unique \( \Phi^P \)-skeleton. Axiom 4 now holds as a special case, when each \( S_i = T_i \) (\( i > 1 \)).

It also follows that when a generalized transformation applies to a complex string, each elementary component of the complex string must be mapped to
the same stage, the stage required by the transformation.
Essentially, this condition means that if we analyze
a generalized transformation into its several branches, each
one originating in a single kernel string, then each such
branch contains a unique $\Phi^P$-skeleton, i.e., each mapping $\Phi^P_{1,1}$
is applied once and only once to each kernel string in the
process of deriving the ultimate string in $W$ that results
from this generalized transformation.

There is no essential difficulty in applying transformational
analysis when this condition is incorporated, but there are
certain complications in this application. Consider for example
the transformation $T$ that carries (137) into (138)

(137)  \textbf{John-knew-it} - $\#$-Sentence

(138)  \textbf{John-knew-that} $\uparrow$Sentence

Suppose that $T$ applies before any part of the mapping $\Phi^P$. Then
the $T$-marker for (138) will be

(139)  "John knew it" $\uparrow k_1^1 I \leftarrow \text{Sentence} \uparrow k_2^2 I \leftarrow T \leftarrow \Phi^P$

where in place of \textbf{Sentence} in (138) and (139), we have some
particular sentence. $\Phi^P$ thus applies to the whole string (138),
but this means that $\Phi^P$ must actually apply twice to (138), once
to the string as a whole, and once to the that-clause, which has
the internal construction of a sentence.

Suppose that in place of \textbf{Sentence} in (138) we choose the
sentence (138), thus deriving

(140)  \textbf{John} - knew - that $\uparrow$John $\uparrow$knew $\uparrow$that $\uparrow$Sentence

in essentially the same way as we derived (138) - i.e., by
a $T$-marker
(141) "John knew it" \( \Phi^p \) \( I \) - "John knew that Sentence" \( \Phi^p \) \( I \cdot T \cdot \Phi^p \).

Then \( \Phi^p \) must essentially apply three times to (140). And since there is no limit to this process, the mapping \( \Phi^p \) will have to be formulated in such a way that it can apply indefinitely many times. It is possible to do this within our present framework. Each sub-mapping \( \Phi^p_{11} \) will be given as an infinite family of transformations (characterized, of course, by a finitely statable condition), and the transformations will have to be carefully formulated with just the right derived constituent structure given so that each mapping will apply correctly to each part of any sentence.

This is the major difficulty associated with the structural characterization proposed for normal strings in condition 5. This effect of condition 5 indicates that very likely, this condition is not the correct one. It seems that it should be possible to have the proper mappings apply to each transform \( X \) before it enters into a further transformation, and still to have these mappings reapply in the correct way to any further transform \( Y \), including \( X \), without disturbing \( X \) internally. I have, however, found no good general formulation that meets these requirements. We will continue the investigation of transformations, then, accepting condition 5 as a tentative and not completely satisfactory formulation. In investigating transformational structure, we will come across several instances of this and related difficulties, and we will see in detail the effects of accepting condition 5. This detailed investigation can be regarded as offering data for some future investigation of a preferable alternative to condition 5.
87.6. With these qualifications, the system of generalized transformations that has been developed in §87 seems adequate for the cases cited in §87.1 and many others. It is interesting to see how it applies to cases like

(142) That John was unhappy was quite obvious \([\cdots](114a)\]

in greater detail.

The \(T\)-marker of (142) can be taken as

\[
(143) \ Z_1^\wedge K_1^\wedge P^\wedge T_{\text{that}}^\wedge Z_2^\wedge K_2^\wedge P^\wedge T_X^\wedge \nonumber
\]

where (i) \(Z_1\) is a string of \(G_0(P)\) such that \(\Phi^P(Z_1, K_1)\) = "John was unhappy"

(ii) \(Z_2\) " " " " " " " " \(\Phi^P(Z_2, K_2)\) = "it was quite obvious"

(iii) \(T_{\text{that}}\) is the deformation that converts \(Y\) into \(\text{that}^*Y\).

(iv) \(T_X\) is the transformation determined by \((Q, t_b)\),

where \(Q = \left\{ (\text{that}^*\text{Sentence}, \#, \text{it}, \text{VP}) \right\}\)

t\_b is the elementary substitution such that \(t_b : Y_1^*Y_2^*Y_3^*Y_4^* \rightarrow U^*U^*Y_1^*Y_4^*Y_1^*Y_4^*\)

Thus \(T_X\) converts anything of the form \(\text{that}^*\text{Sentence}-\#-\text{it}-\text{VP}\) into the corresponding string of the form \(\text{that}^*\text{Sentence}-\text{VP}\).

(143) is a normal string \(S_1^*S_2^*S_3^*S_4^*\), where \(S_1 = Z_1^\wedge K_1^\wedge P^\wedge\), \(S_2^*T_{\text{that}}\), \(S_3^*Z_2^\wedge K_2^\wedge P^\wedge\), and \(S_4^*T_X\). We can trace the operation of (143) in several steps by determining \(\text{Val}_1\) of its initial substrings (cf. Def. 52, §87.3).
\[
\text{(144) } \text{Val}_1(S_1) = \Phi^P(Z_1, K_1); \text{ John was unhappy.}\]

\[
\text{Val}_1(S_1 \uparrow S_2) = T(\text{that John was unhappy}, K_3) = \text{that John was unhappy,}\]

\[
\text{Val}_1(S_1 \uparrow S_2 \uparrow S_3) = (\text{that John was unhappy})\# P(Z_2, K_2) =

(\text{that John was unhappy})\# (\text{it was quite obvious}) (33)

\[
\text{Val}_1(S_1 \uparrow S_2 \uparrow S_3 \uparrow S_4) = T_X(\text{Val}_1(S_1 \uparrow S_2 \uparrow S_3), K_4) = (142) \uparrow (\text{Val}_1(S_1 \uparrow S_2 \uparrow S_3))
\]

Other strings of the same form as (142) can be obtained by changing the P-basis \((Z_1, K_1), (Z_2, K_2)\) of (143). Note that

\begin{itemize}
  \item it is not necessary to take \(Z_1\) and \(Z_2\) as strings of \(\text{Gr}(P)\).
  \item In the grammar corresponding to the level \(T_2\), then, we can state (143) with variables replacing the terms of the P-basis, and
  \item perhaps a condition on the values that these variables can take, instead of listing all the T-markers that differ from (143) only in their P-basis.
\end{itemize}

A sufficiently detailed grammatical statement will of course have to indicate that there is a restriction on the distribution of "that John was unhappy" in grammatical sentences.

This restriction can be given either as a part of the definition of the restricting class of \(T_X\); or, if this is simpler, as a condition on the strings that can form the P-basis in the T-marker schema formed from (143), as suggested above, by taking "Z_1", "K_1", "Z_2", and "K_2" as variables.

In the case we have just discussed, the sentence (142) was formed by substituting the transform "that John was unhappy" for the noun phrase subject of "it was quite clear," which is itself a kernel sentence. It may be that there is no kernel sentence of which the generalized transform is a substitution instance in this sense. In this case a dummy carrier can be added to the set of primes of \(P\), and its distribution given as
a part of the kernel grammar. Then the transformation in
question can be very simply defined as applying only to sentences
containing this dummy carrier. Thus in the case of (142), if
there were no kernel sentence NP was quite clear, we could
adjoin to P a prime p, and give the distribution of p completely
in the phrase grammar. This might be the simplest procedure
even if there is a kernel element (e.g., "it") occurring in
every position where "that John was nervous", etc., can appear.
Of course if we take this approach, we must account for the
case where the transformation T is not applied. The product
of a P-marker must itself map into \( \mu \). Thus p may be carried
by \( \Phi \) into one of the strings that can occur grammatically
in the position where p occurs, e.g., "that John was
unhappy". In one way or another, then, the distribution of
"that John was unhappy" will be given in the description of
phrase structure as the distribution of a simple noun phrase
(with possible modification in terms of conditions on the
P basis of T-marker-schemata), and the internal structure
of this simple noun phrase will be given by a transformation.
Thus we can describe the distribution of a complex element
on the level P, where this is most easily done, and we can
describe its internal construction transformationally, instead
of complicating the level P by requiring that this too be given
directly. It will be recalled that a good many of the
complications that arose in the attempt to construct a grammar
of phrase structure in chapter VII, were due to the many
recursions which were necessary, and to the difficulty of
properly ordering the rules in which complex phrases played
a role (cf. \$64-65\). These difficulties can be avoided now
by generalized transformations.
The general class of \( T \)-markers of which (143) is an instance operate by converting a string \( Y_1 \) into \( Y_1 \), and then substituting \( Y_1 \) for some element \( X \) of a second string \( Y_2 \).

In the case of (143), \( Y_1 = \mathcal{P}(Z_1, K_1) = \) "John was unhappy"; \( Y_2 = \mathcal{P}(Z_2, K_2) = \) "it was quite obvious"; and \( X = \) "it". The result, then, is a string \( \ldots Y_1 \ldots \) -- in the case of (143), it is (142). The derived constituent structure of \( \ldots Y_1 \ldots \) is determined in the following way:

(i) The internal structure of \( Y_1 \) is just that carried over by the transformation from \( Y_1 \) to \( Y_1 \) (cf. especially, §82.6).

(ii) The external constituent role of \( Y_1 \) is just that of the element \( X \) for which it was substituted. If \( X \) is a \( Z \) in \( Y_2 \), then \( Y_1 \) is a \( Z \) in \( \ldots Y_1 \ldots \). If some segment containing \( X \) is a \( Z \) in \( Y_2 \), then this segment, with \( Y_1 \) replacing \( X \), is a \( Z \) in \( \ldots Y_1 \ldots \) (cf. §82.4, 82.5(1)).

Thus in the case of the derivation of (142), above, the internal structure of "John was unhappy" is that of a sentence, and the external structure of "that John was unhappy", is that of the noun phrase "it" for which it substitutes.

83. A grammar corresponding to the level \( T \) for a given language is basically a statement of the set \( \mu^T \) of \( T \)-markers for this language. The simplest way to present such a grammar is to list \( T \)-markers with variables for the elements of the \( P \)-basis, and to set various conditions on these variables to determine which pairs \((Z, K)\) can actually occur in the \( P \)-basis of such a schematic \( T \)-marker. In general, we can expect to find that an order of application is defined on these schematic \( T \)-markers, and we may be able to utilize this fact, as we did in the construction
of grammars on lower levels, to simplify the statement of conditions on \( P \)-bases of \( T \)-markers. For instance, we might be able to order \( T \)-marker schemata in such a way that the \( P \)-basis of each schema can be occupied by any strings of \( Gr(P) \) (and their \( P \)-markers) or by any strings derived by earlier \( T \)-markers (and their derived interpretations). We have not investigated the problem of developing a form of grammatical statement for the transformational level and relating this statement to the grammar corresponding to lower levels. A more urgent task is to investigate the range and power of transformational analysis, and this is the problem to which we turn in the next chapter.

In the course of these constructions, we have incidentally come across potential solutions for a number of the difficulties that originally suggested the desirability of establishing a new and higher level of syntactic analysis. Thus we have pointed out that generalized transformations will enable us to avoid much of the complexity caused by recursions in the grammar of phrase structure. As another instance, note that the conjunction rule, which we saw could not be incorporated into the grammar of phrase structure (cf. \( \tau \), \( \epsilon \)), can now be stated as a generalized transformation (actually, a family of generalized transformations, cf. \( \tau \), \( \epsilon \), below). Thus we can avoid transformationally a difficulty that actually undercut much of the reasoning that led to the particular form of the interpretation of \( P \) for English, i.e., much of the validation of the statement of English Phrase Structure.

We will now go onto a more detailed study of the transformational structure of English, and the effectiveness of transformational analysis in remedying the descriptive inadequacies of the previously established levels.
Footnotes - Chapter VIII

(1) (page 357) Further investigation of grammaticalness might wipe out even this distinction if, e.g., such sentences as
"the factory was owned by \{ a new technique \} the railroad tracks" are excluded as not fully grammatical.

(2) (p. 357) We know that "tire" and "bore" are in a different subclass of verbs from "paint", etc. (cf. footnote 14, chap. VII, and statement 17, \( \frac{\text{b}}{\text{c} \text{a} \text{b} \text{c}} \)), but the en forms of both are adjectives, as we have seen in \( \frac{\text{E} \text{b} \text{b} \text{E} \text{E}}{\text{c} \text{b} \text{c} \text{b} \text{c}} \), and neither of these subclasses of adjectives is, for any formal reason that we have yet established, more 'verbal' than the other.

(3) (p. 359) If we had analyzed verbs differently into three classes, wholly transitive, wholly intransitive, and those that are either transitive or intransitive (cf. \( \frac{\text{b}}{\text{c} \text{a} \text{b} \text{c}} \)), then (13) and (14) would in fact have only single \( \frac{\text{b}}{\text{p}} \) markers.

(4) (p. 360) Or, since some speakers require "him" with one of these interpretations in this position, consider "his drinking is something I don't approve of".

(5) (p. 361) If we analyze (20) in the counter-intuitive manner rejected in \( \frac{\text{E} \text{b} \text{b} \text{E} \text{b}}{\text{c} \text{b} \text{a} \text{b} \text{c}} \), and discussed below in \( \frac{\text{b}}{\text{c} \text{a} \text{b} \text{c}} \), the difficulty remains. We still have two analyses, now both intuitively incorrect, where only one should appear.

(6) (p. 363) It has been pointed out correctly (Trager and Smith, Fries) that intonation cannot serve as a grounds for this distinction. Different intonational patterns can appear fairly freely with the various sentence types.

Fries has carried out certain exploratory investigations of sentence types on the basis of the linguistic and situational context in which the sentence occurs. This approach goes beyond the basis that we have assumed. What we are trying to determine here is whether this distinction can be made by further theoretical constructions on the same limited basis as earlier theoretical constructions, that is, assuming no new kinds of data.
fn. 81. (p. 372a) This amounts to developing $P$ in such a way that $\text{Gr}(P)$ -- the set of products of restricted $\gamma$-derivations -- is actually included in $\mathcal{W}$ -- the set of grammatical strings of words. Cf. § 46.2-3, § 57. More generally, strings of words are taken to result from application of the mapping $\overline{\gamma}$ to $P$-markers, each of which contains one and only one string of $\overline{P}$ as its product. Cf.

fn. 14. (p. 380) i.e., such that $KK'km$ for some $Z'$, $(Z,K,Z')_T$, where $T$ is taken, as in the last paragraph of § 77.2, as a set of ordered triples.
(7) (p. 365) The necessity of a separate selectional statement for passives (statement 18, §66.2) serves as a hint that passives are somehow distinct, but we require a systematic explanation for this distinction.

(8) (p. 366) There are other criteria on the level $P$ for making some of the distinctions that we require. E.g., declaratives are formally distinct from non-declaratives in that they occur after that. This is an important distinguishing feature, and it would lend support to a separate analysis of declaratives, but it does not permit us to distinguish among the various types of non-declaratives, or among the subclasses of declaratives.

(9) (p. 375) Although $P$-markers are based on equivalence classes of derivations, this does not imply that $P$-markers (which are classes of strings) are equivalence classes of strings. In fact, every two $P$-markers overlap at least in the element Sentence $= P_0$, and they may overlap elsewhere.

(10) (p. 376) This generalization gives a further compelling reason why $T$ cannot be simply defined on strings, but only on strings and analyses of them.

(12) (p. 378) We will henceforth use brackets to denote sets. Thus the set containing the elements $X$, $Y$, $Z$ will be denoted $\{X, Y, Z\}$, and the set containing the sequences $(W_1, W_2, W_3)$ and $(W_4, W_5, W_6)$ as members (this is thus a two-membered set) will be denoted $\{(W_1, W_2, W_3), (W_4, W_5, W_6)\}$.

(14) (p. 378) Cf. §49.2 for the definition of "consistent analysis".

(15) (p. 379) Cf. §49.1 for the definition of "\(E_0\)".

(16) (p. 379) The notion 'domain of a transformation' is used ambiguously. It can refer either to the set of elements to which the transformation actually applies, or to the set from which this set is chosen. Henceforth we will use it only in the former sense, as in (C4).

(17) (p. 389) In fact almost all of it is utilized. It is possible for distinct triples to determine the same $T_2^2E_0$, but only
under very special circumstances which might perhaps be excluded for other independent reasons by a more complete axiomatization of $\phi$. Without going on to investigate this question, we may merely point out that this is one of many points where further study can lead to a neater theory. Cf. (C12), 478.3 and Th. 2, 482.2.

(16) (p. 394) This is not quite correct, cf. § 94, but the principle remains.

(17) (p. 395) Cf. below, § 80.1, for a more exact formulation.

(18) (p. 407) Later we will see that considerations specific to English syntactic structure require that the passive be derived from the active, and not vice versa. If we did not permit both possibilities in the theory, we would not be able to make the interesting discovery that almost without exception, conditions inherent in the language under analysis require that transformations be irreversible, i.e., that there is a preferred direction. In general it is wise to make the theory as non-specific as possible (consistent with the demands that motivated its construction), so that the observations that we make in applying the theory will not be mere tautologies about the manner in which we have chosen to describe data, but will be statements with empirical content.

(19) (p. 423) For the definition of this term, cf. Def. 2, § 12.

(20) (p. 433) The details of this analysis will be somewhat different in respects that are not now relevant.

(21) (p. 439) "only if" can be replaced by "if and only if" in this statement if we add the condition that $\pi(\delta) / \neq T$.

(22) (p. 440) There would be certain exceptions to this if the analysis were carried far enough. Cf. § 83.2. But even these exceptions appear to be of a very restricted type, and do not seem to falsify the concluding sentence of this paragraph.

(23) (p. 443) Thus $A_1$ contains $en^V_k$ (cf. statements 17, 23, § 67.2, and fn. 14, chap. VII). $A_2$ contains $en^V_T$ for all members of $V_T$ not in $V_k$, e.g., "accuse". We have "very tired", but not "very accused."
These reasons turn on the fact that "very" can precede "tired" in (88), but cannot precede "accused" in (87). The statement of the distribution of "very" would be complicated if (88) were dropped from the kernel. Furthermore there are positive gains from dropping (87), but not from dropping (88).

I.e., \( \Phi^P \) carries en\textsuperscript{accuse} into acc\textsuperscript{use} en, which by morphological rules (i.e., \( \Phi^M \)) becomes "accused". Cf. §4.61, 62.2, 62.3.

For the distinction between \( \text{Gr}(W) \) and \( \mathcal{M}^W \), cf.

Briefly, \( \text{Gr}(W) \) is the (finite) set of strings of words which are first-order grammatical in terms of syntactic category analysis. \( \mathcal{M}^W \) is the (infinite) set of grammatical strings of words, i.e., the set of \( W \)-markers. We have suggested, in §4.5, a way in which \( \text{Gr}(W) \) can be extended to \( \mathcal{M}^W \) by means of the descriptive potential of the level \( P \). Below, in , we will suggest rather that the major source of this extension lies in the descriptive potential of the level \( T \). Recall that in terms of \( P \) and \( T \) we may not only extend \( \text{Gr}(W) \), but also drop certain parts of it, in forming \( \mathcal{M}^W \). Hence the statement in the text is not quite correct. We will omit this possibility and the attendant conceptual complications here for the sake of simplifying the exposition (though I do not wish to imply that there are no problems here).

We might drop the distinction between \( T_3 \) and \( T_4 \) by interpreting each grammatical transformation \( T \) as identical with the one-termed compound \( [T] \), thus incorporating \( T_3 \) into \( T_4 \).

Note that a string outside of \( \text{Gr}(P) \) may still have a \( P \)-marker, for \( P \)-markers are based on equivalence classes of \( J \)-derivations, which need not be restricted \( J \)-derivations.

This is important, because it means that some transformationally derived strings may have \( P \)-markers, i.e., they may be closer structurally to kernel strings than other derived strings.

The case of the passive, discussed above in §4.83.2, is one important instance of this.
(30) (p. 485) We must be careful to distinguish concatenation in 
P from concatenation in T. Within the parentheses we have concatenation in \( P \) (more generally, in \( S \)). The parenthesized terms are primes of \( T \), and concatenation between parenthesized expressions in (125) and (126) is concatenation in \( T \). Thus these parentheses cannot be dropped, since different concatenation operations (which perhaps should be differently symbolized) are involved.

(31) (p. 493) Note that \( T \)-markers will normally end with \( S_n = T_n \), although we have not given this as an axiom. If they fail to, then the result given by the \( T \)-marker in question will be a complex string containing \( \# \) rather than a string of words.

Note also (cf. § 35.4) that we assume throughout a fixed decomposition of \( \Phi^P \) into \([\Phi_{m+1}^P, \ldots, \Phi_1^P]\).

(32) (p. 494) Literally, based on the pre-images under \( \Phi^P \) of kernel strings.

(33) (p. 500) Cf. fn. 30, p. 485.
Chapter IX - Transformational Analysis of English

99.1. In the preceding chapter, we developed an abstract theory of transformational analysis and formulated a corresponding linguistic level \( T \). This development was motivated by a series of difficulties which arose when we tried to describe English sentence structure exhaustively in terms of the theoretical devices available on lower levels. Certain inadequacies of our earlier constructions were summarized in \( \S \, 72-74 \), and in \( \S \, 75 \) we pointed out that some notion of 'grammatical transformation' might provide means for resolving these not inconsiderable problems. We must now investigate this claim, applying these new conceptions of transformational analysis to English just as we applied the notions of the level \( F \) to English in chapter VII. Before proceeding with this empirical investigation, however, we will briefly summarize the central ideas and technical devices which have been developed in \( \S \, 77-87 \) and which will be taken for granted in the remainder of this chapter.

In chapter VI we showed how the phrase structure of a language be reconstructed from a grammar which has the form of a sequence of conversions \( \alpha \rightarrow \beta \), where \( \beta \) is formed from \( \alpha \) by replacing a single prime of \( \alpha \) by some string. By running through this sequence of conversions over and over, interpreting each as the instruction "rewrite \( \alpha \) as \( \beta \)", we can construct a derivation of a terminal string \( Z \) to which none of the conversions apply further. Thus \( Z \in F \), the set of lowest level strings of \( F \). We have since extended \( F \) to include also words and morphemes (thus 'embedding' \( W \) and \( M \) into \( F \)). This derivation of \( Z \) is a sequence of strings with the string Sentence as its first term and the string \( Z \) as its final term, and such that each term in the sequence is formed from the preceding term by the application of some conversion to this preceding term. The set of strings of \( F \) which were derived in this way we denoted "Gr(\( F \))". We noted that a string \( Z \) will in general have various derivations which are equivalent from the point of view of constituent structure. Given a set \( \{ D_1, D_2, \ldots \} \) of equivalent derivations of \( Z \), we define a "F-marker" of \( Z \) as the set of strings each of which appears as a term in one of the derivations \( D_1, D_2, \ldots \). Given a
P-marker of Z, we found that we were able to reconstruct completely the constituent structure of Z. We noted further that any set of strings containing only a P-marker of Z could be said to provide an 'interpretation' of Z, i.e., a certain constituent analysis of Z. The grammar of phrase structure for a given language is then completed by the statement of a mapping \( \Phi^P \) which converts every string Z of \( P \) which is the product of some derivation into a string of words.

We now propose to limit derivability on the level \( P \) to a certain subset of the grammatical strings in \( P \). That is, we no longer require that every grammatical string of words be the result of application of \( \Phi^P \) to some string of Gr(P), i.e., some string which is actually derived on the level \( P \) and to which, therefore, phrase structure is assigned directly. The set of strings which do result from Gr(P) by application of \( \Phi^P \) we call the kernel of the language, and we require that all other grammatical strings of words be derived ultimately from kernel strings (more correctly, from the strings in Gr(P) which underly kernel strings) by grammatical transformations.

Each grammatical transformation T operates on a string \( \{X\} \) with the constituent interpretation \( K \) (which may or may not be a P-marker) and converts it into a new string \( Z' \) with the derived interpretation \( K' \). \( Z' \) will be denoted \( T(Z,K) \), and \( K' \), which gives the constituent structure of \( Z' \), is called "the derived interpretation of \( T(Z,K) \) with respect to (wrt) \( Z,K,T \)". Both \( Z' \) \( = T(Z,K) \) and \( K' \) are uniquely determined for each \( Z,K \). Thus given a string \( Z \) with the constituent interpretation given by \( K \), we know unambiguously, for each transformation T, just what string results by application of \( T \) to \( (Z,K) \) and what is the constituent structure of this new string \( T(Z,K) \). Furthermore, each grammatical transformation T has fixed structural properties; for every pair \( (Z,K) \), the corresponding transform \( \{X\} \) \( (Z',K') \) differs from \( (Z,K) \) in a fixed way (if it differs at all).
for every pair \((Z, K)\) the corresponding transform \((Z', K')\) differs from \((Z, K)\) in a fixed way (if it differs at all).

Each \(T\) operates significantly only on a certain restricted set of pairs \((Z, K)\). This set is determined by a restricting class \(Q\) which is associated with \(T\). \(Q\) is a set of sequences \((W_1, \ldots, W_r)\) (with \(r\) fixed for \(Q\)), where each \(W_i\) is a string in \(P\).

Suppose that \((W_1, \ldots, W_r)\) is a member of \(Q\). Then \(T\) applies to \((Z, K)\) significantly in case \(Z\) is analyzed by \(K\) into a \(W_1\) followed by a \(W_2\) . . . followed by a \(W_r\). For example, suppose one of the sequences in \(Q\) is \((T, A, N, saw, NP)\). Then the associated transformation will apply significantly to a string \(Z\) with the interpretation \(K\) if \(Z\) is analyzed by \(K\) into an article \((T)\) followed by an adjective \((A)\), followed by a noun \((N)\), followed by \(saw\), followed by a noun phrase. E.g., it will apply to \(Z\) = a single witness saw the accident, with its appropriate \(P\)-marker \(K\). Note that a transformation may apply to a string with one interpretation, but not with another; i.e., it may apply to \((Z, K_1)\) but not \((Z, K_2)\).

If \(Z\) is not analyzed by \(K\) into some sequence of terms which \(\alpha\) corresponds to a member of the restricting class \(Q\) of \(T\), then \(T\) converts \((Z, K)\) into \((Z, K)\) itself. More accurately, it converts it into \((Z, K')\), where \(K'\) is that part of \(K\) relevant to determining the original constituent structure of \(Z\). In particular, the transformation with the restricting class \(Q\) whose sole member is \((U)\) is an identity transformation that converts every string into itself, leaving constituent structure unchanged.

To characterize a transformation \(T\) completely we must determine the pair \((Z, K)\) to which it applies significantly and (by stating the associated restricting class \(Q\)) and we must state the fixed structural change that \(T\) effects on each \((Z, K)\).
To characterize a transformation $T$ completely we must determine the class of pairs $(z, k)$ to which it applies significantly (by stating the constitution of the associated restricting class $Q$) and we must state the fixed structural change that $T$ effects on each $(z, k)$ to which it significantly applies. The latter information is provided by an elementary transformation $t$ associated with $T$. The elementary transformations that interest us are each limited in significant application to $\mathcal{I}$ sequences of strings $n$ terms in length, where $n$ is fixed for $t$. $t$ will have no effect on sequences of terms of length different from $n$. Given a sequence of strings $(y_1, \ldots, y_n)$, $t$ converts $y_1$ into $w_1$, $y_2$ into $w_2$, $\ldots$, $w_n$ into $w_n$, where the form of $w_i$ depends not only on $y_i$ but also, perhaps, on $y_1, \ldots, y_n$. We express this by writing

$$t(y_1, \ldots, y_n; y_1, \ldots, y_n) = w_i$$

for each $i$. Thus $t$ essentially converts the string $y_1 - y_2 - \ldots - y_{n-1} - y_n$ into $w_1 - w_2 - \ldots - w_{n-1} - w_n$. We express this more precisely by associating with $t$ a derived transformation $t^*$ such that

$$t^*(y_1, \ldots, y_n) = w_1 \ldots w_n$$

For each $i$, the form of $w_i$ has a fixed relation to the form of $y_1, (y_1, \ldots, y_n)$. That is, if $$(y_1, \ldots, y_n; y_1, \ldots, y_n) = w_i$$

and

$$t(y_1, \ldots, y_n; \bar{y}_1, \ldots, \bar{y}_n) = \bar{w}_i$$

then $w_1 - \ldots - w_n$ differs from $y_1 - \ldots - y_n$ in the same way as $\bar{w}_1 - \ldots - \bar{w}_n$ differs from $\bar{y}_1 - \ldots - \bar{y}_n$.

A grammatical transformation $T$ is completely determined by its restricting class $Q$ and its underlying elementary transformation $t$. $Q$ tells us to which strings $T$ applies, and $t$ tells us what effect it has on these strings. We can state $Q$ as a finite set of finite sequences; we discuss the method of describing $t$ directly below. The notions of restricting class and elementary transformation are more carefully developed in Ch. 2. Very frequently
we find that there are many transformations all with the same underlying
elementary transformation \( t \), and with restricting classes all of which are
characterized by some fixed condition \( C \). We then say that the pair \((C, t)\)
determines a family \( F \) of transformations \( T_1, T_2, \ldots \), where each \( T_i \) in \( F \) is
determined by a pair \((Q_i, t)\), where \( Q_i \) meets the condition \( C \). This generalization,
given more carefully in \( \text{IX} \), leads to considerable simplification of the
grammatical statement of the level of transformations.

In \( \text{IX} \), we study certain particularly interesting elementary
transformations, including, in particular, all those which will appear in this
chapter. We define a deformation \( \mathcal{G} \) as an elementary transformation whose
only effect on a string \( Y_1 \cdots Y_n \) is to delete terms or to add some constant
string to certain terms. Thus for each \( i \), either

\[
\mathcal{G} \left( Y_1, \ldots, Y_i, \ldots, Y_n \right) = U,
\]

in case \( \mathcal{G} \) deletes the \( i \)th term of \( Y_1 \cdots Y_n \), or

\[
\mathcal{G} \left( Y_1, \ldots, Y_i, \ldots, Y_n \right) = W_i \cdot Y_i \cdot \overline{W}_i,
\]

where \( W_i \) or \( \overline{W}_i \) (or both) may be \( U \) or may be certain constant strings. If we
know \( W_i \) for each \( i = 1, \ldots, n \), then we know everything about \( \mathcal{G} \). Suppose that we
have an element \( \mathcal{V} \) such that if \( W_i = \mathcal{V} \) then \( \overline{W}_i = U \) and \( W_i \cdot Y_i \cdot \overline{W}_i = U \). Thus \( \mathcal{V} \) is a kind of
repeating element. We can thus characterize \( \mathcal{G} \) completely by a sequence

\[
\mathcal{A} = (W_1, \overline{W}_1, \ldots, W_n, \overline{W}_n),
\]

where \( \overline{W}_i = U \) if \( W_i = \mathcal{V} \). In this case, \( \mathcal{G} \) will convert any string \( Y_1 \cdots Y_n \) into the
string \( W_1 \cdot Y_1 \cdot \overline{W}_1 \cdot \ldots \cdot W_n \cdot Y_n \cdot \overline{W}_n \), where if \( W_i = \mathcal{V} \), then \( \overline{W}_i = U \).

For example, let the sequence \( \mathcal{A} = (U, \text{in the window}, U, U, \mathcal{V}, U) \). Then applied to
any string \( Y_1 - Y_2 - Y_3 \), the corresponding \( \mathcal{G} \) yields the transform \( U \cdot Y_1 \cdot \text{in the window} - Y_2 \cdot Y_3 \).

We define a permutation \( \mathcal{P} \) as an elementary transformation whose only
effect on a string $Y_1 \ldots Y_n$ is to permute the terms of this string. Thus

$$ \tau(Y_1, \ldots, Y_n) = Y_{a_1} \ldots Y_{a_n} $$

where $(a_1, \ldots, a_n)$ is a permutation of the integers $(1, \ldots, n)$; and for each $i$,

$$ \tau(Y_1, \ldots, Y_i; Y_1, \ldots, Y_n) = Y_{a_1} \ldots Y_{a_i} $$

is thus completely characterized by the sequence of integers $E = (a_1, \ldots, a_n)$. For example, let $E = (2, 1, 3)$. Then $\tau$ converts $Y_1 Y_2 Y_3$ into $Y_2 Y_1 Y_3$. E.g., it converts \text{happyunhappyimpossible} he - can - come into \text{can - he - come.}


We define an $\eta$-transformation $\eta$ as an elementary transformation whose sole effect is to 'attach' certain terms to other terms. For example, the elementary transformation that converts $Y_1 Y_2 Y_3$ into $Y_1 Y_2 Y_3$ [prefixing $Y_1$ to $Y_2$] would be an $\eta$-transformation. This particular $\eta$-transformation can be characterized in an obvious way by the sequence of integers $(0, 0, 1, 0, 0, 0)$, indicating that it prefixes nothing to the first term, $\tau$ suffixes nothing to the first term, prefixes the first term to the second term, suffixes nothing to the second term, prefixes nothing to the third term, suffixes nothing to the third term. In general, an $\eta$-transformation $\eta$ is characterized by a sequence of non-negative integers $E = (a_1, \ldots, a_{2n})$, where each $a_i$ is less than or equal to $n$. The $\eta$-transformation $\eta$ converts $Y_1 \ldots Y_n$ into $Y_{a_1} Y_{a_2} Y_{a_3} \ldots Y_{a_{2n-1}} Y_n Y_{2n}$, where $Y_0 = \eta$.

In this case,

$$ \eta(Y_1, \ldots, Y_i; Y_1, \ldots, Y_n) = Y_{a_{2i-1}} Y_{a_i} Y_{a_{2n}} $$

For example, let $E = (0, 0, 1, 0, 0, 0)$, as above. Then the associated carries $Y_1 Y_2 Y_3$ into $Y_1 Y_2 Y_3$. E.g., it carries

\text{happyunhappy} that he was unhappy - it - was quite obvious

into

\text{that he was unhappy - that}
For example, let $x = (0, 3, 0, 0, 0, 0, 0)$. Then the associated \( y \) carries

\[ y_1 - y_2 - y_3 \]

into \( y_1' - y_2' - y_3' \). E.g., it carries (10) into (11):

\[
(10) \quad \text{the man was coming this way in the dark suit}
\]

\[
(11) \quad \text{the man in the dark suit was coming this way in the dark suit.}
\]

We can form new elementary transformations by composing from given elementary transformations. Suppose \( t_1 \) and \( t_2 \) are elementary transformations. Then we define \( t_2(t_1) \) as the elementary transformation whose effect on \( Y_1 \ldots Y_n \) is that obtained by application of \( t_1 \) followed by application of \( t_2 \) to the result. More carefully, suppose that

\[
(12) \quad t_1(Y_1, \ldots, Y_n) = W_1, \ldots, W_n \quad [\text{where } W_i = t_1(Y_1, \ldots, Y_n; I_1, \ldots, I_n)]
\]

\[
t_2(W_1, \ldots, W_n) = Z_1, \ldots, Z_n \quad [\text{where } Z_i = t_2(W_1, \ldots, W_n; I_1, \ldots, I_n)].
\]

Then \( t_2(t_1) \) carries \( Y_1 \ldots Y_n \) into \( Z_1 \ldots Z_n \). I.e.,

\[
(13) \quad t_2(t_1)(Y_1, \ldots, Y_n) = Z_1, \ldots, Z_n
\]

\[
[t_2(t_1)](Y_1, \ldots, Y_n; I_1, \ldots, I_n) = Z_1.
\]

For example, let \( \delta \) be the deformation which carries \( Y_1 - Y_2 - Y_3 \) into \( Y_1 - \text{en} - \text{by} - Y_3 \) [where \( \text{en} \) is the morpheme which, e.g., added to \text{take} gives \text{taken} -- cf. ]. Thus \( \delta \) is defined by the sequence

\[
(14) \quad A = (U, U, \text{is}, \text{en}, \text{by}, U).
\]

Let \( \bar{\Pi} \) be the permutation that carries \( Y_1 - Y_2 - Y_3 \) into \( Y_3 - Y_2 - Y_1 \). Thus \( \bar{\Pi} \) is defined by the sequence

\[
(15) \quad B = (3, 2, 1).
\]

Then \( \delta(\bar{\Pi}) \) is the elementary transformation that carries \( Y_1 - Y_2 - Y_3 \) into \( Y_3 - Y_2 - Y_1 \).
$Y_1$, $Y_2$, $Y_3$ and then into $Y_3 - 1e^t Y_2 - e\text{en-by} Y_1$; it carries the students take the book (16) into (17):

(16) the students take the book

(17) the book is taken by the students.

By compounding the deformation which deletes the $i^{th}$ term with the $\eta$-transformation that attaches the $j^{th}$ term to the $i^{th}$ term (new deleted), we can achieve the effect of a substitution of the $j^{th}$ term for the $i^{th}$ term.

More generally, in $\gamma 22$ we defined a substitution as an elementary transformation $t_\gamma$ whose effect is to delete terms or to substitute certain terms for other terms (thus $t_\gamma$ is a compound of deformations and $\eta$-transformations). Combining the method for characterizing deformations and $\eta$-transformations, we characterize $t_\gamma$ by a sequence $D=(a_1, \ldots, a_2)$, where $a_{2i-1}$ may be $\gamma$, and otherwise, $a_{2i}$ is a non-negative integer (but $a_{2i}=0$ if $a_{2i-1}=0$). Then

$$t_\gamma(Y_1, \ldots, Y_i; Y_{i+1}, \ldots, Y_n) = W_1,$$

where $W_1 = Y_2^{a_2_1}$ if $a_{2i-1} = \gamma$, $W_1 = Y_2^{a_{2i-1}} Y_1^{a_{2i}}$ if $a_{2i-1} \neq \gamma$ (and $Y_0 = Y$)

For example, let $D=(\gamma, 0, \gamma, 1, 0, 0)$. Then the corresponding $t_\gamma$ carries $Y_1 - Y_2 - Y_3$ into $U - Y_1 - Y_3 = Y_1 - Y_3$, since

$$t_\gamma(Y_1, Y_2, Y_3) = U Y_1 Y_0 = U$$

$$t_\gamma(Y_1, Y_2, Y_3) = (\gamma Y_2, Y_1) = Y_1$$

$$t_\gamma(Y_1, Y_2, Y_3) = Y_0 Y_3 Y_0 = U Y_3 U = Y_3.$$

Thus $t_\gamma$ has the same effect as the deformation that carries $Y_1 - Y_2 - Y_3$ into $Y_1 - U - Y_3 = Y_1 - Y_3$. However, we will see directly that the derived constituent structure of $Y_1 - Y_3$ is different in this case.

These elementary transformations and their compositions are the only ones that will concern us below. Th. 4 (81.2) and Th. 5 (82.3), and Th. 8, 2 (82.5) give certain grounds for the assumption that
deformations, permutations, and \( \mathcal{S} \)-transformations, and their compounds may in general be adequate for transformational analysis.

In \( \mathcal{S}2.3 \) we studied the problem of assigning defined constituent structure to transforms. We note first of all that if \( t \) carries \( Y_1 \ldots Y_n \) into \( W_1 \ldots W_n \) and \( t \) is a deformation, permutation, or \( \mathcal{S} \)-transformation, there is an obvious sense in which each \( W_i \) has a certain \( Y_i \) as its root. If \( t \) is a deformation or \( \mathcal{S} \)-transformation, we say that \( Y_i \) is the root of \( W_i \). If \( t \) is a permutation which carries the \( i \)th term of \( Y_1 \ldots Y_n \) into the \( j \)th term of \( W_1 \ldots W_n \) (in this case, \( W_i = Y_j \)), then we say that the \( i \)th term of \( Y_1 \ldots Y_n \) is the root of the \( j \)th term of \( W_1 \ldots W_n \) [in this case, if \( i \rightarrow (a_1, \ldots, a_n) \), then \( a_i = j \)]. The notion of root carries over under compounding. Thus if \( t \) is the \( \mathcal{S} \)-transformation discussed above that carries (16) into (17), then the students in (16) is the root of by the students in (17), take in (16) is the root of is taken in (17), and the book in (16) is the root of the book in (17). Note that in the case of a substitution \( t_s \), the root of \( W_i \) is \( Y_i \), where \( t_s \) carries \( Y_1 \ldots Y_n \) into \( W_1 \ldots W_n \), since substitutions are compounded of deformations and \( \mathcal{S} \)-transformations. Hence if a transformation replaces \( X \) by \( Y \), then \( X \) is the root of \( Y \). We define "root" in such a way that the root of each term of the transform is uniquely determined as a term of the transformed string.

We then went on to lay down various conditions on derived constituent structure. Suppose that a transformation \( T \) carries \( Y_1 \ldots Y_n \) into \( W_1 \ldots W_n \).

\( (i) \) Suppose that within the level \( F \) it is the case that \( W_i \) is an \( S \), for some prime \( S \) (i.e., \( S(W_i) \)). Then we say that \( W_i \) is an \( S \) in \( W_1 \ldots W_n \). E.g., even when passives are deleted from the kernel, the kernel grammar will tell us that by the NP is a PP. Hence by the students in (17) is a Propositional Phrase.

\( (ii) \) Suppose that \( Y_1 \ldots Y_{i+1} \) contains all the roots of \( W_1 \ldots W_i \) (whether or not in the same order), and \( Y_1 \ldots Y_{i+1} \) is an \( S \).
for some prime $S$. Then $W_{1}^\ldots W_{n+1}$ is also an $S$. E.g., if $T$ carries
\[Z'=I\text{-}called\text{-}up\text{-}my\text{'friend'}\text{ into }Z'=I\text{-}called\text{-}my\text{'friend'}\text{-}up,\text{ and if called\text{'up'}\text{'my'}\text{'friend'}\text{ is a VP in }Z, \text{ then called\text{'my'}\text{'friend'}\text{'up}} \text{ is a VP in }Z'.\text{ Cf. 781.5 for several other important cases.}

(iii) If part (or all) of $W_{1}$ is carried over unaltered from its root $Y_{1}$,
then the internal constituent structure of this part of $W_{1}$ is unchanged.
E.g., the 'book and the 'students are NP's in (17) just as they are in the
kernel string of (16).

(iv) If $T$ is a substitution replacing $Y_{1}$ by $Y_{1}$, then $Y_{1}$ keeps its original
internal constituent structure. Thus if $T$ is based on
the substitution $t_{a}$ dimmamdimnix in (19), carrying, e.g.,
that 'he was 'unhappy -- it -- was 'quite' obvious into
U -- that 'he was 'unhappy -- was 'quite' obvious, then the internal structure of
that 'he was 'unhappy in the transform is that of the same phrase
in the transformed string. Note that in this case the external
constituent structure of that 'he was 'unhappy in the transform is that
of it, the term for which it was substituted, by virtue of (ii).
Thus that 'he was 'unhappy is a Noun Phrase in the transform (as is its
root it), while he was 'unhappy is a Sentence, he is a Noun Phrase, etc.,
as they are in the transformed original string to which $T$ was applied.

(v) If $X$ is part of $T(Z,K)$, then $X$ is an $X$.

(vi) If $X_{1}$ is an $S_{1}$ in $T(Z,K)$ and $X_{2}$ is an $S_{2}$ in $T(Z,K)$, then $X_{1}X_{2}$ is an
$S_{1}S_{2}$ in $T(Z,K)$.

We then (in §83.8) defined the derived interpretation $\mathfrak{M}K'$ as the set of
strings which gives just that constituent interpretation carried over for
$T(Z,K)$ from $Z,K$.

Note that since a transformation $T$ carries a string $Z$ with
an interpretation $K$ into a string $X'Z'$ with the unique derived
interpretation $K'$, we can compound transformations freely. We
define \([T_1, T_2]\) as the transformation which, when applied to \((Z, K)\) gives \(T_2(Z', K')\), where \(Z' = T_1(Z, K)\) and \(K'\) is the derived interpretation of \(T_1(Z, K)\) wrt \(Z, K, T_1\). Thus \([T_1, T_2](Z, K)\) is the string which is derived by first applying \(T_1\) to \((Z, K)\) and then applying \(T_2\) to the result; i.e.,

\([T_1, T_2](Z, K) = T_2(T_1(Z, K), K')\), where \(K'\) is as above. Since \([T_1, T_2]\) also gives a unique derived interpretation \(K''\), we can go on to define \([T_1, T_2, T_3]\), and in general, \([T_1, \ldots, T_n]\). Thus \([T_1, \ldots, T_n](Z, K)\) is the string which is derived by applying \(T_1\) to \((Z, K)\), applying \(T_2\) to the result \((Z', K')\), \ldots, and finally applying \(T_n\) to the result \((Z^{n-1}, K^{n-1})\).
define \([T_1, T_2]\) as the transformation which, when applied to \(Z'K'\) \((Z, K)\) gives \(T_2(T_1(Z, K), K')\), where \(K'\) is the derived interpretation of \(T_1(Z, K)\) wrt \(Z, K, T_1\). Thus \([T_1, T_2](Z, K)\) is determined by first applying \(T_1\) to \((Z, K)\) and then applying \(T_2\) to the result. Since \([T_2, T_a]\) also gives a unique derived interpretation \(K''\), we can go on to define \([T_1, T_2, T_3]\), and in general, 
\([T_1, \ldots, T_n]\). Thus \([T_1, \ldots, T_n](Z, K)\) is determined by applying \(T_1\) to \((Z, K)\), applying \(T_2\) to the result \((Z', K')\), \ldots, applying \(T_n\) to the result \((Z^{n-1}, K^{n-1})\).

In \(\psi\) we made use of the notions of transformational analysis to clear up the unresolved formal problems connected with the level of phrase structure. In developing \(\psi\), we were able to determine the constituent structure of strings in Gr(P) by deriving them from the string Sentence by \(\Phi\) the use of a sequence of instruction formalas \(\Lambda \rightarrow \gamma \beta\); but we found that the mapping \(\Phi^P\) which converts strings of Gr(P) into strings of words could not be characterized in this way. The fundamental difficulty is that to apply \(\Phi^P\) correctly to a string \(Z\) of \(\text{Sentence}\) of \(\text{Gr}(P)\) it is necessary in general to know the derivation (i.e., the constituent structure) of \(Z\), not just the \(\text{Sentence}\) shape of \(Z\) itself. But an instruction formula of the form \(\Lambda \rightarrow \gamma \beta\) (i.e., "rewrite \(\Lambda\) as \(\beta\") applies or fails to apply to \(Z\) irrespective of its 'history of derivation.' Since transformations apply to a string with a constituent structure, \(\text{Sentence}\) holds this limitation no longer \(\text{Sentence}\) if we interpret \(\Phi^P\) as a transformation which converts a string of Gr(P) with its \(P\)-marker \(K\) into a string \(Z'\) of words with the derived interpretation \(K'\). Though a string of words is not derived directly from \(\text{Sentence}\), we see that it still has a constituent structure by virtue of the fact that transformations impose a derived interpretation on the strings which they yield.
In constructing the grammar of English in \( \phi \) we found it extremely useful to break up \( \Phi \) into successive components \( \Phi_1, \ldots, \Phi_m, \Phi_{m+1} \) which are applied successively to a string of \( \text{Gr}(P) \). The result of applying \( \Phi_1, \ldots, \Phi_m \) is a string of morphemes, and \( \Phi_{m+1} \) converts this into a string of words by inserting word boundaries in the proper places. Let us relabel \( \Phi_2 \), calling it \( \Phi_1^{(m+1)} \).

Transformational analysis \( \Phi \) provides a clear interpretation for this fragmentation of \( \Phi \). We develop each \( \Phi_1 \) as a transformation, and we define \( \Phi \) as the compound transformation

\[ \Phi = [\Phi_1, \ldots, \Phi_m, \Phi_{m+1}] \]

It is very important to observe that this development allows us great freedom in determining the point of application of a grammatical transformation \( T \). It is not necessary that \( T \) be limited in application to either a string of \( \text{Gr}(P) \) or a string of words. Since each \( \Phi_1 \) is itself a transformation, \( T \) can apply anywhere in the sequence \( \Phi_1, \ldots, \Phi_m \). Suppose that we define \( \Psi_1 \) as \( \Phi_1 \), \( \Psi_2 \) as \( [\Phi_1, \Phi_2], \ldots, \Psi_{m+1} \) as \( [\Phi_1, \ldots, \Phi_{m+1}] \). Thus in general, for \( m \geq 1 \leq m+1 \),

\[ \Psi_i = [\Phi_1, \ldots, \Phi_i] \]

In particular, \( \Psi_{m+1} = \Phi \). Then a transformation \( T \) can apply after \( \Psi_i \) for any \( i \).

In this case, \( T \) will be followed by the remainder \( \Psi \) of \( \Phi \) so that the end result is a string of words. Let us define

\[ \Psi_1^i = [\Phi_1, \ldots, \Phi_{i+1}, \Phi_{i+1}] \]

for each \( 1 \leq i \leq m \). Thus \( \Phi = [\Psi_i, \Psi_1^i] \), for each \( 1 \leq m \). For each specific
for each $1 \leq m$. Thus $\Psi^p = [\psi_1^m, \psi_{m+1}^m]$, for each $1 \leq m$. For each specific transformation $T$, we will now select an $i$ such that $T$ applies after $\psi_i$ and before $\psi_{i+1}$. We will see below that specific transformations in English differ widely in their point of application within $\Psi^p$.

The analysis of $\Psi^p$ is carried out more fully in §89.

Up to this point in the development of transformational analysis we seem to have adopted a point of view radically different than that of our earlier discussion of linguistic structure. In §§85, 87, however, we attempted to show that in a very natural way, we can interpret the theory of transformational analysis as a new linguistic level having the same status and fundamental properties as other levels. Just as an utterance can be represented by a sequence of phonemes or a sequence of words, it can be represented by the sequence of operations by which it is derived from a kernel sentence.

To characterize transformational analysis as a linguistic level $T$, we must present a set of primes of $T$, an interpretation for the concatenation operation $\otimes$, a set $\Psi_T$ of $T$-markers, and a mapping $\Phi_T$ that assigns $T$-markers to utterances as their representations or "spellings" on the level $T$.

Among the primes of $T$ will be included, first of all, all transformations $T$ and compound transformations $[T, \ldots, T]$. Thus in particular, $\Psi^p$ will and its components (including $\psi_i$ and $\psi_{i+1}$, for each $i$) are primes of the level $T$.

We only interpret certain strings of these primes as possessing significance.

We extend the set $\Psi$ of 'lowest level' elements of $\Psi$ to include also all strings of words and morphemes, and mixed strings containing such elements indiscriminately. Then any string $Z$ of $\Psi$, in this extended sense, is also a prime of $T$. In particular, then, the strings
of $\text{Gr}(P)$ and of $\mathcal{W}$ (grammatical strings of words) are primes of $T$.

Finally, we include among the primes of $T$ any set $K$ whose members are strings in $\overline{P}$ or in $P$ in this extended sense. Thus, in particular, all $\overline{P}$-markers are primes of $T$.

We only interpret certain strings of these primes as possessing significance. In particular, any string $\overline{Z}X\overline{X}X\overline{X}T$ (where $\overline{Z}$ is a string of $\overline{P}$, $K$ is a set of strings, and $T$ is a transformation — $Z, K, T$ thus are primes of $\overline{T}$) is interpreted as $T$ applied to $\overline{Z}$ with the interpretation $K$. More precisely, we define the mapping $\overline{P}^T$ in such a way that

\[(24) \overline{P}^T(\overline{Z}^K\overline{T}) = T(\overline{Z}, K).\]

That is, $\overline{P}^T$ applied to the string $\overline{Z}^K\overline{T}$ gives the string $T(\overline{Z}, K)$, just as $\overline{P}^P$ applied to a certain set of strings (a $\overline{P}$-marker) gives a string of words, and $\overline{P}^P$ applied to a string of phonemes gives a string of phones.

More generally,

\[(25) \overline{P}^T(\overline{Z}^K\overline{T}_1 \ldots \overline{T}_n) = \left[\overline{T}_1 \ldots \overline{T}_n \overline{P}^P \left[\overline{I}_1 \ldots \overline{I}_n\right]\right](\overline{Z}, K),\]

also a string of $\overline{P}$ (in fact, a string of words, if the compound transformation $[\overline{T}_1 \ldots \overline{T}_n]$ includes $\overline{P}$), as we have seen above. Thus $\overline{T}_1 \ldots \overline{T}_n$ applied to a string in $\overline{T}$ is interpreted as the compound transformation $\overline{P}^P \left[\overline{I}_1 \ldots \overline{I}_n\right]$.

Certain of the strings $\overline{Z}^K\overline{T}_1 \ldots \overline{T}_n$ for which we have provided an interpretation will be $\overline{T}$-markers of grammatical strings of words. We place certain additional conditions on such strings. For one thing, we must be able to extract $\overline{P}$ in a unique manner from any $\overline{T}$-marker. Thus such strings in $\overline{T}$ as

\[(26) \overline{Z}^X \overline{K} T_1 \overline{P} T_2 \overline{I}_4 T_3\]
(26) $Z^X \psi_3^T \psi_4^T$

$$Z^X T_1^T \psi_3^T T_4^T \psi_4^T$$  \[ \text{[where } T_1^T, T_2^T, T_3^T \text{ are transformations, and } \psi_3, \psi_4, \psi_1' \text{ are as in (22), (23)]} \]

e etc., qualify for admission into the set $\mu^T$ of $T$-markers, since the first contains

the sequence $\psi_3^T$ and the second contains the sequence $\psi_3^T, T_4^T, \overline{\psi}_4^T$, and both $[\psi_3^T, \psi_4^T]$ and $[\psi_3^T, T_4^T, \overline{\psi}_4^T]$ are equivalent to $\overline{\psi}_4^T$, as we have seen above. On the other hand, such strings as

(27) $Z^X T$

$$Z^X \overline{\psi}_4^T T$$

$$Z^X T_1^T \psi_3^T T_4^T \psi_4^T$$ \[ \text{[where } \psi_3^T \neq \overline{\psi}_4^T \]}

e etc., are not in general members of the set $\mu^T$ of $T$-markers.

We restate this condition on $T$-markers in the following terms: each $T$-marker must contain a unique $\overline{\psi}_4^T$-skeleton. The reason for this requirement is evident, when we consider the role that $T$-markers play in the grammar. This requirement guarantees that non-kernel strings which are generated by $T$-markers will be properly mapped into strings of words. Note that the minimal $T$-marker will be

(28) $Z^X \psi_4^T$, where $Z$ is in $\text{Gr}(P)$ and $K$ is a $\overline{\psi}_4^T$-marker of $Z$. In this case (and only in this case), the string $\overline{\psi}_4^T(Z, K) = \overline{\psi}_4^T(Z^X \psi_4^T)$ belongs to the kernel.

In [27] we extended the class of potential $T$-markers to enable us to construct new strings from a set of already generated strings. Suppose that

(29) $Z_1$ is the string of $\text{Gr}(P)$ underlying "John was unhappy"

$$Z_2$$  \[ \text{"it was quite obvious"} \]

Then

...
(30) \( \Phi^P(Z_1, X_1) = \text{John was unhappy} \)
\( \Phi^P(Z_2, X_2) = \text{it was quite obvious} \)

where \( X_1 \) and \( X_2 \) are properly chosen \( P \)-markers. Let \( T \) that by the transformation that converts \( Z_1 \) into \( \hat{Z}_1 \). Let \( T \) be the transformation that \( \text{transf} \) converts (31) into (32).

(31) \( X_1 - X_2 - X_3 - X_4 \)
(32) \( U = U - X_1 - X_4 = X_1 - X_4 \)

Then we interpret the string

(33) \( Z_1 \hat{X}_1 \hat{\Phi} T \hat{\text{that}} Z_2 \hat{X}_2 \hat{\Phi} T \hat{X} \)

in the following series of steps.

(34) (1) Apply \( \hat{\Phi} \) to \( (Z_1, X_1) \) forming \( \text{John was unhappy} = Z_3 \) . Let \( X_3 \) be the derived interpretation of \( \hat{Z}_3 \).
(2) " \( \hat{T} \) that to \( (Z_3, X_3) \) forming \( \text{that John was unhappy} = Z_4 \).
(3) " \( \hat{\Phi} \) to \( (Z_2, X_2) \) forming \( \text{it was quite obvious} = Z_5 \).

(4) Form the complex string \( Z_1 \hat{X}_1 \hat{\Phi} Z_2 \hat{X}_2 \hat{\Phi} T \hat{\text{that}} Z_3 \hat{X}_3 \hat{\Phi} Z_4 \hat{X}_4 \hat{\Phi} Z_5 \) (\( \hat{\Phi} \) is a juncture element, i.e., a particular prime of \( \hat{P} \)).

(5) Apply \( \hat{T} \) to \( Z_1 \hat{X}_1 \hat{\Phi} Z_2 \hat{X}_2 \hat{\Phi} T \hat{\text{that}} Z_3 \hat{X}_3 \hat{\Phi} Z_4 \hat{X}_4 \hat{\Phi} Z_5 \) broken up into the segments:

\[ \text{that John was unhappy} - \hat{T} - \text{it was quite obvious}. \]
\[ X_1 - X_2 - X_3 - X_4 \]

(6) By the definition of \( \hat{T} \) (see (31-2), above), the result of step (5) is:

\[ \text{that John was unhappy was quite obvious}. \]

This last string is the result of applying \( \hat{\Phi} \) to \( \text{that was unhappy} \) (33), which is taken as the \( T \)-marker of this sentence. This example is studied more carefully in 87.6, where the constituent structure of the resulting string is discussed.
In general, we attach significance within the level \( T \) to any string

\[(35) \, S_1 \wedge S_2 \wedge S_3 \cdots \wedge S_m \]

where \( S_1 = T_1^z \wedge K_1 \wedge T_1^z \) and each \( S_i \) is either \( T_i^z \) or \( \# T_i^z \). Thus (33) is 'broken up' into the segments

\[(36) \, T_1^z \wedge P \wedge T_{that} \wedge Z_2 \wedge K_2 \wedge P \wedge T_{X^z}. \]

\[S_1 \wedge S_2 \wedge S_3 \wedge S_4 \]

We can now consider \( \cdots \wedge T \wedge P \wedge \cdots \wedge X \) state, by induction, how \( \Phi^T \) maps any significant string of \( \# T \) into a string of \( F \). Suppose that (35) is a string in \( T \).

**Case I**. Let \( m = 1 \). Then \( \Phi^T(S_1) = \Phi^T(T_1^z \wedge K_1 \wedge T_1) = T_1^z(K, F) \)

**Case II**. Suppose that \( \Phi^T(S_1 \wedge S_2 \wedge \cdots \wedge S_m) = Z \), with the derived interpretation \( K \).

**Case IIa**. Suppose that \( S_m = T_m^z \).

Then \( \Phi^T(S_1 \wedge S_2 \wedge \cdots \wedge S_m) = T_m^z(Z, F) \).

**Case IIb**. Suppose that \( S_m = Z \wedge K \wedge T_m^z \).

Then \( \Phi^T(S_1 \wedge S_2 \wedge \cdots \wedge S_m) = Z \# T_m^z(Z, F) \).

Note that cases I and IIa are treated just as in (24), (25), above, and that case IIb is the only innovation. Furthermore, the \( \# T \)-markers of the form (26) (or as in (27)) are special cases of (35), with each \( \Phi^T \) for \( m \geq 1 \).

This inductive characterization enables us to apply \( \Phi^T \) to any string of the form (35). Notice that each such string can be broken up into

\[(37) \, Z_1^z \wedge K_1 \wedge \cdots \wedge Z_2^z \wedge K_2 \wedge \cdots \wedge Z_m^z \wedge K_m \wedge \cdots \]

where \( \cdots \) contains only transformations. The \( Z_i \)'s in this case are the strings which underly the resulting transform. Thus in the example (34) given above the strings which underly the sentence "that John was unhappy was quite obvious" are \# \( Z_1 \) (which underlies "John was unhappy") and \( Z_2 \) (which underlies "it was quite obvious"). This generalization of \( T \)-markers makes it possible to develop sentences of a high degree of complexity from very simple parts.
We have pointed out that every kernel string has a $T$-marker (28). We now require that the set $\mu^T$ of $T$-markers be extensive enough to allow us to generate all non-kernel strings as well. A finite number of $T$-markers will suffice for this purpose, since we can choose already formed transforms as $Z_1, Z_2$, etc. in the $T$-markers $\Phi \Phi \Phi \Phi\Phi \Phi\ (35)$ (including (26), (28), etc. as special cases). Thus every grammatical string of words is represented one way or another on the level $T^i$.

We will see that representation on the level $T$ has the same general character as representation on lower levels. In particular, many cases of ambiguity will be marked by dual representation on this level, and many sentences that intuitively seem 'similar in structure' (are 'understood similarly', etc.) will be shown to have $\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi•
Chapter IX  
Transformational Analysis of English

§ 2.1 In the preceding chapter, we developed an abstract theory of transformational analysis and formulated a corresponding linguistic level L. We now turn to the problem of applying these new conceptions to the description of English syntax, just as we applied the notions of the level P to English in chapter VII. As was pointed out in §65.5, the problem we now face is that of determining which strings to assign to the kernel, and which to derive transformationally from kernel strings.

The criterion that we employ in making this choice is simplicity. We investigate, for each particular set of strings, the effect on the grammar of assigning this set to the kernel, or, alternatively, excluding it and treating the strings as transforms.

In a broad sense, we refer by "simplicity" to the whole mass of formal requirements placed on the statement of grammars. In particular, we defined "simplicity" in chapter III as "maximal degree of generalization" (in a special and extended sense of "generalization"), measured by length under certain notational transformations designed to convert considerations relating to the similarity of statements into considerations of length. But as was noted in §2.1, this concept of simplicity can not be applied directly to determining the relative complexity of distinct levels. The elementary statements of one level may be intrinsically longer than the elementary statements of another. The list of phrases (including words) will be incomparably longer than the list of phonemes. We will not be willing to, say, double the number of phonemes, in order to drop some thirty phrases from the analysis. Thus while this
conception of simplicity (though itself undoubtedly oversimplified and incomplete) is fairly successful within a level, in order to apply it in considerations affecting two levels it is necessary to weight the symbols of each level in a certain way, so that increasing the complexity of one level may considerably outweigh an identical reduction on another level. This weighting must of course be established in the general theory; otherwise all sorts of ad hoc solutions will be possible in particular cases.

How is this weighting to be determined? Clearly, the basis for this decision is the same as the basis for every other construction made in the general theory. Our purpose is to construct an integrated and systematic theory, which, when applied rigorously to linguistic material, gives the correct analysis for the cases where intuition (or experiment, under more desirable circumstances) makes a clear decision. We assign the weighting so that this will turn out to be the case, just as we define "word" or "phrase" so that, in application, this will turn out to be the case. In practice, this means that we must investigate the characteristics of correct solutions in great detail, and then formulate abstractly conditions that lead to these correct solutions. There are several levels on which theory may be successful. On the one hand, it is successful if such abstract and effective formulation is possible. But for real success, of course, this formulation must be motivated in some manner which is not too easy to characterize, and is often disguised by the word "natural".

In our present context, we meet a specific form of this problem. A deformation of $n$ terms will have $2n$ elements in
its defining sequence (cf. Def. 16, §80). Hence the definition of what is conceptually a very simple transformation may contain more symbols than the statement of an intricate and involved construction on the level $F$, with special restrictions and special cases. If the transformation solution is preferable, on extra-systematic grounds, we must so weight statements on the two levels that it is in fact simpler (or equivalently, we must define reductions on the levels $F$ and $T$ (cf. §51) in such a way as to make it come out simpler). As long as this is done in the general theory, this is no more ad hoc than is our attempt to define "word" so that "writer" and not "ter" is a word in English. What is important is that there be a sharp boundary in terms of total simplicity between the cases where we choose a transformational solution and the cases where we reject it. The only way that the validity of this approach can be determined is by attempting transformational analysis of actual language material. In the following pages, we apply the notions of the preceding chapter to English, and we will attempt to show that in the cases where transformational analysis does give revealing and intuitively satisfactory results, it is also the case that it leads to extensive simplification of the grammar, so that the transformational solution can be formally justified by this fact within linguistic theory.

In general, we introduce an element or a sentence form transformationally only when by so doing we manage to eliminate special restrictions from the grammar, and to incorporate many special cases into a single generalization. As a kind of standard of reference for the ensuing discussion, we might consider certain cases where transformational analysis would not be in place, because no such simplification results.
We might, for instance, drop the word "very" from the kernel and introduce it by a certain family of deformational transformations. If we did this, we would see that the kernel grammar would be simplified in no way other than by the fact that "very" is dropped from it, i.e., that a certain list is shorter by one. Similarly if we drop the 'perfective' element have\textsuperscript{en} from the auxiliary verb phrase we can shorten the kernel grammar in \textsuperscript{67.2} to the extent that \textless have\textsuperscript{en}\textgreater can be dropped from statement 21, but in no other way. Since one transformation is added to the transformational part of the grammar, there has been no saving in this case. If, on the other hand, we drop the 'progressive' element be\textsuperscript{ing} from the kernel, we drop from statement 21 of \textsuperscript{67.2} both \textless be\textsuperscript{ing}\textgreater and a restriction, namely "except in env. \textsuperscript{ing} --". But even this is no saving, since this restriction will have to be put right back into the transformational analysis as a special limitation. It does not become an automatic consequence of some more general structural feature revealed by transformational analysis, as will often be the case below with apparent special restrictions of the kernel grammar. Hence in this case, too, there is no gain in transformational analysis. On the other hand, when we drop out the passive element be\textsuperscript{en} from the kernel, not only is a line dropped from Statement 17, \textsuperscript{67.2} (i.e., \textsuperscript{en} \textsuperscript{f}), the final line of this statement, is dropped), but one extra and complex statement, statement 18, the rule of verbal selection for passives, can be eliminated from the kernel grammar. This rule is essentially an inversion of statement 9, since only objects of actives can be subjects of the corresponding passives, etc. And this rule will not have to be reintroduced in some form
into the transformational part of the grammar, since the proper verbal selection in passives is provided automatically by the permutational part of the passive transformation. Dropping statement 18, then, is really a significant saving, and this leads us to introduce passives, though not perfectives or progressives, by transformation.

To develop this reasoning formally, within the framework of our scheme of evaluation, we would have to determine a weighting for statements on the levels $P$ and $T$ in such a way that elimination of passives, but not progressives, will lead to a total simplification. The reasoning we have sketched suggests that this can be done. One might propose a different relative evaluation, which would include both cases, or exclude both formal transformational analysis. A different weighting would lead to a different delimitation of the kernel. This question can be resolved in the same way as all similar questions about the definition of elements in linguistic theory, in the way that we have often described above, i.e., by exploring the empirical implications of the various abstract formulations. It seems to me that the decision outlined in the preceding paragraph is the correct one, in the sense that it is most amenable to natural and systematic formulation in the general theory, and that it is consistent with a characterization of transformational analysis that gives a good deal of insight into linguistic structure and the source of linguistic intuition.

It would be possible at this point to assign a weighting by a formal definition, and to give a precise account of the form that the statement of the grammar of the transformational level must assume, and to do this in such a way that all of the
conclusions of the following pages will be validated. But this seems to me premature. It is much more important, for the present, to investigate informally the potentialities of transformational analysis, the kinds of simplification that it effects, and the kinds of insights that it can reveal. We will therefore make no attempt to present the characterization of transformations in the most concise possible form, but we will point out in detail the kind of simplification that results from each transformational analysis.

Note that the length of the grammar relative to a fixed set of available notations, is not the only formal feature that can be studied and varied in an attempt to provide an evaluation procedure in terms of what can be broadly understood as 'simplicity of grammar'. Below, in , we will see that there are good reasons for imposing a condition of 'finite generation' as a formal requirement on the kernel grammar. And we will see that the problem of evaluating alternative solutions is very much reduced by this condition.

The grammatical sketch of §67.2-3 will be used as a basis for this investigation. The notations of earlier chapters can be readily adapted for use here, and we will employ them without further comment.

90.1. The simplest class of sentences not included in our grammatical sketch in §67.2-3 is the class of interrogative sentences taking a yes-or-no answer. For any sentence of the form \( NP \rightarrow VP_A \rightarrow VP_1 \) (hence any sentence derived in §67.2), we can form a corresponding question by inverting the \( NP \) and
en initial segment of the \( \text{VP}_A \) (auxiliary verb phrase). Thus, such a question will be of the form

\[
1) X' \text{NP} Y' \text{VP}_1
\]

where \( X' Y' \) is the \( \text{VP}_A \) of the corresponding declarative. Recall that at the conclusion of a derivation in \( P \), \( \text{VP}_A \) can be any string of one of the forms

\[
2) \left\{ \begin{array}{ll}
C & M \langle \text{have}^{\text{en}} \rangle \langle \text{be}^{\text{ing}} \rangle \\
\text{ed} & \end{array} \right.
\]

where \( C \) becomes either \( S \) or \( \varnothing \) (by \( \Phi_{1}^{P} \)) and \( M \) may be "will", "can", etc.

The \( X' \)'s of (1) can be any of

\[
3) \left\{ \begin{array}{ll}
\text{do} & \text{when } X = U \\
\text{did} & \\
\end{array} \right.
\]

\[
(i) \left\{ \begin{array}{ll}
C & \text{will} \quad \text{(will being used as the representative of } M) \\
\text{ed} & \\
\end{array} \right.
\]

\[
(ii) \left\{ \begin{array}{ll}
C & \text{have} \\
\text{ed} & \\
\end{array} \right.
\]

\[
(iii) \left\{ \begin{array}{ll}
C & \text{be} \\
\text{ed} & \\
\end{array} \right.
\]

Thus we have "does John like it", "would John like it" (where "would" comes from \( \text{ed} \text{will} \) -- cf. \( \text{\#51.1} \)), "has John seen it" (where "has" comes from \( \text{S have} \) by \( \Phi_{1}^{P} \), \( S \) coming from \( C \) by \( \Phi_{1}^{P} \)), etc., but not "does John have seen it", etc. Aside from (3i), it is correct that \( X' Y' \) is the \( \text{VP}_A \), as asserted directly below (1). But in the morphology we have a statement to the effect that

\[
4) \begin{array}{l}
\text{do} \quad \text{S} \\
\text{ed} \rightarrow \text{did}
\end{array}
\]

This statement is needed to account for the forms of the verb
"do" in its appearance as a main verb, as, e.g., in "I did him a favor". Thus we can rephrase (1) and (3i) slightly, stating that if \( Y \cup u \), then \( X^n Y \) is the \( VPA \), and if \( x = u \), then \( X = \emptyset, S, \) or ed, and do \( X^n Y \) is the \( VPA \). "do" can thus be described as the bearer of \( \emptyset, S, ed \) when these elements are not affixed to a verb. This idea will appear in a slightly different form below, where do will be introduced by a mapping \( \Theta^P \), and further support for it will appear from different sources. It turns out finally that the treatment of "do" as an element automatically introduced to carry an unaffixed affix will have a considerable simplifying effect on the grammar. This effect was the determining factor in the decision of §61.2 to analyze \( C \) into two morphemes of 'number' \( \emptyset \) and \( S \), with the requirement that \( C \) must occur in \( VPA \), rather than to have as the first element of \( VPA \) only the element \( S \), which may or may not occur. The former treatment, with a zero morpheme, permits a uniform treatment of "do", "does", and "did", as the bearer of a displaced affix.

20.2. Before investigating the transformational analysis of yes-or-no questions, we must determine the effect of introducing them into the grammar of the kernel (§67.2) directly. Actually they can be introduced without essential modification of this grammatical sketch. To incorporate these interrogatives into §67.2-2, it is necessary to make the following changes and additions:

(1) \( \text{Sentence} \rightarrow \left( X \right) \text{NP}^0 \text{VP} \)  
(replacing statement 1)

(5) (ii) \[
\begin{align*}
& \{ \text{do} \} \\
& \{ \text{C} \} \\
& \{ \text{ed} \} \\
& \{ \text{M have} \} \\
& \{ \text{be} \}
\end{align*}
\]

(occurring between statements 1 and 2)
(iii) \( \text{VP} \rightarrow \begin{cases} \text{en} \langle \text{be} \, ' \text{ing} \rangle & \text{VP}_1 \text{ in env.} \left\{ \begin{cases} \text{ed} \\ \text{have} \\ \text{be} \end{cases} \right\} \text{ NP} \\ \text{VP}_A^2 \\ \text{VP}_A \end{cases} \) (replacing statement 2)

(iv) The forms in the left hand bracket of (iii) replace "VP_A" in statement 9*, and a similar adjustment is made in statement 18.

It is necessary to add one new component to the mapping \( \Phi^P \), following \( \Phi_{1,2}^P \). Thus to \( 167.2 \) we add

\( \Phi'_{1,2} : \) goes into \( \begin{cases} S \\ \emptyset \end{cases} \) in env. -- \( \left\{ \begin{cases} \text{have} \\ \text{be} \end{cases} \right\} \) \( \begin{cases} \text{NP}_1^p \\ \text{NP}_2^p \end{cases} \)

90.3. For the transformational analysis of yes-or-no questions, the underlying elementary transformation is the permutation \( \Pi_q \) such that

\( \Pi_q (Y_1;Y_1,Y_2,Y_3) = Y_2 \)

\( \Pi_q (Y_1,Y_2;Y_2,Y_3) = Y_1 \)

\( \Pi_q (Y_1,Y_2,Y_3;Y_3) = Y_3 \)

Thus \( \Pi_q^* \) carries \( Y_1,Y_2,Y_3 \) into \( Y_2,Y_1,Y_3 \). The restricting class for the transformation in question will be, in accordance with Def.2, \( 172.2 \), a set of triples of strings \( \{(W_1^{(1)},W_2^{(1)},W_3^{(1)})|l \leq i \leq m\} \)

We can characterize this set \( Q_q \) as the set of such triples meeting the following condition:
(8) (i) \( W_1^{(1)} = \{NP\} \)

(ii) \( W_2^{(1)} = \{S, \emptyset, \text{ed}\} \)

(iii) if \( W_2^{(1)} = \{\emptyset, \text{ed}\} \), then \( W_3^{(1)} = VP_1 \)

Let \( T_q \) be the grammatical transformation determined by the pair \((\mathcal{T}_q, \mathcal{Q}_q)\) (cf. DeK 10 172). Thus \( T_q \) carries any string of the form \( NP - W_2^{(1)} W_2^{(1)} \) into the corresponding string of the form \( W_2^{(1)} - NP - W_2^{(1)} \), where \( W_2^{(1)} \) and \( W_2^{(1)} \) jointly meet (ii), (iii) of (8). We could specify \( W_2^{(1)} \) more closely, but this is not necessary. The notations in (8) have the obvious sense carried over from earlier usages. Thus \( W_2^{(1)} \) can be any of the forms \( S, S\text{have}, S\text{be}, \emptyset, \emptyset\text{have}, \emptyset\text{be}, \text{ed}, \text{ed}\text{have}, \text{ed}\text{be}, M \).

We must now provide for the introduction of "do" by a mapping \( \Phi_1^{P} \) which must appear between \( \Phi_1^{P} \) and \( \Phi_1^{P} \).

(9) \( \Phi_1^{P} : U \) goes into \( \text{do} \) in env. \( \# \rightarrow \{\emptyset, S, \text{ed}\} \)

We will replace (9) by a more general characterization below.

Each transformation requires that the strings to which it applies be mapped out to a certain stage. \( T_q \) must clearly apply after \( \Phi_1^{P} \) to ensure that \( C \) will have the proper form (otherwise we will have to add the \( - - - \) mapping \( \Phi_1^{P} \) of (6) even in the transformational analysis). Thus as a \( T \)-marker we have

(10) \( Z^K \Psi_3^P T_q \Psi_3^P \)
To determine the effect of (10) for a given choice of $Z$ and $K$, we first apply $\psi_3$ (=[\Phi_{13}^P, \Phi_{12}^P, \Phi_{11}^P])$ **cf. Def. 40, 414** to the string $Z$ with the interpretation $K$, then apply $T_q$ to the string $\psi_3(Z, K)$ with its derived interpretation, and finally apply in turn the remaining components $\Phi_{14}^P, \Phi_{15}^P, \ldots$ of $\Phi^P$ to the transform under $T_q$, with its derived interpretation.

Suppose that $\psi_3(Z, K)$ does not have the analysis prescribed by the restricting class for $T_q$, i.e., that it is not analyzeable into $W_1^{(1)} - W_2^{(1)} - W_3^{(1)}$ meeting (8). Then we have an instance of case II, Def. 10, 474.2, so that $T_q$ leaves the string $\psi_3(Z, K)$ and its derived interpretation unaltered. In other words, the effect of (10) on such a pair $(Z, K)$ is to apply the mapping $\Phi^P$ to $Z, K$. As instances of a significant operation of (10), we have

\[(11) \ \emptyset^*John^*\emptyset - \text{come}^*\text{from}^*\text{Boston} \rightarrow \emptyset^*\text{John}^*\emptyset - \text{S-come}^*\text{from}^*\text{Boston} \]

(by $\psi_3$)

\[\emptyset - \emptyset^*\text{John}^*\emptyset - \text{come}^*\text{from}^*\text{Boston} \]

(by $T_q$)

\[\text{do}^*\emptyset - \text{John}^*\text{come}^*\text{from}^*\text{Boston} \]

(by $116^*P$)

\[\text{does}^*\text{John}^*\text{come}^*\text{from}^*\text{Boston} \]

(by $\Phi_{18}^P$ and morphological rules)

\[\emptyset^*\text{John}^*\emptyset - \text{have-en}^*\text{be}^*\text{ing}^*\text{read} \rightarrow \emptyset^*\text{John}^*\emptyset - \text{S-have-en}^*\text{be}^*\text{ing}^*\text{read} \]

(by $\psi_3^*P$)

\[\text{S-have-}\emptyset^*\text{John}^*\emptyset - \text{en}^*\text{be}^*\text{ing}^*\text{read} \]

(by $T_q$)

\[\text{has}^*\text{John}^*\text{been}^*\text{reading} \]

(by $\psi_3^*$)
The transformational analysis given by (8), (9), and (10) (the definition of $\Pi_q$ is not a factor in the complexity, since elementary permutations can be enumerated in the general theory) is clearly somewhat simpler than the extension of the kernel grammar of (5) and (6), even in absolute terms as these analyses now stand. Thus we must accept the transformational analysis of yes-or-no questions, and drop them from the kernel. The difference in complexity between these two analyses would become more compelling if we could show that (8) and (9) are needed anyway, for other transformations. But this is in fact the case.

20.4. Consider the sentences

(12) I saw the play and so did he
    I will see the play and so will he
    I have seen the play and so has he
    I have been seeing the play and so has he

Since no noun can appear as the final element in (12) unless it can also appear before "saw", we see that there is a selectional relation between the main verb of the first conjunct and the noun phrase of the second. Assuming for the moment that the and-transformation has been given, the sentences of (12) are most simply derived from pairs of kernel sentences

(13) I saw the play; he saw the play
    as P-basis. By the and-transformation, (13) is carried into

(14) I saw the play and he saw the play.

There is a selectional relation between the auxiliary phrases of the two conjuncts in (14). Thus we do not have

(15) I will see the play and so did he, etc.
The simplest way to provide for this selectional relation is to require that the auxiliary phrases must be identical in the two conjuncts of (14) for the transformation that gives (12) from (14), call it \( T_{so} \), to apply. But investigating the grammatical cases further, we find that excluded along with (15) are such sentences as

(16) I have been reading the book and so has been he, etc. We find that \( T_{so} \) changes a sentence of the form

(17) \( \text{NP} - Z - W - \text{and} - \text{NP} - Z - W \)

and

(18) \( \text{NP} - Z - W - \text{and} - so - X - \text{NP} \)

where \( X \) can be any of the forms listed in (3), i.e., any of the initial segments \( W_2^{(i)} \) of questions, as in (8). But this means that we can use (8) and (9) to state this transformation.

Let \( T_{so} \) be the grammatical transformation determined by \((Q_{so}, t_{so})\), where

(19) \( Q_{so} = \{ (W_1^{(i)}, W_2^{(i)}, W_3^{(i)}) \, \text{and} \, W_1^{(i)}, W_2^{(i)}, W_3^{(i)} \} \)

where \( W_1^{(i)}, W_2^{(i)}, W_3^{(i)} \) meet (8) (3)

\( t_{so} = \{ \pi_{so} \}, \) where

(i) \( \pi_{so} \) is the permutation that carries

\( Y_1 - Y_2 - Y_3 - Y_4 \) into \( Y_1 - Y_4 - Y_3 - Y_2 \)

(ii) \( t_{so} \) is the defommation that carries

\( Z_1 - Z_2 - Z_3 - Z_4 \) into \( Z_1 - Z_2 - Z_3 - Z_4 \)

Thus \( t_{so} \) carries \( Y_1 - Y_2 - Y_3 - Y_4 \) into \( Y_1 - so - Y_3 - Y_2 \), and \( T_{so} \) carries a string of the form \( \text{NP} - W_2^{(i)} - W_3^{(i)} \) and \( \text{NP} - W_2^{(i)} - W_3^{(i)} \).
the corresponding string of the form $\frac{NP^1 \cap W^1 \cap W^2 \cap W^3 \cap W^4 \cap \cdots}{\cdots \cap W^2 \cap W^3 \cap \cdots \cap NP^*}$

We now extend (9), giving "so--" alongside of "#--" as an initial conditioning context for the introduction of "do" by the mapping. (This will be further generalized below).

Sentences like (12) are thus derived by applying the generalized transformation $T_{\text{followed}}$ by $T_{\text{so}}$ as defined above. Thus we add to $\mu^T$ the generalized $T$-marker

$$(20) Z_1^1 \cap K_1^1 \cap I - Z_2^2 \cap K_2^2 \cap I - T_{\text{and}} - \Psi_3 - T_{\text{so}} - \Psi_3$$

But I is the identity, so that the value of $Z_1^1 \cap K_1^1 \cap I - Z_2^2 \cap K_2^2 \cap I$ is the complex string $Z_1^1 \cap \# \cap Z_2^2$ (cf. Def. 52, §7.3). Thus the first sentence of (12) is derived from the kernel by (20) in essentially the following steps.

$$(21) Z_1 = \text{\textasciitilde} \text{ed see\textasciitilde the\textasciitilde play}\text{; } Z_2 = \text{he\textasciitilde ed see\textasciitilde the\textasciitilde play}\text{ (Z_1 and Z_2 being two sentences of Gr(P) with K_1 and K_2 as respective P-markers.)}$$

2. $\text{\textasciitilde ed see\textasciitilde the\textasciitilde play\textasciitilde \# he\textasciitilde ed see\textasciitilde the\textasciitilde play}$ \text{(Val$_1$ ($Z_1^1 \cap K_1^1 \cap I \cap Z_2^2 \cap K_2^2 \cap I$)})

3. $\text{\textasciitilde ed see\textasciitilde the\textasciitilde play and he\textasciitilde ed see\textasciitilde the\textasciitilde play}$ \text{(by $T_{\text{and}}$)}

4. ------------------------- see\textasciitilde the\textasciitilde play\textasciitilde ed\textasciitilde he \text{(by $T_{\text{so}}$)}

5. ------------------------- so - ed - he \text{(by $\Psi_3$)}

6. ------------------------- so - do\textasciitilde ed - he \text{(by $\Psi_1^P$ -- (9), as extended above)}

7. $\text{I saw\textasciitilde the\textasciitilde play and so did\textasciitilde he}$ \text{(by the component mappings of $\Psi^P$ and the morphological rules)}

In a similar way, all other forms of (12) (and only these) can be derived.
The important thing in this context is that the transformation $T_{so}$ makes use of (8) and (9). Hence given the transformational analysis of yes-or-no questions, sentences of the form (12) can be introduced transformationally in a very simple way, with no independent characterization necessary. If, on the other hand, we had followed the path of §90.2 and retained yes-or-no questions in the kernel with the modifications of (5) and (6), then (8) and (9) would not be available for the formulation of $T_{so}$, and we would be compelled to choose between a transformational analysis which restates (8) and (9), or the equally complex alternative of introducing sentences of the form (12) directly into the kernel. In other words, we have found that two phenomena which on the level of phrase structure are distinct and complex, become in transformational terms, instances of a single generalization. But this naturally leads us to choose the transformational analysis in this case, and in particular, to drop yes-or-no questions from the kernel.

Investigation of sentences like

(22) I saw the play and he did too,

etc., gives additional support to this analysis. These are not quite like (12), since we can have both

(23) I have been reading the book and he has too

I " " " " " " " " been too

but the simplest characterization of (22), (23), etc. is also in terms of (8) and (9), with a certain emendation.

The transformation $T_{so}$ (and, similarly, $T_{too}$, had we developed it properly) provides the first instance in our analysis of a
"pro-element"—a class of elements of which pronouns will later appear as a special case. So (and too) is an element which takes the place of a verb phrase "see the play", etc. This seems quite in accord with our intuitive understanding of (12) (and (22)).

Another case of a transformation that makes use of (8) and (9), and thus supports the transformational analysis of yes-or-no questions, will be discussed below, in §92.

20.5 The consideration of questions has an effect on our analysis of sentences with be as the main verb, i.e., sentences of the form

\[(24) \quad \text{NP} - \text{VP}_{A} - \text{be} - \text{Pred}\]

e.g., "John — will — be — the candidate".

There are several important respects in which be behaves quite differently from other main verbs. The first is with respect to the formation of questions. With all other main verbs, the question is formed by inverting the subject and a certain initial segment of the auxiliary verb, as we have just seen, thus giving, e.g., "can John come" from "John can come". But when the main verb is be, and the \(\text{VP}_{A}\) is minimal (i.e., \(\text{VP}_{A}\) is either \(\emptyset\), \(\text{is}\), or \(\text{ed}\)), then the question is formed by inverting the subject and \(\text{VP}_{A}^{\wedge}\text{be}\). Thus the question formed from "John is your friend" is not (25) (as it would be if the verb were "see"), but (26)

\[(25) \quad \text{does John be your friend} \]
\[(26) \quad \text{is John your friend}.\]

In other words, be in (24) is treated exactly as if it were
a part of the auxiliary phrase, as it is in "John is eating lunch".

Another respect in which be in (24) is treated as an auxiliary is in the placing of not, a problem which we will discuss in more detail in \( \text{§92} \). Not is usually placed after the first word of the auxiliary phrase, as in

(27) John has not (hasn't) come here

It certainly never appears after the main verb — we cannot have

(28) John came not (camen't) here

But when the main verb is be, this is exactly what happens. Thus we have

(29) John was not (wasn't) here

Considering only the first case, that of questions, there are two ways of handling this anomaly.

1. We can continue to treat be as the main verb, and formulate (8) so that \( W_{2}^{(1)} \), the preposed element in the question transformation, covers any instance of "was", "is", etc., whether the instance of be in question is the main verb or the auxiliary.

2. We can consider be to be an auxiliary always, and we can regard (24) as a certain kind of 'nominal sentence', with no main verb.

The difficulty with the second solution is first, that a special statement is necessary to account for the fact that the element ing of the auxiliary being is missing in (24), and second, that it will be difficult to account properly for such sentences as

(30) John is being nice about it
where the second be is clearly the main verb. Nevertheless, this approach may be possible with some juggling.

The first solution can be carried through quite readily. We revise (8), leaving (i) and (ii) unaltered, and replacing (iii) by

$$\text{(31) If } \mathcal{W}_2^{(1)} = \begin{cases} S \\ \emptyset \\ \text{ed} \end{cases}, \text{ then } \mathcal{W}_3^{(1)} = \mathcal{V}_x^n \ldots$$

where a notational convention will indicate that "\(\mathcal{V}_x\)" denotes any of the subclasses of \(\mathcal{V}\) listed in \(\textbf{62.2}\), statement 23.

Before continuing with the discussion of be, it is worth noting that this replacement of (8iii) by (31) is actually necessary for independent reasons. One of the forms of \(\mathcal{V}_{P_1}\) is \(D_2 \mathcal{V}_x\) (cf. statement 3, \(\textbf{62.2}\)), as in "John will certainly object". But we cannot form questions such as "will John certainly object" from such sentences. But such transforms would be permitted by (8iii), though excluded by (31) (they could easily be readmitted in (31) by replacing "\(\mathcal{V}_x\)" by \(\langle D_2 \rangle \mathcal{V}_x\).

Since be is not in any of the subclasses of \(\mathcal{V}\) (be is introduced directly as a main verb in statement 3, \(\textbf{62.2}\), for quite independent reasons), be is not represented by \(\mathcal{V}_x\); and thus (31) indicates that no sequence \((\mathcal{NP}, \mathcal{W}_2^{(1)}, \text{be})\) can be in the restricting class \(Q_q\) of \(T_q\), where \(\mathcal{W}_2^{(1)}\) is \(\emptyset\), \(S\), or \(\text{ed}\). But this excludes the possibility of (25), etc. But the correct form (26) results automatically from (8ii) as formulated above, since there was no requirement given in stating (8ii) that the element be in question be the auxiliary. To have required this, would have led to an additional and special restriction on (8). We thus note the interesting fact that this apparently irregular
behavior of be actually simplifies the grammar. If we had (25) but not (26) as a grammatical sentence, then a special condition would have to be added to (8ii) to indicate that the be involved in that statement is not the main verb be. In addition, it would be necessary to replace $V_A$ in (31) (which, we have seen, must replace (8iii) for independent reasons) by $\{ V_A \}$ be. But to say that be behaves in such a way as to simplify the grammar is to say that its behavior is not irregular, but systematic. This gives an important and unexpected justification for the transformational approach. If we had continued to analyze English structure on the basis of the devices presented in chapter VI, the behavior of be would have been exceptional, and would have required special statement in the grammar, as can easily be determined by attempting to revise 467.2 and (5) to cope with this phenomenon. If what is exceptional from one point of view becomes systematic and regular from a second, then this may be interpreted as giving some indication that the latter is more fundamental.

Before continuing, it is important to note that the considerations of this section give further support to the transformational analysis of yes-or-no questions, since these considerations fall into place automatically in the transformational treatment, but they create further difficulties for the approach of 490.2.

One potential trouble spot remains, however, Just as the main verb be is not excluded in (8ii), neither is the main verb have. And have is also not excluded in the reformulation (31) of (8iii), since have, being a transitive verb, is a $V_A$. 
Thus if we add no further qualifications, it will follow from the simplest formulation of the question transformation (namely, (8i-ii) and (31)) that have as a main verb should share the normal characteristics of transitive verbs on the one hand, and the 'peculiarities' of be on the other. But in fact, this is exactly the case. Though we cannot have (25), we do have (32) does John have a ticket parallelling the normal "does John play the piano". But it is an interesting fact that have, alone among transitive verbs, shares both of the special features of be mentioned above. Thus we can have (33) and (34), but not (35) or (36).\(^4\)

(33) has he a chance to live
(34) he hasn't a chance to live
(35) plays he the piano
(36) he playsn't the piano.

We see then that two apparent anomalies, the irregular behavior of be and the irregular (and different) behavior of have as main verbs, turn out to be higher level regularities. That is, they turn out to be direct consequences of the simplest way of stating the facts for the 'regular' main verbs.

Another special feature which appears relevant here is the peculiar behavior of the word got in colloquial English. We will not attempt to give a serious account of this -- for one thing, such an attempt would lead at once into the study of special dialects -- but we note the following facts. First, we have (37) but not (38).

(37) has John got a chance to live
(38) will John have got a chance to live
Second, "got" is not the participial form of "get" (this being "gotten", as we can tell from the normal uses of "get" as a main verb) or any other verb. It appears from this that the simplest way to describe "got" is as a dummy verb that occurs in the normal main verb position when "have" is treated by transformation as an auxiliary (i.e., by (8ii)), though it is actually the main verb of the kernel sentence. In transforming (39) into "has John a chance to live", "have" is treated as an auxiliary, but in transforming (40) into "will John have a chance to live" it is not, since here the element "will" is inverted.

(39) John has a chance to live
(40) John will have a chance to live

Hence in (39), but not (40), the main verb position is left open, and can be filled by a second transformation by the element "got". This accounts for the possibility of (37), but not (38).

But we have seen that both (8ii) and (31) cover the case of "have" as a main verb. If we transform (39) by (31), rather than by (8ii), the result is

(41) does John have a chance to live

just as with any other transitive verb. Hence (37) and (41) are essentially in a relation of free variation -- the question transformation permits either one or the other, freely. But it is a striking intuitive fact that we interpret (37) as meaning roughly the same thing as (41), not as having the same structure as

(42) has John found a place to live.

If the suggested interpretation of "got" carries through, then
the explanation for this will be simply that (37) and (41) are alternative transforms of the same sentence (39), under the question transformation, while (42) is a transform of the quite different sentence

(43) John has found a place to live

in which "has" is in fact the auxiliary.

The $VPA$ of (39) is simply $S$, while the $VPA$ of (43) is $S^{\text{have}^\text{en}}$. This accounts for the fact that (37) and (41) are 'present' in meaning, while (42) is 'present perfect', although (37) superficially appears to be present perfect in form.

A technical requirement that must be met here is that a transformation must be single-valued, and $T_q$ as we have stated it in (8) and (31) is ambiguous in the case of have, as we have just seen. This can be settled quickly by regarding $T_q$ as a family of transformations (cf. h 85 ). We specify this family by stating a pair $(C,t)$, where $C$ is a condition on the restricting class and $t$ is an elementary transformation.

(44) Let $F_q$ be the family of transformations defined by the pair $(C_q, \pi_q)$, where

(1) $\pi_q$ is the elementary permutation that carries $X_1-X_2-X_3$ into $X_2-X_1-X_3$ (cf. (7))

(II) $C_q(\alpha_1, \alpha_2, \alpha_3)$ is the schema: (I) and either (II) or (III)

where (I) $\alpha_1=NP$

(II) $\alpha_2=\left\{\left[\frac{C}{\text{ed}}\right], \left[\frac{\text{be}}{M}\right]\right\}$ (as in (8ii))—note that $S$ is a $C$ and $\emptyset$ is a $C$, where $S$ and $\emptyset$ are the auxiliaries given by $\pi_{13}$

(III) $\alpha_2=\left\{\frac{C}{\text{ed}}\right\}$ and $\alpha_3=V^\text{en}$ (as in (31))
The $T$-marker (10) for yes-or-no questions can now be rewritten:

\[(45) \quad z^K\psi_3^F\psi_3^t\]

20.6. It is important to note that the transformation $T_q$ is irreversible in the sense that we could not have chosen questions as the kernel, and considered declaratives to be transforms of them. There are a variety of reasons for this, all turning on the relative complexity of the two alternatives. For one thing, the kernel grammar is more simply stated for declaratives than for questions, as can easily be seen by comparing (5) with the replaced statements of 467.2 -- particularly, by comparing (5ii-iii) with statements 20, 21, which are essentially their parallels for declaratives. It is always easier to describe a continuous element than a discontinuous one, and in interrogatives, the auxiliary phrase is discontinuous. Secondly, almost all other transformations are derived from declaratives, not questions. Thus if questions are taken as the kernel, most transformations will have to be compound transformations, with resulting complication in the formulation of the level $T$. Third, and most important, we have seen in chapter VII fundamental reasons for assigning the constituent structure of 467.2 (or something very similar) to declaratives. But if we choose interrogatives as the kernel sentences and derive declaratives from them by transformation, we will find that this constituent interpretation will not be carried over by the interrogative-to-declarative transformation, from the constituent structure which will be assigned to interrogatives as the new kernel. In fact, even the basic NP-VP division would not be established. On the other hand, the reasons adduced
in chapter VII for assigning a systematic constituent structure to declaratives do not hold in general for questions, and in fact, \( T_q \) does not assign a complete derived structure to questions. That is, questions will not have \( \Xi \)-markers conferred on them by transformation, as will, e.g., passives (cf. \$73,2\), and \$94\) below). From these few remarks it is easy to see that a detailed study would show that there is a considerable gain in overall simplicity of the grammar if declaratives are taken as the kernel sentences and interrogatives derived from them, as opposed to the alternative possibility of choosing interrogatives as the kernel and deriving declaratives by transformation. Thus there is a clear and forceful systematic motivation for the feeling that declaratives are 'more basic' and questions a 'derived' phenomenon, from the point of view of sentence structure, as well as for the fact that a syntactic study of English would normally discuss simple declarative sentences, the actor-action form, etc., as the fundamental sentence types and grammatical relations of the language.\(^{(5)}\) We can offer as a reason for this the fact that the simplest grammar takes declaratives as the kernel, and derives questions from them by transformation.

It is also important to observe that we made no appeal here to semantic or other non-distributional considerations, nor, for that matter, to detailed selectional relations which are within the realm of distributional study. Even the most rudimentary level of grammaticalness would have sufficed to establish \( T_q \). We can regard this transformation, then, as being quite firmly grounded in distributional terms. Considerably fewer assumptions about grammaticalness were required here than for the establishment of the system of phrase structure in chapter VII.
A second class of sentences not included in \( \S 67.2 \) is the class of interrogatives receiving other than a yes-or-no answer, i.e., interrogatives of the form

(46) whom did he see
    whom has he seen

(47) who saw him
    who has seen him

On the surface, there seems no special structural reason at all for calling these "interrogatives", thus classifying them along with the sentences investigated in \( \S 90 \), though intuitively, this is a correct classification. Furthermore, the sentences of (46) and (47) seem to be of quite different formal types, (46) containing inversions, (47) none; and there seems little formal reason to classify (46) and (47) as belonging to a single subclass of interrogatives, though again this is intuitively the case quite obviously. But in transformational terms, it will appear that both of these intuitions are formally well grounded.

Investigating sentences of the form (46), we see immediately that the relation between the subject and the auxiliary phrase is exactly that of yes-or-no questions, i.e., the subject is inverted with an initial segment of the auxiliary in exactly the same way. Hence the simplest way to describe sentences of the form (46) is as double transforms of kernel sentences, as transforms of yes-or-no questions where this inversion is already 'built in'. The kernel sentences leading to (46) are thus
(48) he\textsuperscript{ed} see him
he have\textsuperscript{en} see him

\( T_q \) (preceded by \( y_3 \), cf. (45)) applied to (48) gives

(49) ed he see him
S have he en see him

(46) is formed by applying to this result \( \Psi_3 \) a new transformation \( T_w \) which permutes the final noun phrase "him" with the preceding string, then replacing this noun phrase with "whom", thus giving

(50) whom ed he see
whom S have he en see

This becomes (46) by applying the remaining mappings \( \Psi_3 \), with a further generalization of the mapping \( \Phi_3^P \) that introduces "do".

Investigating \( T_w \) in more detail, we note first that its underlying elementary transformation is a compound of a permutation and a deformation. Since the permuted noun phrase may be non-final, as in the transformation of "he saw him yesterday" to "whom did he see yesterday", we see that the underlying permutation is again \( T_q \), as in (441). We can choose the underlying deformation in various ways. The simplest way, perhaps, is to assume that "whom" is analyzed into a morpheme wh (6) plus him, and that "who" is analyzed into wh plus he. That is, we add morphological rules to the effect that

(51) wh he \rightarrow who
wh him \rightarrow whom

We accordingly revise \( \Phi_2^P \) so that word boundary does not fall after wh.
As the underlying deformation we now select $\delta_q$ such that

\[(52) \quad \delta_q: X_1 - X_2 - X_3 \to \text{wh}^r Y_1 - Y_2 - Y_3\]

The defining sequence for $\delta_q$ is thus $(\text{wh}, U, U, U, U)$. Thus $\delta_q(Q_q)$ will be the elementary transformation such that

\[(53) \quad \delta_q(Q_q): X_1 - X_2 - X_3 \to \text{wh}^r Y_2 - Y_1 - Y_3\]

If $X_1 = \text{ed} \text{ he } \text{ see}$, $X_2 = \text{him}$, and $X_3 = U$, then $t^*(Y_1, Y_2, Y_3)$ is the first sentence of (50), where $t_w = \delta_q(Q_q)$.

In §67.2 we did not trouble to make the distinction between he and him, but this is a trivial matter of added detail. The simplest way would be with a mapping $\Phi^P_1$ which may precede $\Phi^P_{11}$.

We can then limit the application of $T_w$ to the noun phrase he. That is, we can take $T_w$ as the transformation which carries $X_1 - \text{he} - X_3$ into $\text{wh}^r \text{he } - X_1 - X_3$. As the restricting class for $T_w$, then, we might consider $Q_w$ defined as the set of triples

\[(54) \quad \{(X_1(i), \text{he } S, X_3(i))\}, \text{ where } X_1(i) \text{ and } X_3(i) \text{ are strings (or U)}\]

But in this case we have not defined a grammatical transformation, since he may occur several times in a string (e.g., (46)), and thus $(Q_w, t_w)$ does not determine a single valued transformation.

To limit $T_w$ to exactly the cases (46) we would have to specify $X_1(i)$ further. But if we take (54) as a condition $C_w$ on restricting classes, then $(C_w, t_w)$ does define a family of transformations $F_w$. A grammatical transformation belongs to this family just in case its elementary transformation is $t_w = \delta_q(Q_q)$ and its restricting class contains only strings meeting (54).

If it were the case that any transformation belonging to
F_w can be applied wherever T_w can be applied (i.e., if T_w can be applied to any occurrence of he in a sentence) then we need not specify T_w any further, and in stating the grammar of the level T we can simply state a T-marker with F_w. But we will see directly that (within certain limitations — cf. 91.2) this is in fact the case.

Assuming then that we have a family F_w defined by \((C_w, t_w)\), where C_w is (54) and t_w is the elementary transformation (53), we can construct a T-marker containing F_w. We have seen that T_w is best construed as applying after the application of T_q. The T-marker for T_q was given in (45). We can now generalize (45), replacing it by the statement that any string in T of the form (55) is a T-marker.

\[(55) \mathcal{Z}^\mathcal{K}^\psi_3^\psi F_q \langle F_w \rangle \psi_3^i.\]

91.2. We have seen that one of the members of the family F_w is the transformation T_w which carries (49) into (50), permuting the second occurrence of him in (49) with the preceding string. We now turn to other grammatical transformations in F_w, and determine their effects on strings to which F_q has been applied, i.e., strings of the form (49). Consider now the transformation T_w_1 that applies to the first occurrence of he in (49).

T_w_1 analyzes (49) into the proper analysis

\[(56) \text{ed-he- see^him}\]

\[S' \text{have} \quad \text{he} \quad \text{en^see^him}, \text{etc.}\]

Applying T_w, the underlying elementary transformation, to (55), we derive first (57) (by applying T_q) and then (58), (applying \(\delta_q\) to the result of applying \(\bar{q}\)).
(57) he - ed - see him
    he - S have - en see him

(58) wh he - ed - see him
    wh he - S have - en see him

Applying the remaining mappings we derive

(59) who saw him
    who has seen him

in the usual fashion. But (59) is exactly (47). We see then that when we apply $T_w$ to (49) we derive (46), and when we apply $T_{w_1}$ (another member of $F_w$) to (49), we derive (47). Hence despite their apparent structural differences, (46) and (47) (one set with inversions, and one without) are instances of the same structural type in transformational terms, both having instances of (55) as their $T_m$-markers. Hence there is formal support, on the level $T$, for the fact that intuitively, these sentence types seem to have the same form. An apparent difference in formal structure between two types of sentence once more turns out to be the effect of an underlying regularity, in the sense that the simplest description of one of the types automatically describes the other as well. We found, in constructing the level $P_m$, that we were able to account for the intuitive fact that sentences distinct in terms of lower levels seemed similar in structure, by showing that these distinct sentences had similar $P_m$-markers (e.g., "John read the book" and "I saw him", though completely distinct in terms of words, have $P_m$-markers with very similar construction, are both represented by $NP\ VP$, etc.). The case of (46) and (47) is quite analogous, on the level $T_m$. The similarity of $T_m$-markers (both being instances of (55)) provides a formal
justification for the classification of (46) and (47) into the same subclass of interrogatives. The only difference between the two cases is that in one case the transformation is applied to the subject, and in the other, to the object.

The reason why we do not have any inversion in (47), while we have a complex inversion in (46) becomes evident when we examine the details of the derivation of these sentences. Reducing this to essentials, we see that the transformational derivation of (55) is based on a permutation $\Pi_q$ from $T_q$ which is repeated a second time (from $F_w$) and then followed by a deformation $\sigma_q$ (from $F_q$). The development of the first sentence of (46) and the first sentence of (47) can be compared by the following chart

<table>
<thead>
<tr>
<th>(60)</th>
<th>$T_w(T_q)$</th>
<th>$T_{w_1}(T_q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The kernel sentence:</td>
<td>he ed see him</td>
<td>he ed see him</td>
</tr>
<tr>
<td>is transformed by $\Pi_q$ of $T_q$ into:</td>
<td>ed he see him</td>
<td>ed he see him</td>
</tr>
<tr>
<td>which is transformed by $\Pi_q$ of $F_w$ into:</td>
<td>him ed he see</td>
<td>he ed see him</td>
</tr>
<tr>
<td>which is transformed by $\sigma_q$ of $F_w$ into:</td>
<td>wh him ed he see</td>
<td>wh he ed see him</td>
</tr>
<tr>
<td>transformed by $r_1^w$ into:</td>
<td>wh him ed he see</td>
<td>wh he see ed him</td>
</tr>
<tr>
<td>transformed by $r_1^w$ into:</td>
<td>wh him do ed he see</td>
<td>wh he see ed him</td>
</tr>
<tr>
<td>transformed by the remaining rules into:</td>
<td>whom did he see</td>
<td>who saw him</td>
</tr>
</tbody>
</table>

The case of the kernel sentence "he S have en see him" is perfectly analogous. In the case of $T_{w_1}$, the second application of $T_q$ cancels out the first, i.e., row 3 is identical with row 1. This accounts for the fact that there is no inversion in the derived string of words.
91.3. We have seen that two transformations in $F_w$ as defined by $(C_w,t_w)$ ((54) and (53), respectively) give grammatical strings. There will be a distinct transformation in $F_w$ for every position in which he can occur. But there are certain occurrences of he which cannot be transformed into who-questions, and restrictions must be placed on (54) to rule out the possibility of transformation in these cases. Thus we have (61) but not (62) (61) your interest in him seemed to me rather strange (62) whom did your interest in seem to me rather strange

On the other hand, we have both cases of (63), and both cases of (64).

(63) you lost interest in him < this year >
(64) whom did you lose interest in < this year >

We might account for this by adding to (54) the condition

(65) $X_1^{(i)} \neq \mathbf{Z}^{\text{Prep}}$ unless $X_2^{(i)} = \left\{ \begin{array}{l} \mathbf{U} \\
\mathbf{F_{\text{Prep Phrase}}} \\
\mathbf{Adverbial Phrase} \end{array} \right\}$.

It is interesting to note in this connection, that though (62) is not grammatical, (66) is.

(66) in whom did you lose interest

This possibility can be accommodated quite neatly in our framework by taking Prep he as one of the permitted forms of the second term of the restricting class in (54), and adding to $\mathbf{Z}_1^{\text{ Prep}}$ the requirement that

(67) $\mathbf{Wh}^{\text{Prep}} \rightarrow \mathbf{Prep} \mathbf{Wh}$.

91.4. Alongside of the who-transformation discussed in §91.1–2, we have an exactly parallel case of what-transformations, giving
(68) what did you see
    what has he seen
    what hit him
    what has hit him

These what-questions can be introduced quite readily by noting that just as an occurrence of he (or him) can be preposed and prefixed by wh to give a grammatical sentence (46) or (47), an occurrence of it can be preposed and prefixed by wh, by exactly the same transformation, to give a sentence of the form (68). To account for what-questions, then, it is only necessary to add to (54) the condition that the second term of the restricting class may be it, and to add to (51) the rule (69) wh^it \rightarrow what.

This statement is correct for the uses of it as an inanimate pronoun, but a sufficiently sensitive analysis of grammaticalness might show that what-questions cannot be formed in cases where it is used as an animate pronoun (i.e., where it is the subject of a verb that takes only N\_anim as subject, etc.), e.g., for animals, babies, etc. In this case we would have to add a condition (cf. Condition K\(_A\), (37-7), below) that it be inanimate in the \(P\)-basis of the given \(T\)-marker. There is an analogous problem for inanimate uses of he she, etc., if who-questions cannot be formed in these cases. But cf. \(§29\), for a possible way out.

91.5. We might continue this analysis with the investigation of such wh-questions as

(70) what plane did he take
    what plane arrived today
    whose book is that
    where was it
    when did he come, etc.
This would necessitate adding to (54) the condition that the second (proposed) term of the restricting class may be the $\langle AP \rangle N \left\{ \begin{array}{c} 2 \\ \epsilon \end{array} \right\} \langle PB \rangle$, he $\sim S_1 \sim NP$ (cf. statement 17 and following discussion, §57.2), there, then, etc., and to (51) and (69) the further rules

$(71)$ \text{wh} \text{the} $\rightarrow$ \text{what} \text{wh} \text{there} $\rightarrow$ \text{where} \text{wh} \text{then} $\rightarrow$ \text{when}, etc.

No serious modification seems to be necessary to accommodate these further sentences, though certain further restrictions are necessary on (54). This requires a detailed study, and we will go into it no further here.

91.6. In §90, (9), we introduced the element do by the mapping $\Phi^p_{16}$ as the bearer of a displaced affix at the beginning of a sentence. In §90.4, we extended this to cover also the environment so--, and in §91.1 we say that this must be extended to cover the contexts wh$\sim$he --, wh$\sim$him --. It is apparent then that this is quite a general phenomenon -- that do appears as a dummy verb bearing the affix when no verbal carrier appears for the affix in question. We can take this statement as the informal definition of $\Phi^p_{16}$. It can obviously be given in the proper form as the definition of a grammatical transformation.

91.7. In §91.1 we mentioned that we had as yet found no formal grounds for the intuitive classification of (46) and (47) as interrogatives, along with the yes-or-no questions of the type "did he come", etc., investigated in §90, or for the feeling that (46) and (47) constitute a single subclass of interrogatives. We have already seen that the second intuitive
judgment can be explained on the formal grounds that (46) and (47) are in fact derived from the kernel by the same family of transformations. As a support for the notion that all of these sentences constitute a single class of interrogatives, we note the fact that (46) and (47) are derived from yes-or-no questions by a further transformation. Hence if we define sentence types and subtypes in terms of the sequence of transformations by which sentences are derived from the kernel, it will follow that (46) and (47) are a subtype of the general type to which these and yes-or-no questions together belong; i.e., the first step in deriving all of these sentences is the same, although later steps are different. This suggested formal explanation for the intuitive classification would acquire further plausibility if we could discover a case where \( F_w \) is applied directly to kernel sentences, with no intervening \( T_q \), and if in this case we did not intuitively consider the result to be a class of interrogatives. Relative clauses are just such a case. In such sentences as
(72) The man whom I visited was ill

The people who visited me were pleasant, etc.

the \( \text{who} \)-clause is derived by \( F_w \) directly from a kernel sentence, with no intervening interrogative transformation (as we see from the fact that there is no inversion of the auxiliary phrase), and the resulting sentence has, of course, no intuitive interrogative status. Thus it seems correct to explain the feeling that (46) and (47) are a kind of question on the grounds that \( T_q \) actually appears in their \( T \)-markers, i.e., that they are in fact constructed from yes-or-no questions.

We will return to sentences of the form (72) below.
21.8. Since \( F_q \) was determined to be 'irreversible' (cf. \( \S 20.6 \)) then, a fortiori, (46) and (47), which are secondary transforms of \( F_q \), cannot be kernel sentences. An independent investigation of the possibility of taking these as the kernel sentences instead of declaratives would give clearly negative results, even more strongly than in \( \S 20.6 \). In fact, it is impossible to derive all kernel sentences by a single family of transformations from sentences of this type, since a different transformation will be needed for each noun phrase that can replace "who" in the corresponding declarative. We conclude, then, that declarative sentences constitute the kernel, and that interrogatives of both the types discussed in \( \S 20.1 \) are derived from them by transformation.

22.1. The distribution of the word not was not given in \( \S 57.2 \) in the description of phrase structure. not occurs, in auxiliaries, in exactly the position marked by the definition of the restricting class for \( T_q \), that is, it occurs at the \( \alpha_2 \rightarrow \alpha_3 \) break as this is stated in (44). This break, it will be recalled, has two cases. When the auxiliary phrase consists of just the single morpheme \( \emptyset \), \( S_i \), or \( \text{ed} \), then not occurs after the auxiliary (as in \( \S(44)ii-III \)). If the auxiliary phrase consists of any several-morpheme form (as in \( \S(44)ii-II \)), then not occurs after the first word of the auxiliary, i.e., after \( \alpha_2 \) of (44)ii). For instance, we have

(73) John did not come (from John-ed-not-come, by \( \Phi^p_1 \))
John has not come
John has not been coming,

etc.
This distributional statement could have been detailed in §62.2 for the kernel grammar, but only as a unique and complex phenomenon. Even if questions had been introduced by (5), the position where not occurs would not be marked. But once we have stated the definition of T_q, as in (44), it is very easy to state the occurrence of not transformationally. T_not will be based on a deformation, introducing the word not and a restricting class meeting condition C_q, (44ii). Thus we have a family of transformations F_N, just as in the case of questions. The fact that (44) appears once more as a condition on transformations lends further support to the position taken in §90 that yes-or-no questions require a transformational analysis.

The deformation which underlies T_not introduces the word not at this break, but we have a choice as to whether to assign the not to α_2 or to α_3. Thus we have a choice between (U,U,U,not,U,U) and (U,U,U,U,not,U) as the sequence defining the deformation. The choice is resolved in favor of the former alternative by noting that T_q applied to (74) gives (75)(8)

(74) I can not see it
(75) can't I see it

Thus can not must be an M in (74), since T_q carries NP-M into M-NP, and in general, not must be assigned to α_2, as in (44ii). This result is achieved by basing F_N on a deformation

\[ \begin{align*}
\not \leftrightarrow U, U, U, not, U, U
\end{align*} \]

which carries I_1-I_2-I_3 into \[ U \overline{U}_{I_1} \overline{U} U \overline{U}_{I_2} \overline{not-U} \overline{U}_{I_3} \overline{U} = \overline{U} \overline{I_1} \overline{I_2} \overline{not-I_3} \].

The restricting class for F_N is given exactly by (44ii). The proper derived constituent structure is assigned, as we see from
the discussion of \( \psi 3.2 \), (ii). There is no particular reason for \( F_N \) to apply after \( \psi \), as does \( F_q \), and in order to have other transformations apply freely to negative sentences, we give the \( T \)-marker for \( F_N \) as \(^9\)

\[
(77) \; \tilde{Z}^k \tilde{T}^k F_N \tilde{F}^P
\]

92.4 In \( \psi 3.5 \) we discussed the apparent irregularity of \textit{be} and \textit{have} as main verbs in questions, with \textit{be} being treated as if it were the auxiliary \textit{be}, and \textit{have} sharing the features of \textit{be} and transitive verbs. Since the restricting class for \( T_q \) has been carried over unchanged in the \textit{not}-transformation, it follows that \textit{be} and \textit{have} should behave similarly with respect to the position of \textit{not}. And in fact we do have (78) but not (79).

\[
(78) \quad \text{John wasn't here}
\]
\[
\quad \text{John hasn't a chance}
\]
\[
\quad \text{John doesn't have a chance}
\]
\[
\quad \text{John doesn't live here}
\]

\[
(79) \quad \text{John doesn't be here}
\]
\[
\quad \text{John livesn't here}
\]

Exactly as in the case of questions, this situation turns out to be a consequence of the simplest statement of the rules for the 'regular' verbs \textit{live}, etc.—hence to be in actuality a higher level regularity.

\( \tilde{F}^P_{15} \) is the mapping that carries \( Af \langle D \rangle V \) into \( \langle D \rangle V \tilde{A}f \), where \( Af \) stands for any of the affixes \( \emptyset, S, ed, en, ing \), \( D \) is an adverb, and \( V \) a verb, and where angles co-occur. Thus \( S \text{have} \rightarrow \) (ultimately, has) \( \text{have's, certainly have's, certainly have's (ultimately, certainly has), etc., in such contents as John -- a chance. But not is not a member of D/ Hence } \tilde{F}^P_{15} \text{ does not apply to the string} \)
(80) John’s not have a chance.

But this means that S is a displaced affix in (80), so that $\overline{q}_{16}$ carries (80) into

(81) John’s do S not have a chance ( $\rightarrow$ John does not doesn’t have a chance)

Thus no further specification of the mappings is necessary to give the correct forms of negative sentences. Cf. also fn. 9.

In the case of $T_7$ and $T_{not}$, then, the apparent irregularities of be and have disappear. We have given one further transformation based on (44), namely $T_7$ (cf. $\S 90.4$). In this case too, we would expect to find that the main verb be is treated as an auxiliary, and that the main verb have is treated either as an auxiliary or a main verb. And in fact this is exactly the situation that we find. We have (82) but not (83), as grammatical sentences, just as we have (78) but not (79) in the case of not, and (84) but not (85) in the case of questions.

(82) I am here and so is he
I have an appointment and so has he
I have an appointment and so does he
I kept the appointment and so did he

(83) I am here and so does he
I kept the appointment and so kept he

(84) is he here
has he a chance
does he have a chance
does he live here

(85) does he be here
lives he here

We see that in all three cases (the same is true of $T_{too}$—cf. $\S 90.4$), the apparently irregular behavior is actually
systematic. These three superficially quite different cases all turn out to be instances of the same general formulation (i.e., (44)), and in fact, they turn out to be consequences of the simplest statement of the rules for regular verbs.

22.3. The statement we have given does not account for all occurrences of not. not also occurs with certain noun phrases, as in

(86) (a) not a person was there
     (b) not many people were there
     (c) not coming was a mistake.

The third case does not intuitively seem parallel to the first two, but as things stand now there is no reason not to account for all such cases by a statement in the kernel grammar to the effect that NP → not^NP under certain circumstances. However, we will see below, in 95.10, that there are grounds for the distinction between (a)-(b) on the one hand, and (c) on the other.

Another difficulty in the placing of not appears in sentences like

(87) he tried not to fail

We have not yet account for this. Intuitively, there seems to be a constituent break after "tried", and "not to fail" seems to be a noun phrase of the same type as the subject of (86c), but we have no real basis for this analysis as yet. In fact, the analysis of V-to-V expressions to which we were led in chapter VII, 63 (which had, it will be recalled, a certain intuitive correspondence), would preclude this intuitive correct analysis, since "try to" was treated there
as being a single constituent, i.e., the auxiliary verb \( V_B \).

We will see below ( ) that this difficulty, too, is resolved
when a better analysis of \( V \)-to-\( V \) constructions results from
transformational analysis.

However sentences like (86) are analyzed, they clearly
associate not with the noun phrase, not the \( V P_A \). (86c) thus
provides a case of constructional homonymity for sentences
of the form

\[(88) \, NP^\prime \text{is}'not'\text{ing} \, VP_1 \quad (10)\]

There are three sources for such sentences. They can be
derived from (89a-b-c).

\[(89) \, a \, NP_1 \text{-is-}NP_2, \quad \text{with} \, NP_2 = \text{not'}\text{ing'}VP_1\]

\[(b) \, NP_1 \text{-is'}not'\text{-}NP_2, \quad \text{with} \, NP_2 = \text{ing'}VP_1\]

\[(c) \, NP_1 \text{-is'}not'\text{-ing'} \, VP_1, \quad \text{where} \, is'\text{ing'}=VP_A\]

(b) results from the application of \( T_{\text{not}} \) to \( NP_1 \text{-is-}NP_2 \). (c)
results from the application of \( T_{\text{not}} \) to \( NP_1 \text{-}VP_A\text{-}VP_1 \), where
\( VP_A = is'\text{ing} \) (in the case of (b), \( VP_A = S \)). As examples of these
three constructions, we have, respectively,

\[(90) \, (a) \, \text{The important thing is - not answering the questions} \]
\( \quad \text{(that are unfairly put)} \)

\[(b) \, \text{The important thing is not - answering the questions} \]
\( \quad \text{(so much as knowing the answers)} \)

\[(c) \, \text{John is not answering the questions.} \]

93. Our analysis of questions paired questions and declaratives
in the following way.
(93)  he comes  -- does he come
he came  -- did he come
he will come  -- will he come
he has been coming -- has he been coming
etc.

This is intuitively correct, and our theoretical conceptions
are supported if they lead to this analysis. However, there is
another analysis that we have not considered, but which,
superficially, seems at least as a simple as (91). We are led
to consider this alternative by noting that our analysis of
declaratives in §67.2 left one form of the declarative sentence
out of consideration, namely,

(92)  he did come, he does come, they do come

If we bring (92) into consideration, as of course we must in
a complete grammar, then (93) seems to be a possible analysis
of the declarative-question transformation, in place of (91).

(93)  he does come  -- does he come
he did come  -- did he come
he will come  -- will he come
he has been coming  -- has he been coming
etc.

(93) leaves "he comes", etc., unpaired; but (91) leaves
(92) unpaired. If (93) is taken as the analysis of \( F_q \), then
(44ii-III) can be dropped from the characterization of \( F_q \), and
"do" will be added to (44ii-II) along with "have" and "be". In
this case, \( \Phi_{16}^p \) will not have to apply to questions. These
alterations constitute no real saving, however. \( \Phi_{16}^p \) and (44ii-III)
must still be stated in the grammar for \( T_{\text{not}} \) and \( T_{\text{so}} \). On the
other hand, we have not yet shown that (93) is a distinctly
less simple analysis.
One argument against the analysis (93) is just this fact that it disturbs the uniformity of the analysis of $T_q$, $T_{eq}$, and $T_{not}$, thus leading to certain additional complexity in the statement of the grammar. However, the transformational analysis would be more compelling if we could show that this uniformity is a consequence of what is independently the best analysis, instead of appealing to the resulting uniformity as the criterion for the analysis. And there are indeed independent arguments for the inadequacy of the analysis (93).

For one thing, we note that if "do" is regarded as an auxiliary, it will be subject to certain distributional limitations of a special and unique nature. As distinct from the elements $M$, do does not occur freely with other auxiliary verbs. Thus we do not have

(94) They did have come, etc.

Similarly, do is distinct in distribution from the other elements of the auxiliary phrase. Hence the auxiliary phrase in the kernel grammar will become more complex if do is treated as an auxiliary. Furthermore, do as an auxiliary cannot occur with the main verb be -- we cannot have "he did be here", etc.-- and this too requires a special restricting statement.

Investigation of (92) shows that these sentences have certain other peculiarities. For one thing, "do" in (92) must occur with heavy stress, whereas "do" in other positions (as an auxiliary) always occurs unstressed. Secondly, all other auxiliaries appearing in the position of "do" in (92) are always unstressed. A complete description of the structure of declaratives would have to account for this with a special statement. The only way
to avoid an excessively complicated statement to this effect in the morphology, is to set up an element $Ac$ of accentuation as a prime of $P$ (i.e., a morpheme, in this case), occurring along with $do$ in sentences like (92). As one of the forms of the auxiliary phrase, then, we have

\[
\{ S \} \quad ^\circ do^\circ Ac
\]

And in the morphological rules, i.e., the mapping which carries word and morpheme strings into strings of phonemes, we will have a statement

(96) $V \ldots Ac \rightarrow \ldots V \ldots$, where $V$ is the vowel just preceding $Ac$.

But this solution in fact rules out (93) as an analysis of the question transformation, since in questions "does" does occur without heavy stress.

Perhaps an even stronger argument against (93) comes from noting that "do" is also a main verb, as in

(97) He does the crossword puzzle every day

If "do" also appears as an auxiliary, then "do" will have the same status as "have", and thus the special distributional features noted above in the case of "have" will have to hold as well for $do$. But in fact, this is not the case. We do not have

(98) does he the crossword puzzle every day

he doesn't the crossword puzzle every day

though we do have the parallel forms (33) and (34) in the case of $have$. Thus if (95) is admitted as a form of the auxiliary, we will have to add a set of special conditions to (44).
(89) is thus clearly established as the correct analysis. The considerations adduced above suggest that do^Ac be analyzed in transformational terms, so that these various complications can be avoided in the kernel grammar.

We note at once that only Ac need be introduced by a special transformation. Then the appearance of do will be accounted for automatically by \( \Phi^P_{15} \). For suppose that Ac is introduced into the string C^V (or ed^V), where V is any verb, giving C^Ac^V. But Ac is not an adverb of D. Thus \( S^Ac^V \) (or \( \Phi^P_{15} Al^V \), both of which result from C^Ac^V by \( \Phi^P_{15} \)) will not be converted by \( \Phi^P_{15} \) into Ac^V^S (or V^Ac^S -- see fn. 9). But this means that S (or S or ed) is a displaced affix in ..S^Ac^V.., and \( \Phi^P_{15} \) will thus introduce do as a bearer of the affix. This reasoning is perfectly analogous to the case of not discussed in §92.2.

Thus we see that the forms (95) result automatically when Ac is introduced transformationally in the position where not occurs.

We have seen above that Ac and not share the further property that neither occurs before be, though both occur before all other verbs. The final indication that Ac and not are elements of the same type comes from noting that either one or the other may occur, but not both. That is, we do not have (99) he did not come.

Clearly the simplest way to handle the forms (92), then, is to leave both the kernel grammar and Tq unchanged, and to define a transformation \( T_{Ac} \) differing from \( T_{not} \) only in that Ac appears in the underlying deformation instead of not. (14)

This parallel treatment of Ac and not certainly has intuitive appeal. It assigns to the paired sentences of (100)
the same phrase structure and a parallel transformational structure, the only difference between them being that in one the element Ac appears, in the other the element not. And in fact these sentences do seem paired intuitively.

(100) he did come - he did not come
    he has come - he has not come

etc.

The systematic effect of this analysis is that all asymmetry in the auxiliary verb phrase and in the pairing of questions and declaratives disappears.

24.1. So far we have investigated methods of extending the grammatical sketch of §67.2 transformationally with the ultimate goal of covering the set \( W \) of grammatical strings of words. It is also necessary to examine §67.2 itself, to determine whether some of its complexities can be removed by dropping some of the sentences there generated from the kernel, and reproducing them transformationally. We have a hint that transformational analysis might be called for wherever two distinct statements are required in the grammar for what is essentially the same selectional relation. Since selectional relations can be preserved under a suitable transformation, there is always the possibility of avoiding the duplication by considering one of the classes of sentences for which the selectional relation holds to be transforms of the other class.

The most obvious trouble spot of this kind in §67.2 is statement 18, the rule of verbal selection for the transitive verbs which appear as adjectives in the form \( \text{en} \gamma \nu \text{r} \), i.e., the passives. Statement 18, had we given it, would be at least as complex as statement 9, since every selectional relation in
statement 9 must be restated for passives in statement 18. Every
active has a corresponding passive. In fact Statement 18 would
be considerably more complex than statement 9, since many
prepositional phrases (e.g., "by a new method" —cf. 72.1)
do not function as conditioning contexts for the choice of
the verb \(V_T\), and this will have to be stated.

The passive transformation \(T_p\) is based on an underlying
elementary transformation \(t_p\) which is the product of a permutation
and a deformation. That is, we form a passive by interchanging
subject and object (by a permutation) and adding be\`en between
the auxiliary and the verb, and by\(\) after the verb. \(T_p\) will
be applied to any string of the form \(NP_1-VPA-(D)V_T-NP_2\)
and will convert it into a string of the form \(NP_2-VPA-\text{be\`en }en-(D)V_T\)
by \(\text{by } NP_1\). Thus it will pair sentences in the following way:

\[
\begin{array}{c|c}
Z = NP_1 - VPA - V_T - NP_2 & T(Z, K) = NP_2 - VPA - \text{be\`en } en - V_T \\
\hline
\text{John-ed-meet-Mary} & \text{Mary-ed-be en-meet-by John} \\
(\text{John met Mary}) & (\text{Mary was met by John}) \\
\text{John-will\`have\`en-meet-Mary} & \text{Mary-will\`have\`en-be\`en-meet-by John} \\
(\text{John will have met Mary}) & (\text{Mary will have been met by John}) \\
etc. & \\
\end{array}
\]

The permutation that underlies the passive transformation is

\[
(102) T_p: Y_1 - Y_2 - Y_3 - Y_4 \rightarrow Y_4 - Y_2 - Y_3 - Y_1
\]

We may select the underlying deformation so as to assign be\(\), en\(\)
and by\(\) in various ways to the root elements \(NP, VPA, V_T, NP\).
This assignment will determine the derived constituent structure
of passives (since it will determine the place of the constituent
breaks in the transform), and it will thus determine which further
transformations can be applied to passives.

Since we have

(103) he was seen by John and by Mary

we see that by John is a constituent in (103) (by the conjunction criterion), so that by must be assigned to the following NP, not to the preceding \( V_T \). In §62.2-3, we noted several reasons for assigning be\( \text{en} \) to the verb phrase, rather than the auxiliary phrase (thus treating be as the main verb). Several inadequacies were noted in this analysis, but we will see below that all of them disappear under further transformational analysis.

It follows that the underlying deformation \( \delta_p \) must be

\[
(104) \quad \delta_p : Y_1-Y_2-Y_3-Y_4 \rightarrow Y_1-Y_2-\text{be}\text{en}Y_3-\text{by}Y_4
\]

Thus the defining sequence for \( \delta_p \) is \((U,U,U,U,\text{be}\text{en},U,\text{by},U)\).

Summing up, the passive transformation is defined as follows

(105) \( T_p \) is determined by \((Q_p,t_p)\),

where \( t_p = \delta_p(T_p) \) (as in (102), (104))

\[
Q_p = (\text{NP},\text{VP}_A,(D)Y_M,\text{NP})
\]

Thus \( T_p \) carries any string of the form \( \text{NP}_1-\text{VP}_A-V_T-\text{NP}_2 \) into the corresponding string \( \text{NP}_2-\text{VP}_A-V_T-\text{NP}_1 \) (by \( \Pi_p \)), finally into \( \text{NP}_2-\text{VP}_A-\text{be}\text{en}V_T-\text{by}\text{NP}_1 \) (by \( \delta_p \)).

We must now state a \( T \)-marker for \( T_p \). This means that we must determine where \( T_p \) applies in the sequence of mappings \( \delta_1 \). The only requirement here is that \( T_p \) must precede the mapping \( \delta_1 \), which carries \( G \) (from the auxiliary verb) into \( S \) after a singular noun phrase, and into \( \emptyset \) after a plural noun phrase. The reason for this order of application is that the subject must agree in number with the verb in passives. Thus as a \( T \)-marker we can state
No further conditions need be placed on the restricting classes of the interrogative and negative transformations to cover the case where $T_p$ applies before them. That is, questions and negatives can be formed freely from passives. In fact, any transformation that applies to declaratives in general, will apply to passives as well, since, as we noted in §62-2, passives have the full constituent structure of declaratives conferred on them by transformation, even to the extent that their derived interpretations are actually $P$-markers.

94-2. One simplification resulting from the transformational analysis is, naturally, that $en^{\wedge}V^T_2$ can be dropped from statement 17, §67-2. This becomes important when we find, below, that $in^{\wedge}V_{11,2}$ can also be introduced transformationally. Thus the entire third part of the analysis of the adjective phrase in the kernel grammar (statement 17) can be eliminated. These were the adjectives which were marked in the grammar of phrase structure as a separate subclass by virtue of the fact that they could not occur freely with adverbs such as "very", but only with the special subclass of adverbs $D_2$ containing "certainly", "soon", etc. Thus we can have "the mistakes were soon found", but not "the mistakes were very found". But when passives (and adjectives of the form $in^{\wedge}V_{11,2}$ -- e.g., "barking") are introduced transformationally from declaratives, this result will be an automatic consequence of the analysis, since only $D_2$ can occur with verbs. I.e., we have "he soon found the mistakes" but not "he very found the mistakes". Thus one support for the transformational analysis is that this special restriction on certain adjectives need not be separately stated.
The main motivation for the transformational analysis of passives, however, is the desirability of avoiding the complex statement 18, \( \frac{57.2}{57.2} \), the rule of verbal selection for passives. As our analysis becomes more detailed, and our level of grammaticalness becomes more refined, this statement, along with statement 9, will become more complex. Thus the support for the transformational analysis grows with increasing refinement of the notion of grammaticalness. It is interesting and important to determine what is the crudest possible degree of grammaticalness that would support the transformational analysis of passives. This transformational analysis will be called for on any level of grammaticalness for which it is the case that for some choice of noun phrases \( NP_x \) and \( NP_y \) is grammatical, but not (108).

\[
\begin{align*}
(107) \quad NP_x & \rightarrow V \rightarrow NP_{2x} \\
NP_y & \rightarrow is^{\sim}Ven \rightarrow by^{\sim}NP_x
\end{align*}
\]

\[
\begin{align*}
(108) \quad NP_y & \rightarrow V \rightarrow NP_{2y} \\
NP_x & \rightarrow is^{\sim}Ven \rightarrow by^{\sim}NP_y
\end{align*}
\]

Given a distinction between abstract and proper nouns, we have (109) but not (110) as highest degree grammatical sentences (16)

\[
\begin{align*}
(109) \quad \text{John appreciates sincerity} & \quad \text{sincerity frightens John} \\
\text{sincerity is appreciated by John} & \quad \text{John is frightened by sincerity}
\end{align*}
\]

\[
\begin{align*}
(110) \quad \text{sincerity appreciates John} & \quad \text{John frightens sincerity} \\
\text{John is appreciated by sincerity} & \quad \text{sincerity is frightened by John}
\end{align*}
\]

Thus the distinction between abstract and proper nouns (and the subclasses of verbs that occur with them in first degree grammatical sentences) would suffice to establish this transformation.
We will see below (§97.2) that the distinction between singular and plural is actually sufficient to establish it.

94.3. The passive transformation is irreversible in the same sense as the question transformation (cf. §92.5). If we were to choose passives as the kernel sentences, and were to derive actives from them by a transformation $T_a$, it would require a great many fairly complex conditions to establish the fact that $T_a$ applies to (111) but not (112).

(111) the wine was drunk by the guests
(112) John was drunk by midnight.

If passives are derived from actives, there is no analogous complication. (Cf. ) Thus passives and not actives must be deleted from the kernel and reintroduced by transformation. We are left with active declarative sentences as our kernel of basic sentences.

94.4. The transformational analysis of the passive provides us with an excellent criterion for constituent analysis. Since every passive must be derived from a sentence of a form $NP - VP_T - V_T - NP$, we can determine that an active sentence is of this form by noting the existence of a passive presupposing that the corresponding active have this analysis. This criterion has the immediate effect of ruling out the counter-intuitive possibility suggested in §64.3 (and §72.2) that

(113) John - is eating - lunch

be analyzed as a case of $NP$-is-$NP$, like "John is a politician". The correct analysis is now dictated by the existence of

(114) lunch is being eaten by John
which requires the analysis $NP - VP_A - V_T - NP$ for its corresponding active (113). The passive transformation thus gives us our first really convincing reason for considering (113) to be of the same form as "John ate lunch", the only difference being in the choice of the auxiliary verb ($\text{be}\_\text{ing}$, in the case of (113), $\text{ed}$ in the case of "John ate lunch").

Another effect of this criterion is that it solves correctly the difficulty discussed in §64-5, and recapitulated in §72-1, where it was pointed out that there are no grounds for distinguishing (115) and (116) as strings with distinct constituent structure.

(115) I knew the boy studying in the library
(116) I found the boy studying in the library

But now we note that (117) are grammatical, but not (118).

(117) in the boy studying, the library was known ($\text{by the teacher}$)

(i) the boy studying in the library was found ($\ldots$)

(ii) the boy was found studying in the library ($\ldots$

(118) the boy was known studying in the library ($\ldots$)

Hence it must be the case that (115) is an instance of

(119) $NP_1 - VP_A - V_T - NP_2$

with a compound object $NP_2 \rightarrow NP_3 \_\text{ing} VP_1$, while (116) must be a case of constructional homonymity, having (119) as one analysis (as in "I found the boy studying in the library, but not the one in the gym"), and (120) as an alternative analysis,

(120) $NP_1 - VP_A - V_T - NP_2 - \_\text{ing} VP_1$,

with $NP_2$ the object, and $\_\text{ing} VP_1$ a discontinuous compound verb. This would be the analysis, e.g., in "I found the boy studying in the library, not running around in the streets." We have
not yet shown that the passive transformation, as it now stands, carries (116) into (117c). This will be shown below, when a more serious study of sentences of the form (120) is undertaken in 

The analysis of (115) as an instance of (119), and of (116) as a case of constructional homonymity in the overlap of the patterns (119) and (120) is the intuitively correct analysis of these sentences, but there is apparently no support for it except in transformational terms.

In both the case of (113) and the case of (115-6), the passive transformation furnishes a criterion of analysis in the sense of §52, §60.1. If what happens to be the intuitively correct analysis of constituent structure is given for (113), (115) and (116), then (114) and (117) are generated automatically, i.e., the grammar automatically produces grammatical sentences. If a different constituent analysis is provided for (113), (114), and (115), then (114) and (117c) remain as kernel sentences of a unique structure, and special added statements are required to provide for them, with resulting complexity of the grammar. Thus the criterion of simplicity of grammar leads to (and formally grounds) the intuitively correct analysis, by way of the passive criterion.

It will appear below that this criterion has a rather wide application in determining constituent structure.

24.5. Given any passive, e.g., "Mary was met by John", we have a sentence without the by-phrase, in this case

(121) Mary was met.
We account for this possibility with a transformation $T_{pd}$ defined by

\[(122) \quad T_{pd} \text{ is determined by } (\delta_{pd}, Q_{pd}), \text{ where} \]

\[\delta_{pd}: X_1 X_2 \rightarrow X_1 - U = Y_1 \]

\[Q_{pd} = \{(X_i \text{ en } V, \text{ by } NP)\}, \text{ where } X_i \text{ is any string} \]

Thus the defining sequence for $\delta_{pd}$ is $(U, U, O, U)$, and

$T_{pd}$ converts $X \text{ en } V - by \text{ NP}$ into $X \text{ en } V$, e.g., "Mary was met by John" ($= \text{ Mary was en meet - by John}$)

\[(121) = \text{ Mary was en meet}. \]

Since this transformation follows $T_p$, we can state the corresponding $T$-marker together with the $T$-marker for passives. Thus in the grammar of the level $T$, we replace (106) by

\[(123) \quad Z K T_p < T_{pd} \Phi^P.\]

This will be revised below in 7 99.2.

24.6. There are other forms similar to the passive that should be discussed at this point. Thus there are 'passives' with "with" instead of "by", and with "get" instead of "be", as in

\[(124)\]

(a) he was laid up with the grippe
(b) he has been occupied with this problem
(c) all of his time has been taken up with their complaints etc.

\[(125)\]

(a) he got thrown by the horse
(b) he got recommended for the job etc.

A detailed study of the passive and related forms should prove quite interesting. Many further problems could be cited at this point. But our present purpose is not to present a complete picture of any one transformation, but rather to explore the general scope of this kind of analysis.
94.7. We cannot simultaneously retain the following four requirements, each of which might be considered a desideratum for constituent and transformational analysis.

(126) (i) Non-overlap of constituents, for a given P-marker (i.e., consistency of P-markers -- cf. 46.3, 42.3, Def.13, Th.15)

(ii) The passive transformation $T_P$

(iii) The question transformation $T_q$

(iv) Every term of the proper analysis of a transformed sentence is an elementary constituent (i.e., a constituent represented by a prime of $P_A$, as NP, VP, etc.)

This is clear from the fact that the passive and question transformations analyze the auxiliary verb phrase in different ways. Our analysis of $VP_A$ allows (iv) to be satisfied for $T_P$. An alternative analysis, allowing it to be satisfied for $T_q$, would make it impossible to satisfy it for $T_P$. We have retained (i-iii) in our construction of transformations. If we had retained (iv), transformational analysis would produce a direct and powerful criterion for constituent analysis, but the cost is far too great, since many transformations which greatly simplify the grammar would be impossible. Thus transformational analysis can provide only various indirect criteria for constituent analysis (e.g., the passive criterion discussed in 94.4). Nevertheless, we will see that these are the major criteria for constituent analysis.

95.1. The decision to drop passives from the kernel constraints to delete certain other sentences as well. The noun phrase subject of

(127) being elected certainly surprised John
for instance, was formed in §67.2 by running through the
grammar twice (cf. §51.3, Def. 26) in the following series of steps:

(128) $N_A \rightarrow In^*VP_1$
    $\rightarrow ing^*VP_1$
    $\rightarrow ing^*be^*Predicate$ (* 3
    $\rightarrow ing^*be^*AP$ (* 5
    $\rightarrow ing^*be^*en^*V_T$ (* 17
    $\rightarrow ing^*be^*en select$ (* 23
    $\rightarrow being^*elected$ (by $\Phi^*_P$ and morphological rules)

But this path of development is excluded now, since the
substatement "$AP \rightarrow en^*V_T$" has been dropped from statement 17
of the kernel grammar.

Since at least (127) must be derived by transformation, it
is simplest to delete all cases of $In^*VP_1$ (i.e., all noun
phrases of the form "proving that theorem", "to prove that
theorem", etc.) from the kernel and to introduce them
transformationally. It turns out that this treatment will
effect various other simplifications in the kernel grammar,
and finally, a condition on the kernel grammar suggested in
below, will make the transformational treatment absolutely
necessary, apart from all questions of relative simplicity.

We thus have two transformations $T_{ing}$ and $T_{to}$. $T_{ing}$
is based on a deformation $\delta_{ing}$ with the defining sequence
$(0, U, ing, U)$. Thus

(129) $\delta_{ing}: Y_1 \rightarrow (\sigma^*Y_1^*U)-(ing^*Y_2^*U) = U-ing^*Y_2 = ing^*Y_2$

The restricting class for $T_{ing}$ is

(130) $Q_{ing} = \{(NP^*VP_{A1}, < have^*en > VP_1)\}$

where $VP_{A1}$, it will be recalled, is any string of one of the
forms \( \left\{ \begin{array}{c} S \\ \hat{S} \\ \hat{S}_d \end{array} \right\} \langle M \rangle \) Cf. final remarks of \( \S 51.3 \), and first few paragraphs of \( \S 61.5 \) for further discussion of ing-phrases, connected with this characterization of the restricting class.

Thus \( T_{\text{ing}} \) as determined by \((Q_{\text{ing}}, \delta_{\text{ing}})\) carries a string of the form \( NP \hat{V}_{P_{A1}} \hat{V}_P \) (e.g., John's prove that theorem) into the corresponding string of the form \( \text{ing} \hat{V}_P \) (e.g., 

\[ \text{ing} \text{prove that theorem} \] which becomes, by \( \hat{P}_{15} \), prove ing that theorem, ultimately "proving that theorem"; and \( T_{\text{ing}} \) carries a string of the form \( NP \hat{V}_{P_{A1}} \hat{V}_P \) (e.g., John's have en \( \hat{V}_P \) (e.g., John's prove that theorem) into \( \text{ing} \hat{V}_P \) (e.g., ultimately, "having proved that theorem").

The result of applying \( T_{\text{ing}} \) to a sentence is not another sentence, but rather a noun phrase \( \text{ing} \hat{V}_P \). Hence \( T_{\text{ing}} \) must be combined with a second transformation that substitutes this noun phrase \( T_{\text{ing}}(Z,K) \) for some noun phrase of a second sentence. E.g., \( T_{\text{ing}}(\text{John proves that theorem}, K) \) = proving that theorem, and we may follow \( T_{\text{ing}} \) by a transformation \( T_X \) that substitutes "proving that theorem" for "it" in "it was difficult", yielding "proving that theorem was difficult". We thus have our first instance of a generalized transformation (cf. \( \S 87 \)).

In fact, the case of \( T_{\text{ing}} \) is exactly like the case of the sentence "that John was unhappy was quite obvious" (= (114a), chap. VIII), so that the pattern of its construction is just that described in detail in \( \S 87.6 \).

\[ \text{ing} \] must apply before \( \hat{P}_{15} \), and it is simplest to have it apply before \( \hat{P} \) altogether. Hence the T-marker for \( T_{\text{ing}} \) will differ from (143), \( \S 87.6 \) in that \( S \) of (143) is replaced by
"I". The $T$-marker for $T_{\text{ing}}$ will thus be

(131) \( Z_1^T K_1^T I^T_{\text{ing}} Z_2^T K_2^T I^T_{\text{X}} \theta^P \)

The effect of (131) is to apply $T_{\text{ing}}$ to $I(Z_1, K_1) = Z_1$; and to apply $T_{\text{X}}$ to $Z_3^T \#^{\text{ing}} Z_2$, where $Z_3$ is the ing-phrase resulting from the application of $T_{\text{ing}}$ and $Z_2 = I(Z_2, K_2)$ (since $I$ is the identity transformation). Letting $Z_1 = \text{John proved that theorem}$ and $Z_2 = \text{it was difficult}$ (and assuming $K_1, K_2, K_3, K_4$ to be properly chosen) we can trace the operation of (131) in the following series of steps, which match (144), §87.6:

(132) $I(Z_1, K_1) = Z_1$

$T_{\text{ing}}(Z_1, K_3) = \text{proving that theorem}$ (note that $K_3 = K_1$)

$I(Z_2, K_2) = Z_2 = \text{it was difficult}$

$T_{\text{X}}(\text{proving that theorem} \# \text{it was difficult}, K_4) = \text{proving that theorem was difficult}$

(note that $K_4 = K_3 \# K_2$, cf. Def. 50, §87.2)

$T_{\text{X}}$ is the as yet unspecified transformation that substitutes $T_{\text{ing}}(Z_1, K_1)$ for some noun phrase of $Z_2$. On the level of grammaticalness assumed in §67.2, ing-phrases are abstract nouns and hence have a distribution similar to $N_{ab}$. ing-phrases as abstract nouns are introduced in §67.2 in statement 15. There we saw that there are two types of ing-phrases, $\text{ing}( \text{VP}_2 )$ (like "reading") and $\text{ing}( \text{VP}_2 ) \text{ VP}_2$ (like "reading books", "having read books"), and in statement 21, we saw that $\text{VP}_2$ becomes have-en in this position. The transformation $T_{\text{ing}}$ that we have constructed produces instances only of the second type. We see that these ing-phrases with a full $\text{VP}_1$ are excluded from certain environments, namely, before "of" and after the article. There is in fact one
abstract noun that is also excluded from these environments, namely "it". "it" is an \( N_{ab} \), since it occurs in all the contexts of \( N_{ab} \) and in contexts where only \( N_{ab} \) can occur, but we do not have "it of", "the it", etc. In this case, then, we are forced to choose "it" as the element of \( Z_2 \) which is replaced by the generalized transformation \( T_X \).

There is one apparent difficulty in the choice of "it" as the element for which \( ing \)-phrases are substituted. \( ing \)-phrases can occur after the possessive adjective phrase \( NP^S_1 \) (cf. statement 17, p. 67-2), as in "John’s flying planes is something I don’t approve of", but "it" can not occur here — we do not have "John’s it". But we will see below that this discrepancy is only apparent, since \( NP^S_1^ {ing} VP_1 \) will also be deleted from the kernel and introduced transformationally as a unit.

\( T_X \), then, is the transformation that replaces \( it \) in \( Z_2 \) by \( T_{ing}(Z_1, X_1) \), where \( it \) is an \( N_{ab} \). Since any such \( it \) can be replaced (on the level of grammaticalness we are now assuming), we have to do here with a family of transformations \( F_A \).

The transformations of this family convert

\[
\begin{align*}
(133) \quad & \text{ing}\langle \text{have}^\text{en}\rangle VP_1^- \# - X_1 - N_{ab} - X_2 \\
\end{align*}
\]

(where dashes separate elements of the proper analysis) into

\[
\begin{align*}
(134) \quad & U - U - X_1 - \text{ing}\langle \text{have}^\text{en}\rangle VP_1 - X_2 = X_1 - \text{ing}\langle \text{have}^\text{en}\rangle VP_1 - X_2 \\
\end{align*}
\]

\( T_X \) must then be based on an elementary substitution \( t_A \) such that

\[
\begin{align*}
(135) \quad & t_A: \ Y_1 - Y_2 - Y_3 - Y_4 - Y_5 \rightarrow U - U - Y_3 - Y_1 - Y_5 = Y_3 - Y_1 - Y_5 \\
\end{align*}
\]

The defining sequence for \( t_A \) is thus \( (\tau_0, 0, 0, 0, 0, 0, \tau_1, 0, 0) \) (cf. Th. 6 and (85) of §82-5); the derived constituent structure of \( Y_1 \) is internally that of \( Y_1 \) of the transformed string, and externally, that of \( Y_4 \), the element for which it substitutes (cf. §87-6). Thus
the ing-phrase in (34) (or "proving that theorem" in "proving that theorem was difficult") is an \(N_{ab}\) with the internal structure of a verb phrase.

The restricting class for any transformation in \(F_A\) must meet the following condition:

\[(36)\ C_A(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)\text{ if and only if}\]

\[\alpha_1 = \text{ing} <\text{have}^\text{en}> \text{VP}_1\]

\[\alpha_2 = \#\]

\[\alpha_3 = X_i\quad (X_i\text{ any string (including } U))\]

\[\alpha_4 = N_{ab}\]

\[\alpha_5 = X_j\quad (X_j\text{ any string (including } U))\]

Thus a transformation of \(F_A\) can be applied only to a string of the form (133), and \(t_A\) will convert this string into the corresponding string of the form (134). But \(C_A\) does not yet require that the replaced element \(\alpha_4\) be it. We will have to construct this requirement as a condition on \(T\)-markers, since our presently available theory does not allow us to impose two conditions simultaneously on the terms of the proper analysis of the transformed string. We now define \(F_A\) as the family of transformations determined by \((C_A, t_A)\) and we construct the \(T\)-marker

\[(137)\ Z_1^\wedge K_1^\wedge I^T \text{ing}^Z_2^\wedge K_2^\wedge I^F_A^\wedge P^D\]

We add to the grammar of the level \(T\) the requirement that for \(Z_1\) and \(Z_2\) to appear in the \(P\)-basis (cf. Def. 22, §27.3) of (137), they must meet the condition \(K_A\)

\[(138)\ \text{Condition } K_A: \text{ there is a } Y \text{ and a } W \text{ such that } Z_2^\xrightarrow{Y} \text{it}^W \text{ and the result of applying } F_A \text{ to } T_{\text{ing}}(Z_1, K_1)^\wedge \#^Z_2 \text{ is}\]

\[Y^T \text{ing}(Z_1, K_1)^\wedge W.\]
In other words, for \( Z_1 \) and \( Z_2 \) to appear in (137), it is necessary that \( F_A \) replace an occurrence of \( \text{it} \) in \( Z_2 \) by the \( \text{ing} \)-phrase formed from \( Z_1 \). We know from (136) that this occurrence of \( \text{it} \) must be an \( N_{ab} \).

Condition \( K_A \) can be phrased more precisely in terms of the notion of "reduced correlate" (Def. 54, §37.4). The reduced correlate of a \( T \)-marker is obtained by dropping from it the final component of the mapping in the \( T \)-marker. In the case of (137), the reduced correlate is formed by dropping \( \Phi^P \).

Let \( Z \) be any actual \( T \)-marker containing a transformation belonging to \( F_A \) and of the form (137). Let \( Z_{\text{red}} \) be the reduced correlate of \( Z \). Then

\[
(139) \quad \text{Condition } K_A: \text{ There is a } Y \text{ and a } W \text{ such that } Z_2 = Y^{\text{it}} W \quad \text{and } \quad \text{Val}_1(Z_{\text{red}})^{\text{ing}}(Z_1, K_1)^{\text{W}}
\]

(cf. Def. 52, §37.3)

The necessity for giving a separate statement of the condition that the replaced element of \( Z_2 \) be \( \text{it} \) suggests that it might be worthwhile to generalize our conception of transformations to allow several conditions to be imposed simultaneously on the constituent structure of a term of the proper analysis of a transformed string.

As matters now stand, questions and passives can be formed from sentences containing transformationally introduced \( \text{ing} \)-phrases, and \( \text{ing} \)-phrases can be formed from passives, but not questions, as \( Z_1 \) in (137). This result follows from the manner in which derived constituent structure is assigned in these cases, and from condition 4, §37.4, which permits transforms to appear in the \( P \)-basis of a \( T \)-marker (i.e., it permits compounding of
separately established transformations). In §87.5 we laid down condition 5 which limits the possibilities of compounding by imposing certain conditions on the $P$-skeletons of the $T$-markers figuring in the compound. But in the case of the transformations so far established, condition 5 does not restrict the possibility of compounding in any undesirable way. The potential difficulties envisaged in §87.5 do not arise at this point.

95.2. In a perfectly analogous way we can provide for the introduction of to-phrases such as "to prove that theorem", etc. We construct $T_{to}$ such that

$$(140) \quad T_{to} \text{ is determined by } (Q_{to}, \delta_{to})$$

where $Q_{to} = (NP^a VP_{A1}, \langle VP_{A2}, VP_1 \rangle)$

$$\delta_{to} \rightarrow (\nu, U, to, U)$$

Thus $\delta_{to}$ carries $Y_1 - Y_2$ into $to^\nu Y_2$. It differs from $\delta_{ing}$ only in that $to$ replaces $ing$ (cf. (129)). $Q_{to}$ differs from $Q_{ing}$ (cf. (130)) only in that it permits $T_{to}$ to apply to strings of the form $NP^a VP_{A1} - be^\nu ing^\nu VP_1$, whereas $T_{ing}$ cannot. Thus we have "to be eating dinner when they arrive would be quite impolite", but not "being eating dinner when they arrive would be quite impolite". (Cf. §81.5).

The distribution of to-phrases is more limited than that of ing-phrases, however. Only sentence-initial $N_{ab}$ can be replaced by a to-phrase (though in a larger class of grammatical sentences than we are now considering, further emendation would be necessary). Hence in the case of to-phrases there is no need to set up a family of generalized substitutional transformations, as in the case of ing-phrases. As the substitutional transformation which follows $T_{to}$, we construct
(141) $T_A$ is determined by $(Q_A, t_A)$

where $t_A$ is as in (135)

$$Q_A = \{(to < \nu P_{Ah} > \nu P_1, \# V, \nu V_{Ah}, X_2)\}$$

Thus with the revision that $to$ replaces $ing$, $Q_A$ meets $C_A$ of (136) with $\alpha_2 = U$. The $T$-marker for $to$-phrases can be stated together with the $T$-marker for $ing$-phrases. Thus replacing (137) we have

(142) $$Z_1 \wedge K_1 \wedge I \left\{ \begin{array}{l}
T_{to} \\
T_{ing}
\end{array} \right\} \left\{ \begin{array}{l}
T_A \\
F_A
\end{array} \right\} \beta \gamma$$

We extend condition $K_A$ (139) in the obvious manner to apply to the $P$-basis of both of the $T$-marker schema in (142). (17)

In §95.1-2 we have accounted only for such $ing$-phrases and $to$-phrases as are noun phrases, i.e., for $inf$ as it appears in statement 15, §67.2. But $inf$ is also introduced in statement 10, §67.2, in sentences like "he wants to read", "he likes reading", in which (as we saw in §63) the $ing$-phrases and $to$-phrases are not to be treated as noun phrases. We will return to the analysis of these sentences below, supplying a transformational analysis which will dispense with the element $inf$ along with statements 15 and 16 completely. Meanwhile we note that the passive criterion gives a further support, now on the transformational level, for the decision of §63 that these $to$-phrases and $ing$-phrases cannot be regarded as noun phrases. Thus we do not have "to read is wanted by him" from "he wants to read." But this situation is complicated by the homonymy pointed out in §63.4.

95.2. As an example of the functioning of these transformations,
we can derive the sentence

(143) being elected certainly surprised John  \( = \) (127)

This is derived by several transformations from the kernel sentences

(144) \( Z_1 = \text{They} \wedge \text{ed} \wedge \text{elect} \wedge \text{John} \)  
     (with the \( P \)-marker \( K_1 \))

\( Z_2 = \text{it} \wedge \text{ed} \wedge \text{certainly} \wedge \text{surprise} \wedge \text{John} \)  
     (with the \( P \)-marker \( K_2 \))

Applying the transformations \( T_p \) and \( T_{pd} \) (as in (123)) to \( (Z_1, K_1) \), we derive

(145) \( T_p(Z_1, K_1) = Z_3 = \text{John} \wedge \text{ed} \wedge \text{be} \wedge \text{en} \wedge \text{elect} \wedge \text{by} \wedge \text{them} \), with \( K_3 \) as its derived interpretation (in this case, a \( P \)-marker)

\( T_{pd}(Z_3, K_3) = Z_4 = \text{John} \wedge \text{ed} \wedge \text{be} \wedge \text{en} \wedge \text{elect} \), with \( K_4 \) as derived interpretation

We do not apply \( \Phi^p \) to \( (Z_4, K_4) \), as required by (123), since by condition 4, 87.4, we know that only the value of the reduced correlate figures in further transformations. We now construct the \( T \)-marker

(146) \( Z_4 \wedge K_4 \wedge I \wedge \text{ing} \wedge Z_2 \wedge K_2 \wedge I \wedge \text{ing} \wedge \Phi^p \)

conforming to the bottom line of (142), where \( T_x \) is a member of the family \( F_A \), having the restricting class \( (\alpha_1, \alpha_2, \cup, \alpha_4, \text{VP}_1) \), meeting (136).

To determine the value of (146), we apply \( T_{\text{ing}} \) to \( I(Z_4, K_4) = Z_4 \)
with the derived interpretation \( K_4 \).

(147) \( T_{\text{ing}}(Z_4, K_4) = Z_5 = \text{ing} \wedge \text{be} \wedge \text{en} \wedge \text{elect} \), with derived interpretation \( K_5 \).

\( I(Z_2, K_2) = Z_2 \) with \( K_2 \) as derived interpretation. Hence the
next step in determining the value of (146) is to apply $T_2$ to the complex string $Z_2 \# Z_2$.

(148) $T_2(Z_2 \# Z_2, K_5 * K_2) = T_2(Z_2 \# - U - I t - e d \text{ certainl} y \text{ surprise} \text{ John}, K_5 * K_2)$

$= Z_2 \# \text{ certainl} y \text{ surprise} \text{ John}$

$= \text{ ing} \text{ be} \text{ en} \text{ elect} \text{ - ed} \text{ certainl} y \text{ surprise} \text{ John}$

Applying $\Phi^P$, we derive (143), with $\Phi^P_1$ applying in three places, i.e., to the affixes ing, en, and ed.

Closer investigation of grammatical sentences shows that there are restrictions on $Z_1$ and $Z_2$ in (142), beyond that stated in condition $K_A$. Thus we do not have

(149) being elected tomorrow certainly surprised John

Apparently, a relation of tense holds between $Z_1$ and $Z_2$. Furthermore, the object of $Z_1$ and the object of $Z_2$ are related. Since (150) is not grammatical, we also do not have (151)

(150) They built John

(150) being built surprised John

The simplest way to state these limitations in general seems to be to require that $Z_1$ and $Z_2$ have the same auxiliary verb phrase (though this may be too strict) and the same object. This condition, if correct, would be added as a condition on the $P$-basis of (142), alongside of condition $K_A$, in a detailed grammar.

If this reasoning stands in a detailed study, it will provide us with a structural analogue to the intuition that (143) is related in content to (152), and not, e.g., to (153), from
which it might also be derived, given only its morphemic constitution.

(152) They elected John. It certainly surprised John.
(153) They will elect me. It certainly surprised John.

There are a variety of other limitations on the T-markers of the type (142) which should be investigated in detail with a view towards determining structural grounds for the intuitive interpretation of these sentences.

25.4. In §66.1 we noted that the selectional relation between a possessive adjective NP^S1 and the verb in a following ing-phrase, as in (154), is just that between subject and verb.

(154) John's flying is something I don't approve of.

Because of this selectional relation, we can, using a transformation closely related to T^ing, delete sentences like (154) from the kernel, thus avoiding some of the complexities which would appear in statement 9, §67.2, had we given this statement in detail. (It will be recalled that we gave only a subpart of statement 9, namely 9*, so as to avoid some of these complexities, having noted there that 9* would be shown to be sufficient for the kernel).

Flying is derived from John^flies by deleting the subject with the deformation δ^ing defined by (U, U^ing, U). Similarly, John's^flying is derived from John^flies by a deformation δ^g-ing which instead of deleting John and the initial part of the auxiliary verb (in the case of John^flies, the element S), deletes only the segment of the auxiliary, and affixes S1 to John. The transformation in question (call it "T_g-ing") is more precisely defined as follows:
(155) \( T_{-ing} \) is determined by \( (\delta_{-ing}, Q_{-ing}) \)

where \( \delta_{-ing}: Y_1 \rightarrow Y_1 \wedge Y_3 \rightarrow Y_1 \wedge Y_3 \)

\( Q_{-ing} = (NP, VP_{A1}, <have\,en>, VP_1) \)

Thus \( T_{-ing} \) converts a sentence of the form \( NP - VP_{A1} - <have\,en> VP_1 \) into the corresponding sentence of the form \( NP \wedge Y_3 - ing <have\,en> VP_1 \). The defining sequence of \( \delta_{-ing} \) is \( (U, S_1, \wedge, U, \wedge, U) \). Just as in the case of \( ing \)-phrases, the transform substitutes for abstract nouns, but with the distributional limitations of \( it \). Thus we do not have "real John's flying" or "the John's flying", though we have "real sincerity" and "the sincerity". We thus take \( it \) as the element for which these transforms are substituted, just as in the case of the simple \( ing \)-phrases. No further statement about the distribution of \( T_{-ing} \) transforms is necessary. In place of (142), then, we now state

(156) \[
\begin{pmatrix}
Z_1 \wedge K_1 \wedge I
\end{pmatrix}
\begin{pmatrix}
T_{to} \\
\{T_{-ing}\}
\end{pmatrix}
\begin{pmatrix}
Z_2 \wedge K_2 \wedge I
\end{pmatrix}
\begin{pmatrix}
T_A \\
F_A
\end{pmatrix}
\vec{P}
\]

(with condition \( K_A \))

where \( F_A \) is revised to include \( T_{-ing} \) transforms, i.e., \( NP \wedge S_1 \wedge ing <have\,en> VP_1 \) is given as one of the forms of \( \alpha_1 \) in (136).

We can now revise the definitions of \( T_{to} \) and \( T_{-ing} \) to bring out the parallel with \( T_{-ing} \). Summing up, we replace (129), (130), (140), and (155) by

(157) \( T_{-ing} \) is determined by \( (\delta_{-ing}, Q_{-ing}) \)

\( T_{-ing} \) is " " \( (\delta_{-ing}, Q_{-ing}) \)

\( T_{to} \) " " \( (\delta_{to}, Q_{to}) \)
(156) now gives the complete set of \( T \)-markers of this type, where \( F_A \) (136) is revised as mentioned below (156).

We can now avoid reference in statement 9, §67.2, to the selection of verbs by \( \mathbf{NP} \) in \( \mathbf{NP}^*\mathbf{S}_1 \). We can also drop the conditioning context "\( \mathbf{ing} \mathbf{VP}_1 \)" in the first line of statement 17. We will see below that this entire first third of statement 17 can be dropped (just as the third third of this statement is dropped, cf. first paragraph of §64.2).

We also omit the section of statement 15 that introduces \( \mathbf{Inf} \langle \mathbf{VP}_2 \rangle \mathbf{NP}_1 \), with its restricting contexts, and we omit all restrictions in statement 16 except "\( \mathbf{inf} \mathbf{V}_2 \)". We will see below that statements 15 and 16 can be dropped completely. The element \( N_A \) which is introduced in statement 8 and developed in statement 15 can also be eliminated, being replaced by \( N_{ab} \) wherever it occurs. Thus the first line of statement 15 is also dropped, and only "\( N_{ab} \rightarrow \mathbf{Inf} \mathbf{V}_T \)" remains. Finally, we eliminate the restriction in statement 21. The elimination of the restrictions in statements 15 and 21 is important, since actually we had no way to state them within the framework of the grammar corresponding to \( \mathbf{P}_3 \) as this had been formulated in chapter VI and chapter III. Cf. fn. 33, chapter VII. If we had attempted to state these restrictions formally, in terms of the available notations, the complexity of
the grammatical statement would have risen considerably. The length of $\beta_1$-derivations now drops considerably. For instance, Derivation 2 of §57.4 formerly required running through the grammar three times in 32 steps. Now we can apply one of the generalized transformations of (156) to the pair of kernel sentences "John was elected", "the voters sincerely approved of it". The second of these sentences requires the first 15 steps of Derivation 2, with $NP \to N_{ab} \to it$. The order of statements can be rearranged in such a way that no recursions are necessary in the derivation of this sentence. The significance of this fact becomes clear in , below.

25-5. In the concluding paragraphs of §72.1, in discussing (18), (19) of that section, we noted that the noun phrases "John's drinking" and "John's cooking" as in

(158) I don't approve of \[
\begin{align*}
\text{John's drinking} \\
\text{John's cooking}
\end{align*}
\]

can be viewed in two ways. On the one hand, these phrases are single units, transforms of "John drinks", "John cooks"; and being transforms of sentences, they are represented by the prime Sentence (cf. Def. 32, §83.4). But if we investigate their derived constituent structure in greater detail, we see that since "John's" is an AP in the kernel grammar (by statement 17, §67.2), these phrases are instances of AP-N, like "John's book". Cf. Def. 30, §82.2. The dual interpretation of (158) discussed in the final paragraphs of §72.1 seems to correlate with this dual-structural picture. This is not, then, a case of constructional homonymity, since only one T-marker is involved, but a case of an overspecified derived constituent structure (i.e.,
a derived interpretation which contains too much information to qualify as a \( P \)-marker. Thus the derived interpretation of (158) assigns "John's drinking", "John's cooking" to the pattern \( AP-N \) (hence \( NP \)) on the one hand; and sets off the transform as a single unit, having the structure \textbf{Sentence} on the other.

95.5. We still have not accounted for the noun phrases of the type \( \text{Inf}^V_T \) (cf. statement 15, \( \S 65.2 \)). Actually, \( \text{Inf}^V_T \) is a special case of \( \text{Inf}^V_P \), with \( V_P \rightarrow V_T \), so we need only account for these \( \text{Inf}^V_T \) phrases in the positions where \( \text{Inf}^V_P \) in general cannot occur, i.e., before of, and after the article. In the position before of we have cases like

(159) **growling of lions**

(160) **reading of good literature**

These cases caused a good deal of trouble in the system of phrase structure discussed in chapter VII, since in (159) we find the selectional relation verb-subject, and in (160), the selectional relation verb-object, holding between the italicized parts. Cf. the third case discussed in \( \S 66.1 \). Because of these selectional relations, we see that we must derive \( \text{Inf}^V_T \) from sentences of the form \( NP-V \) on the one hand (for 159) and from the \( V-N \) part of \( NP-V-N \), on the other (for 160). Accordingly we set up the transformations \( T_{\text{sub}} \) (for (159), where the selectional relation is verb-subject, i.e., subject-verb inverted), and \( T_{\text{ob}} \) (for (160), where the selectional relation is verb-object). First we define the elementary transformations \( T_{\text{p}}, T_{\text{ing-of}} \).

(161) \( T_p : Y_1 - Y_2 - Y_3 - Y_4 \rightarrow Y_4 - Y_2 - Y_3 - Y_1 \quad (= \text{(102)}) \)

\[ T_{\text{ing-of}} : Y_1 - Y_2 - Y_3 - Y_4 \rightarrow U-U-\text{ing}^Y_{3} - \text{of}^Y_{4} = \text{ing}^Y_{3} - \text{of}^Y_{4} \]
Thus the defining sequence for $\varphi_P$ is $(4,2,3,1)$ and the defining sequence for $\delta_{\text{ing-of}}$ is $(\sigma, U, \sigma, U, \text{ing}, U, \text{of}, U)$. The compound elementary transformation $\delta_{\text{ing-of}}(\varphi_P)$ thus carries $Y_1 - Y_2 - Y_3 - Y_4$ into $\text{ing}^\top \text{Y}_3 - \text{of}^\top Y_1$. $T_{\text{sub}}$ and $T_{\text{ob}}$ are defined as follows:

(162) $T_{\text{sub}}$ is determined by $(Q_{\text{sub}}, \delta_{\text{ing-of}}(\varphi_P))$

$T_{\text{ob}}$ = $(Q_{\text{ob}}, \delta_{\text{ing-of}})$

where the elementary transformations are as in (161)

and:

$Q_{\text{sub}} = (NP, VP_A, V_T, U)$

$Q_{\text{ob}} = (NP, VP_A, V_T, NP)$

(162) can obviously be abbreviated, and coalesced with the very similar (157) to the advantage of both.

$T_{\text{ob}}$ thus converts a sentence of the form $NP_1 - VP_A - V_T - NP_2$ into the corresponding sentence of the form $U - U - \text{ing}^\top V_T - \text{of}^\top NP_2$. For instance, it converts $\text{he} - \sigma - \text{read} - \text{good}^\top \text{literature}$ into $\text{ing}^\top \text{read} - \text{of}^\top \text{good}^\top \text{literature}$, which becomes (160) by $\varphi_P^{1.5}$.

$T_{\text{sub}}$ converts a sentence of the form $NP_1 - VP_A - V_T - U$ into the corresponding sentence of the form $U - U - \text{ing}^\top V_T - \text{of}^\top NP_1$. Thus it converts $\text{lions} - \sigma - \text{growl} - U$ into $\text{ing}^\top \text{growl} - \text{of}^\top \text{lions}$, which becomes (159) by $\varphi_P^{1.5}$. We define $Q_{\text{sub}}$ with the vacuous term $U$, imposing a four-term proper analysis, so as to be able to use the same underlying transformations in stating $T_{\text{sub}}$ and $T_{\text{ob}}$.

Alongside of $T_{\text{ob}}$ we have the transformation $T_{\text{s-ob}}$ which has the same relation to $T_{\text{ob}}$ as $T_{\text{s-ing}}$ (cf. (155)) has to $T_{\text{ing}}$.

Thus $T_{\text{s-ob}}$ carries "he reads good literature" into "his reading of good literature". $T_{\text{s-ob}}$ is thus defined by
(163) $T_{s-ob}$ is determined by $(Q_{ob}, \delta_{s-ing-of})$

where: $Q_{ob}$ is as in (162)

$$\delta_{s-ing-of}: Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4 \rightarrow Y_1 \wedge S_1 \wedge U \wedge \text{ing} \wedge Y_3 \rightarrow \text{of} \wedge Y_4 \rightarrow Y_1 \wedge S_1 \rightarrow \text{ing} \wedge Y_3 \rightarrow \text{of} \wedge Y_4$$

Thus the defining sequence of $\delta_{s-ing-of}$ is $(U, S_1, U, \text{ing}, U, \text{of}, U)$

and $T_{s-ob}$ converts a sentence of the form $NP_1 \wedge VP_A \wedge Y_T \wedge NP_2$ into the corresponding sentence of the form $NP_1 \wedge S_1 \wedge \text{ing} \wedge Y_T \wedge \text{of} \wedge NP_2$;

e.g., it carries John - s - read - good' literature into

John $\wedge S_1 \rightarrow \text{ing} \wedge \text{read} \rightarrow \text{of} \rightarrow \text{good} \rightarrow \text{literature}$, which becomes

"John's reading of good literature" by further mappings, from $\mathcal{P}$

(163) too can be assimilated to (162) and (157).

These three transformations $T_{sub}$, $T_{ob}$, and $T_{s-ob}$ produce noun phrases which substitute for abstract nouns, just as in the case of $T_{ing}$ and $T_{to}$. Thus we must extend the condition (136) on $F_A$ so that the products of these transformations can appear as $\alpha_{1}$. (156) need be expanded only to the extent of adding $T_{sub}$, $T_{ob}$ and $T_{s-ob}$ to the bracket containing $T_{ing}$ and $T_{s-ing}$.

Condition $K_A$ (cf. (139)) must be extended to cover $T_{s-ob}$, since we do not have "the John's reading of good literature", etc., but $T_{sub}$ and $T_{ob}$ are not limited in this way, since we have "the growling of lions", "the reading of good literature", etc., so that condition $K_A$ does not apply to these transformations. We will sum up these revisions in $\S$ 95.8, below.

Given this transformational analysis, we can drop from statement 9, $\S$ 67.2, all references to "of" and all statements about the distribution of transitive and intransitive verbs with respect to the nouns of following of-phrases, and
preceding possessive adjectives. This permits a considerable
simplification of statement 9, since the information briefly
diagrammed in (153) and (156), §66.1, need not be built into
statement 9 in all its detail. We see then, that statement 9*,
§62.1, suffices for the kernel grammar, given the transformational
analysis of these phrases. We will see below, that statement
9* can be somewhat simplified. It will also appear below (cf.
that the simplification introduced by these transformations
grows even greater as more complicated verb phrases are
considered. It stands to reason that the transformational
apparatus should remain fairly constant in complexity as more
detail is added (or as a higher level of grammaticality is
discussed), whereas the system of phrase structure should increase
sharply in complexity, since transformational analysis
stepwise permits the formation of complex phrases from already constructed
simple phrases, while in terms of phrase structure, simple and
complex phrases must in general be constructed simultaneously
by a single set of rules.

In §72.1 we noted that (159) and (160) have intuitively
different interpretations, though both have the same structure
NP - PP, where NP is the abstract ingV. The reason can
now be clearly stated in transformational terms. Though they
have the same structure on the level P, they have different
transformational structures. That is, they are differently
represented in terms of the operations by which they are
derived from the kernel, and in terms of the kernel sentences
from which they are derived. (159) is derived from
the kernel sentence "lions growl", and (160), from the kernel
sentence "he reads good literature", or the like. This provides
a systematic explanation in the terms of linguistic theory for the fact that "lions" seems to be the subject of "growling" in (159), and "good literature" the object of "reading" in (160). These interpretations are determined by the fact that we do not have "he growled lions" or "good literature reads", so that the alternative interpretation is impossible in each of these cases. However, such sentences as "the rearming of Germany" (= (13), § 72.1) and "the shooting of the hunters" (= (14), § 72.1) are subject to either interpretation, since we have both "Germany rears" and "the West rears Germany", both "the hunters shoot" and "they shoot the hunters", as kernel sentences. These are thus real cases of constructional homonymity on the level $B$, with different $T$-markers, and a different $P$-basis in the kernel. These ambiguous phrases have the same status on the level $A$ that "old men and women" has on the level $P$. Just as "old men and women" can be represented in two conflicting ways in terms of phrase structure, and has associated with it two $P$-markers in the simplest grammar of phrase structure, so "the rearming of Germany" and "the shooting of the hunters" are represented in two different ways on the level $A$, as an automatic consequence of the attempt to construct the simplest transformational analysis. "our rearming of Germany", on the other hand, is unambiguous. It must be derived from "we rearm Germany" by $T_{s\text{-ob}}^*$.

25.7. It remains to account for the fact that we have "the growling", but not "the playing the piano". The latter is excluded by condition $K_A$ (cf. (139)). The simplest way to account for the possibility of the former seems to be to set up another transformation related to $T_{\text{ing}}$ and $T_{\text{sub}}$. We thus define $T_1$,
(164) \( T_I \) is determined by \((Q_{\text{sub}}, \delta_{\text{ing}})\)

where: \( Q_{\text{sub}} \) as in (162)

\[ \delta_{\text{ing}} : X_1 \rightarrow X_2 \rightarrow \cdots \rightarrow X_4 \rightarrow U \rightarrow \text{ing} \rightarrow Y_3 \rightarrow Y_4 \]

Thus the defining sequence for \( \delta_{\text{ing}} \) is \((\sigma, U, U, \text{ing}, Y, Y, U, U)\), and \( T_I \) carries \( \text{NP-VP}_A \rightarrow Y_1 \rightarrow U \) into \( \text{ing} \rightarrow Y_2 \); e.g., it carries \text{lions} \rightarrow \emptyset \rightarrow \text{growl} \rightarrow U \rightarrow \text{ing} \rightarrow \text{growl} \), which is carried by \( \Phi_{15}^P \) into "growling". \( \delta_{\text{ing}} \) is very similar to \( \delta_{\text{ing}} \), (157).

We now extend (156) so as to include \( T_I \). The family of substitutions \( F_A \) is already defined in such a way as to include \( \text{ing} \rightarrow Y_2 \) as one possible case of \( \alpha_1 \) in (136). Condition \( K_A \) does not apply to \( T_I \), i.e., \( T_I(Z, K) = \text{ing} \rightarrow Y_2 \) can freely replace abstract nouns, even in such contexts as \text{the---}.

One bad feature of this analysis is that it assigns two such \( T \)-markers to sentences as (165) reading is fun "reading" can be derived by \( T_{\text{ing}} \) or \( T_I \) from "they read". But there is no dual interpretation of (165). This was also the case in \( \theta_{67.2} \), where (165) was derived from \( \text{Inf} \rightarrow Y_2 \) or \( \text{Inf} \rightarrow \text{VP}_1 \). This case of constructional homonymy with no intuitive correlate suggests that something may be amiss in this analysis, and that further investigation of this matter is necessary.

95.8. This winds up the discussion of \( \text{ing} \)-phrases and \( \text{to} \)-phrases occurring as abstract nouns. We can sum up the constructions of \( \theta_95.1-7 \) as follows:

As \( T \)-markers we have all strings \( Z \) in \( \mathbb{T} \) such that \( Z \) is formed from (166) by choosing strings and interpretations for the variables
"Z_1", "K_1", "Z_2", "K_2" and by selecting one of the grammatical transformations belonging to F_A, and such that the strings and interpretations chosen in constructing Z meet condition K_A.

\[(166) \begin{bmatrix} \text{T_{to}} \\ \text{T_{ing}} \\ \text{T_{s-ing}} \\ \text{T_{sub}} \\ \text{T_{ob}} \\ \text{T_{s-ob}} \\ \text{T_I} \end{bmatrix} \begin{bmatrix} \text{Z_1} \wedge K_1 \wedge I \\ \text{Z_2} \wedge K_2 \wedge I \\ \text{F_A} \end{bmatrix} \begin{bmatrix} \text{T_A} \\ \text{F_P} \end{bmatrix}\]

Condition K_A: Where W is one of T_{to}(Z_1, K_1), T_{ing}(Z_1, K_1), T_{s-ing}(Z_1, K_1), T_{s-ob}(Z_1, K_1):

(i) \[Z_2 = X \wedge \text{it} \wedge Y\]

(ii) Val_1(Z^{\text{red}}) = X \wedge W \wedge Y, where Z^{\text{red}} is the reduced correlate of Z.

The elements of (166) are analyzed as follows:

\[(167) \begin{array}{l}
\text{T_{to}} \\
\text{T_{ing}} \\
\text{T_{s-ing}} \\
\text{T_{sub}} \\
\text{T_{ob}} \\
\text{T_I-ob} \\
\text{T_I} \\
\text{T_A} \\
\text{F_A} \\
\end{array} \text{ is determined by } \begin{array}{l}
(Q_{to}, \delta_{to}) \\
(Q_{ing}, \delta_{ing}) \\
(Q_{s-ing}, \delta_{s-ing}) \\
(Q_{sub}, \delta_{ing-of}) \\
(Q_{ob}, \delta_{ing-of}) \\
(Q_{ob}, \delta_{s-ing-of}) \\
(Q_{sub}, \delta_{s-ing}) \\
(Q_A, t_A) \\
(C_A, t_A) \\
\end{array}\]

The description of the underlying deformations can be simplified by a notational convention. Given the number of terms in the restricting class of a deformation we know automatically how...
how many terms there are in the defining sequence of its underlying deformation, under the single condition that the deformation in question is not the identity, i.e., that it is not insignificant in the sense that every case to which it applies is an instance of case II, def. 10, §79.2. Hence assuming significance for all stated transformations, we can unambiguously state that a defining sequence \((a_1, \ldots, a_k, U, U, U, \ldots)\), where \(a_k\) is the last non-unit term, can be taken as defining any elementary deformation whose defining sequence is any initial part of this sequence, excluding only unit terms. Thus the defining sequence for \(\delta_{1, \text{ing}}\), which is \((\sigma, U, \sigma, U, \text{ing}, U, U)\), can simultaneously be taken as the defining sequence for \(\delta_{\text{ing}}\), which formerly had the defining sequence \((\sigma, U, \sigma, U, \text{ing}, U)\).

We can now define the elements of (167) as follows:

\[
(168) \quad \begin{cases}
\{Q_{\text{to}}\} & \{V_{\text{PA}_2}\} \\
\{Q_{\text{ing}}\} & \{V_{\text{have}^\text{en}}\} \\
\{Q_{\text{sub}}\} & \{V_{\text{A}}, V_T, U\} \\
\{Q_{\text{ob}}\} & \{NP\}
\end{cases} = \text{NP},
\]

\(Q_A = (\text{to}, \{V_{\text{PA}_2}\} \text{ VP}_1, \#, U, N_{\text{ab}}, X)\)

\(C_A(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)\) if and only if

\(i\) \(\alpha_1 = \langle NP \rangle S_1 \text{ ing } \langle \text{have}^\text{en} \rangle \text{ VP}_1\)

\(ii\) \(\alpha_2 = \#\)

\(iii\) \(\alpha_3 = X\)

\(iv\) \(\alpha_4 = N_{\text{ab}}\)

\(v\) \(\alpha_5 = X\)

\(t_A \longleftarrow (\sigma, 0, \sigma, 0, 0, 0, \sigma, 1, 0, 0)\)

\(\Pi_p \longleftarrow (4, 2, 3, 1)\)
If we had defined reductions for the level ℙ, as we did for ℙ in §51, we could simplify this statement in various ways. E.g., the entire list (167) can be eliminated, and the right hand side introduced directly into the proper place in (166). Further coalescence is also possible.

In , we will find a set of very similar transformations.

25.2. The element Inf has been eliminated from the analysis of noun phrases, but it is still introduced by statement 10, §67.2, to account for the V-to-V (e.g., "want to come") and V-ing-V (e.g., "like reading") constructions discussed in §63. But now statement 15 has been dropped completely, so that statement 16 applies only to the element Inf introduced in statement 10. We can thus drop the element Inf completely from the system of phrase structure, and with it, the remaining parts of statement 16; and we can then reformulate statement 10 as:

(169) \( VP_B \rightarrow Z_1 \langle Z_2 \langle \ldots \langle Z_m \rangle \ldots \rangle \)

where \( Z_1 \) is one of the elements of the form:

\[
\begin{align*}
&\{ V_a \xrightarrow{\text{to}} \langle VP_A \rangle \} \\
&\{ V_a \xrightarrow{\text{ing}} \langle \text{have}^\text{en} \rangle \}
\end{align*}
\]

In §67.2 we were able to give a single statement (viz., the restriction in statement 21) covering the distinction between
to and *ing* with respect to the following auxiliary phrase. But now we have found it necessary to present this distinction once for the *ing*-phrases and *to*-phrases introduced transformationally as noun phrases, in (168), and once for the elements *to* and *ing* occurring in \( VP_B \), in (169). We cannot compare the simplicity of the analysis in \( P \) and the analysis in \( T \) in this respect, since the restriction in statement 21 was not literally admissible into the grammar of phrase structure (cf. fn33, chapter VII). But in any event, it appears that transformational analysis is not efficient in this respect. But we will see below, in , that this inadequacy will be remedied with a deeper analysis of \( VP_B \).

We are still left with a statement that is not literally admissible into the kernel grammar, since in developing the form of grammatical statement in chapters III, VI, no allowance was made for recursive statements of the form of (169) or of statement 10 which it replaces. But we will see below that this statement can be eliminated altogether.

With this alteration, then, statements 15 and 16 are dropped completely from the kernel grammar.

95.10. The set of transformations that we have discussed in §92 provide a criterion for the assignment of a string to \( VP_1 \). Whenever we have a phrase *ing* \( \langle \text{have}\_\text{en} \rangle \_W \), *to* \( \langle VP_{A2} \rangle \_W \), etc., functioning as a noun phrase, the statement of the transformations that appear in (165) will be simplified if \( W \) is analyzed as a \( VP_1 \). Otherwise the definitions of \( Q_A \) and \( C_A \) in (168) will have to be modified to include \( W \) as a new form. This criterion will be of some use below.

95.11. We have not yet accounted for such *ing*-phrases as
(170) (a) not to see him
(b) not seeing him
(c) John's not seeing him

It is possible to include them, by slight revisions of (166) and (168). Considering only the case of (170a), suppose that we restate $\delta_{to}$, $Q_{to}$ as

(171) $\delta_{to}: Y_1 - Y_2 - Y_3 - Y_4 \rightarrow U - U - Y_3 - to \rightarrow Y_4 = Y_3 - to \rightarrow Y_4$

$Q_{to} = \langle NP, VP_{A1}, \{ U, not \}, \langle VP_{A2} \rangle, VP \rangle$

Thus the defining sequence for $\delta_{to}$ is now $(\overline{U}, U, \overline{U}, U, U, to, U)$. $T_{to}$ has exactly the same effect as before on strings without not, and it will carry, e.g., "John will not see him" into (170a). (166) will then fit the resulting phrase (170a) properly into sentences.

Analogous alterations of $T_{ing}$ and $T_{s-ing}$ will give (170b-c). This is quite a simple emendation. It seems intuitively adequate, and in particular, it accounts for the feeling noted in §92.2 that (86c) is not a parallel form to (86a-b), as well as for the homonymy mentioned in §92.2.

There is, however, one difficulty connected with this extension. All such phrases as (170) will have to be derived from negative sentences where the $VP_{A1}$ entirely precedes not, i.e., where $VP_{A1} = -M, S, or ed$. And when the main verb is be, only $VP_{A1} = M$ will qualify. These restrictions can be understood by noting that not in all other cases falls after have, be, etc., so that the sentences in question will not be of the proper form for $T_{to}$ to apply, i.e., they will not be analyzed in such a way as to meet $Q_{to}$. This is likely to cause difficulty in
a more refined analysis than ours, when certain considerations concerning the conditions on the auxiliary phrases of \( Z_1 \) and \( Z_2 \) in (166) are introduced (cf. \( \S 95.3 \)). Because of this possible difficulty, we have not included this revision above. It may be that some alternative analysis may ultimately prove to be preferable.

95.1. In chapter VI we used as examples sentences of the form (172) (a) the detective brought in the suspect

(b) the detective brought the suspect in

but we did not include these in the analysis of English syntax in chapter VII. The reason for this exclusion was that it did not seem possible to arrive at any adequate analysis of these sentences at that point. On the basis of the criteria of analysis which were then established there seemed to be no reason to relate these sentences, and no way to express the relationship between them. The only way to analyze sentences such as (172) would have been as instances of a new type of \( VP_1 \), namely,

(173) \( VP_1 \rightarrow V^NP \)

Had we adjoined (173) to statement 3, \( \S 67.2 \) (the analysis of \( VP_1 \)), we would have discovered in stating the restrictions on occurrence for this position that \( V_1^1 \) and \( P_1^1 \) appear together in the construction \( V_1^1 NP P_1^1 \) just in case \( V_1^1 P_1^1 \) also appears in the construction \( NP V_1^1 P_1^1 NP \) (e.g., (172a)). Thus the whole set of restrictions that must be stated for constructions of the latter type would have to be repeated for verb phrases of the form (173). This suggests that we relate these constructions transformationally, either deriving (172a) from (172b) or vice
versa. There are a variety of reasons for choosing to derive (172b) from (172a), rather than the other way around.

Note first that if (172b) is taken as a kernel sentence, then we need one new sentence form, namely, (173). However there is good reason for analyzing (172a) as an instance of the already familiar form NP-V-NP with a compound verb "bring in". For one thing, the conjunction criterion favors this analysis. We have "they brought in and questioned the suspect", but not "they brought in the suspect and in his accomplice". The impossibility of the latter rules out the analysis NP-V-PP, with the verb "brought". The existence of the former requires that "brought in" be in the same class as "questioned", or in the same class as "questioned the suspect". But we do not have "they brought in" as a sentence, and we do have "they brought in the suspect". Hence we cannot analyze this sentence as an instance of (NP-VP) \( \text{and} \) (NP-VP) with the first conjunct NP-VP- they brought in; but we can analyze it as an instance of NP-(V and V)-NP, with "brought in" the first V and "questioned" the second. Hence the conjunction criterion favors the analysis of "brought in" as an instance of V\( _\text{T} \), along with "questioned" (actually, "bring in" and "question" receive this analysis, with ed being the affixed auxiliary phrase in this instance).

Further support for this analysis appears in the criterion of parenthetical intrusion (cf. §50.2). We have "he looked, as you can see, in every possible place", but not "he looked, as you can see, up the case in the records". Hence the latter case must have the constituent break after "look up", not after "look". I.e., it cannot be an instance of NP-V-PP, like "he looked in every possible place". The fact that "he looked up, as you can see, the case in the records" is impossible, is irrelevant here,
since, as we noted in §60.2, parenthetical intrusion can in general not occur between verb and object.

A further reason for the analysis of such sentences as (172a) as instances of NP-V-NP with a compound verb, is given by the passive criterion. This analysis is necessary in order to provide for the generation of such sentences as (174) the suspect was brought in by the detective.

Application of the ing-phrase criterion discussed in §95.10 leads to the same conclusion. Since we have the phrase (175) ing - be\(^^\text{en}\)brought\(^^\text{in}\) (=being brought in)

we see that "be brought in" (=be\(^^\text{en}\)bring\(^^\text{in}\)) is a VP\(_1\), so that bring\(^^\text{in}\) must be a V\(_T\). All of these criteria thus converge on the analysis NP-V-NP for (172a).

But if (172b) is selected for the kernel with the analysis (173), (172a) being derived from it by transformation, then the analysis NP-V-NP will not be conferred on (172a) as its derived constituent structure, and it will be necessary to provide for this by some special condition. If (172a) is given in the kernel, then it is only necessary to list "bring in" in statement 23, §57.2, as a V\(_T\), along with all other transitive verbs, for the proper constituent structure to be assigned.

Applied to (172b), the conjunction criterion determines that "brought the suspect in" must be a VP\(_1\), since we have "the detective brought the suspect in and questioned him", where "questioned him" is a VP\(_1\). But this analysis of (172b) will be provided as its derived constituent structure if it is transformationally derived from (172a), as we saw in §57.4 (we found in §57.5 that the condition on derived constituent
structure under which this analysis is assigned to (172b) is a very general one, which is needed for many other cases as well. Thus all considerations lead us to derive (172b) from the sentence (172a) which has the form \textit{NP-V-NP}, "bring in" being the compound transitive verb.

We must therefore add to statement 23, §67.2, a set of statements of the form

\[(176) \mathcal{V}_T \rightarrow \mathcal{V}_{sep} \wedge \mathcal{P}, \text{ where...}\]

where \(\mathcal{V}_{sep}\) is a certain subclass of verbs, and ... gives the restricting conditions on the choice of \(\mathcal{P}\), given an element of \(\mathcal{V}_{sep}\). We then construct the transformation \(\mathcal{T}_{sep}\) such that

\[(177) \mathcal{T}_{sep} \text{ is determined by } (Q_{sep}, \pi_{sep})\]

where: \(Q_{sep} = (\text{NP, VP}_A, \mathcal{V}_{sep}, \mathcal{P}, \text{NP})\)

\[
\pi_{sep}: Y_1-Y_2-Y_3-Y_4-Y_5 \rightarrow Y_1-Y_2-Y_3-Y_4-Y_5
\]

\(\pi_{sep}\) is thus the permutation with the defining sequence \((1,2,3,5,4)\), and \(\mathcal{T}_{sep}\) converts a string of the form \(\text{NP-VP}_A-\mathcal{V}_{sep}-\mathcal{P}-\text{NP}\) into the corresponding string of the form \(\text{NP-VP}_A-\mathcal{V}_{sep}-\text{NP-VP}\). In particular, it converts (172a) into (172b). As a \(\mathcal{T}\)-marker we have

\[(178) Z^K \mathcal{T}_{sep} \wedge \mathcal{P}\]

By Def. 32, §83.4, we see that \(\mathcal{V}_{sep}-\text{NP-P}\) is a \(\text{VP}_1\) in the transform, and that \(\text{VP}_A-\mathcal{V}_{sep}-\text{NP-P}\) is a \(\text{VP}\). This accords perfectly with the conjunction criterion and with the \textit{ing}-phrase criterion, since we have "they brought him in and questioned him" and "bringing him in".

\(\S 6.4\) There is one feature of the usage of sentences of the form (172) that is not brought out by our analysis, and that may indicate a limitation of our whole approach. While in the
case of (172) both (a) and (b) are grammatical, in general the separability of the preposition is determined by the complexity of the NP object. Thus we could scarcely have

(179) The detective brought the man who was accused of having stolen the automobile in.

It is interesting to note that it is apparently not the length in words of the object that determines the naturalness of the transformation, but rather, in some sense, its complexity. Thus "they brought all the leaders of the riot in" seems more natural than "they brought the man I saw in". The latter, though shorter, is more complex on the transformational level since it has the infixed sentence "I saw". We will see below that this is a transformational construction. A good deal of further study is needed here to determine the nature of this process and to define properly the relevant sense of complexity of the object.

As the object becomes more complex, then, the naturalness of the transform decreases. This is systematic behavior, and we might expect that a grammar should be able to state it. But it may turn out to involve probabilistic considerations for which our system has no place as it now stands. This is of course not the only case where a grammatical rule, if applied over the full scope of its domain, leads to certain unacceptable results. But it is a particularly clear instance of a phenomenon which may prove to be important in a more general treatment of linguistic structure.

26.3. The separation of the preposition is obligatory when the object is a pronoun. Thus we have
(180) The detective brought him in
but not "the detective brought in him". There are two ways
in which this situation might be handled. Either we can
require that \( V_{\text{sep}}^P \) never occur with pronominal objects, and
can add \( V_{\text{sep}}^P \text{Pronoun}^P \) as a new sentence form, or we can add
a new mapping \( \Phi_{11}^P \) which operates exactly like \( T_{\text{sep}} \). Independently
of how we choose to analyze (172) we must choose the second
alternative, because of the passive criterion. Since we have
(181) he was brought in by the detective
we must have the active "the detective-brought in-him" as the
product of a derivation in the kernel grammar, i.e., as the
product of a \( P \)-marker, with (180) derived by a mapping. Note
that we cannot rely on the transformational analysis with \( T_{\text{sep}} \)
in this case, since the inversion is obligatory. We give this
mapping as \( \Phi_{11}^P \).

(182) \( \Phi_{11}^P \) is determined by \( (Q_{\text{sep}}^i, T_{\text{sep}}) \)
where: \( Q_{\text{sep}}^i = (NP, VP_A, V_{\text{sep}}^P, \text{Pronoun}) \)
\( T_{\text{sep}} \) is as in (177)

We will see below that \( \Phi_{11}^P \) must precede \( \Phi_{15}^P \). Since
this mapping must be given anyway, and since \( T_{\text{sep}} \) is so similar
to it, it follows that setting up \( T_{\text{sep}} \) adds almost nothing to
the complexity of the grammar.

We noted in §25.2 that as the object becomes longer and
more complex, the naturalness of the transform decreases.
Conversely, as the object becomes shorter, its naturalness
increases. There is a certain sense in which pronouns are the
'shortest objects, namely, they never carry stress (except the contrastive stress which can occur quite freely with almost any element). The fact that the transformation is obligatory for pronouns might be considered as the limiting case of the increase in naturalness with decrease in length. Thus the treatment of $T_{se}$ and $\Phi^p_{1}$ might somehow be unified. We will discover further suggestions to this effect below.

27.1. We have seen that the grammatical sketch of §67.2 must be extended to include statements of the form (176). By the same reasoning as in §96, we will now see that there are a great many other instances of 'compound verbs' with separable elements.

We saw that one form of the verb phrase, namely (173), can be struck out of the kernel grammar by transformational analysis. Consider now the $VP_{1}$ of the form

(183) $V_{n}^{a}NP^{a}Predicate$

as introduced in statement 3, §67.2. An instance is

(184) they - consider - John - a fool $(NP_{1} - V_{n} - NP_{2} - Predicate)$

We must analyze this sentence in such a way as to admit the possibility of the passive

(185) John is considered a fool by them

There seem to be three alternative solutions available for this problem:

I. We may add a new transformation $T_{p}$ which converts $NP_{1}-V_{n}-NP_{2}-Predicate$ into $NP_{2}-is^{en}-V_{n}-Pred-by-NP_{1}$. $T_{p}$ is related to, but distinct from, the passive transformation $T_{p}$.
We must then revise $T_p$ (§94.5) so that it can apply to

$$\text{X}_1^\wedge \text{en}^\wedge \text{V}^\wedge \text{Predicate} - \text{by}^\wedge \text{NP} \quad (\text{cf. } (122))$$

as well as to $\text{X}_1^\wedge \text{en}^\wedge \text{V}^\wedge \text{by}^\wedge \text{NP}$, since we must account for "John was considered a fool."

II. We may leave the passive transformation unchanged

(i.e., \( \text{NP}_1\wedge \text{VP}_1\wedge \text{V} - \text{NP}_2 \rightarrow \text{NP}_2\wedge \text{VP}_1\wedge \text{be}^\wedge \text{en}^\wedge \text{V} - \text{by}^\wedge \text{NP}_1 \)) and consider \( \text{NP}_2\wedge \text{Predicate} \) to be the complex object of \( \text{V}_1\). The result of applying $T_p$ will then be "John a fool is considered by them"

$$\text{NP}_2\wedge \text{Predicate} - \text{is}^\wedge \text{en} - \text{V}_1 - \text{by}^\wedge \text{NP}_1$$

We can then add a mapping \( \Phi_{1X}^P \) which carries this expression into (185).

III. We may leave $T_p$ as is and consider (184) to be formed by application of a mapping \( \Phi_{1/3}^P \) from

(186) They-considered a fool - John

(186) is then the product of a $P$-marker, i.e., a sentence derived in the kernel grammar. "consider - a fool" is then a compound verb like "bring in", and (183) is struck from the $\text{VP}_1$-analysis, statement 3, §67.2. $T_p$ now applies directly to (186), analyzed as $\text{NP} - \text{V} - \text{NP}$, giving (185). We have already seen that $T_p$ applies prior to any mapping.

The first alternative adds two new transformations. The second and third each add a single new mapping.

Alternative II is ruled out immediately by the $\text{ing}$-phrase criterion of §95.10. Since we have

(187) being considered a fool \( (= \text{ing}^\wedge \text{be}^\wedge \text{en}^\wedge \text{consider}^\wedge \text{a}^\wedge \text{fool}) \),

we know that $\text{be}^\wedge \text{en}^\wedge \text{consider}^\wedge \text{a}^\wedge \text{fool}$ must be a $\text{VP}_1$. Hence consider and $\text{a}^\wedge \text{fool} \quad \text{must constitute a single $\text{en}$-marker element in } (185)$. Now it is possible to formulate $\Phi_{1X}^P$ so that "a fool" in "John a fool is considered by them" is adjoined by the mapping to "consider";
the derived constituent structure of "consider a fool" assigns this phrase to $\bar{V}_\pi$, just as "consider" is a $\bar{V}_\pi$ ("consider" being the root of "consider a fool"—cf. Def. 32, §83. 4). This would be a case almost exactly like case (iii), §83. 5. But $\bar{T}_\text{ing}$, which gives (187), must apply before $\bar{\Phi}_1^P$ and after $\Phi_1^P$; hence $\Phi_1^P$ must apply before $\bar{\Phi}_1^p$. Hence if the result of $\bar{\Phi}_1^p$ is that "consider a fool" is a $\bar{V}_\pi$ (like its root "consider"), then $\bar{\Phi}_1^p$ will carry $\text{ing}^{\prime}\text{consider}^{\prime}\text{a}^{\prime}\text{fool}^{\prime}$ into $\text{consider}^{\prime}\text{a}^{\prime}\text{fool}^{\prime}\text{ing}$, $\text{ed}^{\prime}\text{consider}^{\prime}\text{a}^{\prime}\text{fool}^{\prime}$ into $\text{consider}^{\prime}\text{a}^{\prime}\text{fool}^{\prime}\text{ed}$, etc. Thus if we choose alternative II we are either forced to complicate $\bar{\Phi}_1^p$ somehow so as to exclude this possibility, or else to formulate $\bar{\Phi}_1^p$ in such a way that (187) will not be forthcoming. Neither of these difficulties arises in alternative III, so that III at least is simpler than II.

The choice between I and III is resolved in favor of III when we note the parallel between the solution of III and the considerations of §96. $\bar{\Phi}_1^p$ as introduced in III is a slight extension of $\bar{\Phi}_1^p$ as defined in §96. 2. We thus replace (182) by

(188) $\bar{\Phi}_1^p$ is determined by $(Q_{\text{sep}}, T_{\text{sep}})$

where $Q_{\text{sep}} = \{(\text{NP}, \text{VP}_A, \Phi_{\text{sep}}^{\prime}, \text{P}, \text{Pronoun}, \text{NP}) \}$

Thus $\bar{\Phi}_1^p$ carries $\text{NP-VP}_A-\text{V}_{\text{sep}}-\text{P}-\text{Pronoun}$ into $\text{NP-VP}_A-\text{V}_{\text{sep}}-\text{Pronoun-P}$

and it carries $\text{NP-VP}_A-\text{V}_h-\text{Predicate-NP}$ into $\text{NP-VP}_A-\text{V}_h-\text{NP-Predicate}$.

In the first case, it carries "they-brought in-him" (= they-edbring-in-him) into "they brought him in" (= they-edbring-him-in); in the second case, it carries (186) into (184). $T_p$ and $\bar{T}_{pd}$ remain unchanged, and no new mappings or transformations are
added. We note as an additional argument for III over II that the mapping $\Phi^P_{1\alpha}$ introduced in II is new, while the mapping $\Phi^P_{1/2}$ introduced by III already appears, in part, in (182).

The total effect on the complexity of the grammar of alternative III, then, is to replace (182) by (188), and to delete (183) from the kernel grammar. Alternative II requires that we add to the set of grammatical transformations two new transformations, $\tilde{T}_p$ and $\tilde{T}_{pd}$. But no matter how $\tilde{T}_p$ is formulated it will face the same difficulties that made a simple formulation of $\Phi^P_{1\alpha}$ impossible. As we saw in discussing alternative II, be "en consider" a "fool" must be considered a $VP_1$, so that "en consider" a "fool" must be an element of the form $en^TV_T$ (since we have no other forms of $VP_1$ beginning with $be^{en...}$). Further support for this analysis of $en^consider^a^fool$ comes from the conjunction criterion.

Since we have

(189) John was mistreated and considered a fool

we know that "considered a fool" (= $en^consider^a^fool$) must be the same type of element as "mistreated", i.e., an $en^TV_T$. But the only way to achieve this result in terms of $\tilde{T}_p$, just as in the case of $\Phi^P_{1\alpha}$, is to formulate $\tilde{T}_p$ so that it attaches "a fool" to "consider" as the root term, with the result that "consider a fool" is an $X$ wherever "consider" is an $X$. Thus $\tilde{T}_p$ will be the transformation such that, under $\tilde{T}_p$,

(190) $NP_1-VP_A-VP_2-NP_2$-Predicate $\rightarrow NP_2-VP_A-be^{en^TV_P}$-Predicate-by-$NP_1-U$

where dashes indicate the terms of the proper analysis. Thus $\tilde{T}_p$ will be a complex transformation of a new type, with a deformation, an $f$-transformation, and a permutation figuring in the definition of its underlying elementary transformation. Furthermore, the
difficulties faced by $\Phi_1^P$ now reappear. Since Predicate has been attached to $V_h$ by an $\epsilon$-transformation, it follows that $V_h^\wedge$ Predicate is a $V_h$, so that $en^\wedge V_h^\wedge$ Predicate is carried by $\Phi_1^P$ into $V_h^\wedge$ Predicate$^\wedge$en (i.e., en$^\wedge$consider$^\wedge$a$^\wedge$fool$^\wedge$consider$^\wedge$a$^\wedge$fool$^\wedge$en).

This difficulty is avoided only by alternative III. Here $V_h^\wedge$ Predicate is listed (along with $V_{tl}$, $V_{sep}$, etc.) in statement 23, §67.2, as a $V_T$. Since "consider a fool" is not formed by attaching "a fool" to "consider", "consider a fool" is not a $V_h$, though it is a $V_T$, just as "eat", "bring in", etc. are $V_T$'s. The passive transformation $T_p$ is formulated to apply to $V_T$, and $\Phi_1^P$ is formulated so as to apply to all initial subclasses $V_{tl}$, $V_{sep}$, $V_h$, etc. of $V_T$.

Thus various considerations lead to III as the correct solution. This seems to be intuitively correct. (185) does appear to be a passive, intuitively, in the same sense as "John is mistreated by them". In (184) and (185), "a fool" seems intuitively closely related both to "consider" and to "John". Its relation to "consider" is explained by the fact that "consider a fool" is in both cases the verbal element. The formal reason for the relation to "John" will appear below in

As a result of this transformational analysis, certain (e.g., statement 5 must be revised) changes are necessary in the kernel grammar §67.2A. But as these alterations will themselves be superseded below (we will not construct them here. We will see in that compound verbs of the form $V_h^\wedge$ Predicate are themselves formed transformationally, so that they need not actually be listed in Statement 23.
97.2. In §94.2 we considered the problem of determining the crudest possible grammaticalness assumptions from which it will follow that the passive transformation, in its correct form, must appear as a grammatical transformation in English. We saw there that the distinction between abstract and proper nouns would be sufficient to establish this transformation. But now we find that the distinction between singular and plural is actually sufficient.

We noted, in §94.2, that the passive transformation in the form we have given will be necessary (19) if it is the case that for certain choices of \( \text{NP}_x \), \( \text{NP}_y \), and \( V \), (191) \((= (107))\) is grammatical, but not (192) \((= (108))\).

\[(191)(a) \text{NP}_x - V - \text{NP}_y \]
\[(b) \text{NP}_y - \text{is}^{\text{en}}V - \text{by}^{\text{en}}\text{NP}_x \]

\[(192)(a) \text{NP}_y - V - \text{NP}_x \]
\[(b) \text{NP}_x - \text{is}^{\text{en}}V - \text{by}^{\text{en}}\text{NP}_y \]

As instances of (191) and (192), we have

\[(193)(a) \text{They consider John a fool (=}\text{they}\text{-consider}^{\text{en}}\text{a fool} - \text{John}) \]
\[(b) \text{John is considered a fool by them (=}\text{John}\text{-is}^{\text{en}}\text{consider}^{\text{en}}\text{a fool}\text{-by}^{\text{en}}\text{them}) \]

\[(194)(a) \text{John considered them a fool} \]
\[(b) \text{They were considered a fool by John} \]

with \( \text{NP}_x = \text{they} \), \( \text{NP}_y = \text{John} \), and \( V = \text{consider}^{\text{en}}\text{a fool} \). Since the passive transformation applies before any mapping, it applies in particular before the mapping \(\Phi^p_{11}\) which carries the
parenthesized forms of (193) into the non-parenthesized forms. Hence it operates on the parenthesized form of (193a). Since (193a-b) are grammatical, and (194a-b) are not, the transformational analysis of passives is required. The point here really is that we have found a case of numerical agreement between the verb and the object. The verb contains an element "a fool" which must agree in number with the object "John". This verb-object relation is carried over into the passive. The subject of the passive (= the object of the underlying form) must agree in number with the verbal form which was the main verb of the underlying active.

We can put this somewhat differently. One might ask: how can we tell that the passive carries NP1-V-NP2 into NP2-\text{is}^{\text{Ven}}-\text{by}^{\text{NP1}}, and not into NP1-\text{is}^{\text{Ven}}-\text{by}^{\text{NP2}}. We could not tell this from cases like "John loves Mary", since we have both "John is loved by Mary" and "Mary is loved by John". Harris has pointed out (20) that we have "Casals plays the cello", "the cello is played by Casals", but not "Casals is played by the cello". This will clearly be the case if our analysis of grammaticality is at all adequate -- if it is, then the third sentence will be shown to be only partially grammatical (cf. chapter IV) and will thus not be generated by the grammar (cf. fn.16, this chapter). But now we see that we have (193a-b) but not (194b) as grammatical sentences. Thus any grammar powerful enough to recognize the distinction between singular and plural would correctly solve the problem of stating the passive transformation. This transformation, then, can be considered very firmly established on syntactic grounds.

This argument appears circular in one sense. We relied in
part on the passive transformation for the analysis of such sentences as (183), and we have now used the resulting analysis of such sentences to indicate how limited are the assumptions required by the correct version of the passive transformation. But in this case we have precisely the kind of 'circularity' that marks a correct analysis. This is a case where various parts of the analysis support one another — what is the simplest solution for one problem turns out at the same time to be the simplest solution for another. The correct analysis of all the interdependent cases of passives, ing-phrases, and sentences of the form (183) follows, with no circularity, from the criterion of simplicity applied to grammar construction. In we will consider a formal condition on grammars from which this support for the passive transformation will be derived in a more direct manner.

98. Exactly the same line of reasoning that we applied in the case of sentences of the form (183) leads us to strike out (195) and (196) from the VP₁-analysis in statement 3, § 67.2.

(195) \( v \in NP_1 \text{ to be Predicate} \) (they know him to be a fool)
(196) \( v \in NP_1 NP_2 \) (they elected him an officer)

Since we have

(197) he is known to be a fool
(198) he was elected an officer

we must add "know— to be a fool", "elect—an officer", etc., to the list of compound verbs. (195) and (196) are thus reduced to the general form \( NP-v-NP \), with \( NP_1 \) being the object in both cases. This explains, incidentally, why the pronoun appears as "him", "her", "them", in this position.
Instead of adding still further qualifications on \( Q'_{\text{sep}} \) (cf. (188)), we can recognize a subdivision of \( V_{\text{complement}} \) (21) and, replacing (188), we can define \( \Phi'_{11} \) as

\[
(199) \Phi'_{11} \text{ is determined by } (Q'_{\text{sep}}, T_{\text{sep}}) \text{ where } Q'_{\text{sep}} = \{ \text{NP, VP}_A, V, \{ \text{P, Pronoun} \}, \text{Complement, NP} \}^3
\]

In the analysis of \( V_{\text{complement}} \) we will then indicate, by the necessary restricting statements, that if \( V \rightarrow V_{\text{h}}, \text{Complement} \rightarrow \text{Predicate} \); if \( V \rightarrow V_{\text{e}}, \text{Complement} \rightarrow \text{to be Predicate} \); if \( V \rightarrow V_{\text{e}}, \text{Complement} \rightarrow \text{NP, etc.} \) In introducing further instances of this pattern, it will not be necessary to extend the transformational analysis, but only to expand the subdivision of \( V_{\text{complement}} \).

99.1. These developments permit an important distinction, which could not have been made hitherto, among sentences of the form

\[
(200) \text{NP}_1 - V - \text{NP}_3 - PP
\]

In \( \S 67.2 \), there was no reason to analyze sentences of this kind other than as \( \text{NP}_1 - V - \text{NP}_2 \), with \( \text{NP}_2 \rightarrow \text{NP}_3 \rightarrow PP \). But it is intuitively clear that while this analysis fits such sentences as

\[
(201) \text{the police suspect the man behind the counter}
\]

it is not proper for either (202) or (203) .

\[
(202) \text{the police put the man behind bars}
\]

\[
(203) \text{the police questioned the man behind closed doors}
\]

The same distinctions can easily be found with different prepositions.

The distinction between these three types of cases can be made in terms of the passive transformation. The passives
corresponding to (201-3) are, respectively,

(204) the man behind the counter was suspected by the police
(205) the man was put behind bars by the police
(206) The man was questioned by the police behind closed doors.

The fact that "the man was questioned behind closed doors
by the police" also occurs as a grammatical
sentence is irrelevant here. (202) and (203) are distinguished
by the fact that (206) is permissible, while "the man was put
by the police behind bars" is not. There is another distinguishing
transformation which we have not discussed. From (203) we can
form "behind closed doors, the police questioned the man", but
there are no comparable forms for (201-2). It is interesting
to note that the where-transformation forms "where did they
{put
{question} the man" from (202-3), but it does not apply
to (201). It is clear that we have only begun to scratch the
surface in the analysis of these forms.

From comparing (201-3) with the passives (204-6), it follows
that (201) is of the form NP₁-V-NP₂, with NP₂ "the man behind
bars", and that (202) is another instance of a compound verb
V^Complement, with the complement, in this case, being the
prepositional phrase behind bars. Thus the verb in (202) is
"put - behind bars". (203) requires an extension of the analysis
of §67.2 to include prepositional phrases added to VP₁ as a whole.
It might appear that the PP's could just as well be added to the
whole sentence, i.e., that "<PP>" could be added to statement 1,
§67.2, instead of statement 3. But the ing-phrase criterion for \( VP_1 \) rules this out, since we have "questioning him behind closed doors". The conjunction criterion leads to the same conclusion.

With this alteration, and with the various deletions that have taken place until now, statement 3, §67.2, appears as

\[
(207) \quad VP_1 \rightarrow \langle L_2 \rangle \langle VP_B \rangle \left\{ \begin{aligned}
V_f \langle ^{\text{NP}} \text{NP} \\
V_g \langle \text{that} > \text{Sentence} \\
V_b \langle \text{be} > \text{Predicate} \\
V_t \langle \text{NP} >
\end{aligned} \right\} \langle PP \rangle
\]

where \( PP \) is a certain class of prepositional verbs that are 'detached' from the verb.

The constituent structure of (201), (202) is thus basically \( NP_1 - V - NP_2 \), with \( NP_2 \) in (201) being "the man behind the counter", and \( V \) in (202) being "put-behind bars"; and the analysis of (203) is \( NP-V-NP-PP \). This seems to be in accord with the intuitive analysis of these sentences.

§99.2. It is now necessary to revise the passive transformation \( T_p \) to account for sentences of the type (203). It seems that any such sentence can have either a form like (205) or (206) in the passive, as we noted for (203) directly below (206). This fact leads us to replace \( T_p \) by a family of transformations \( F_p \).
The underlying elementary transformation for $F_p$ is $t_p^i$, where

$$t_p^i = \delta_p^i (\pi_p^i)^i,$$

where

$$\pi_p^i : I_1 - I_2 - I_3 - I_4 - I_5 - I_6 \rightarrow I_4 - I_2 - I_3 - I_5 - I_1 - I_6$$

$$\delta_p^i : Z_1 - Z_2 - Z_3 - Z_4 - Z_5 - Z_6 \rightarrow \neg Z_1 - \neg Z_2 - \neg Z_3 - \neg Z_4 - \neg Z_5 - \neg Z_6$$

Thus the defining sequence for $\pi_p^i$ is $(4, 2, 3, 5, 1, 6)$, and the defining sequence for $\delta_p^i$ is $(U, U, U, U, \neg \text{en}, U, \text{by}, U, U, U, U)$ and $t_p^i$ carries $I_1 - I_2 - I_3 - I_4 - I_5 - I_6$ into $I_4 - I_2 - \neg \text{en} \neg I_3 - \neg \text{by} \neg I_5 - I_1 - I_6$.

We can then define $F_p$ as follows:

(209) $F_p$ is determined by $(C_p, t_p^i)$

where $t_p^i$ is as in (208)

$$Q_p(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) \text{ if and only if }$$

$$\begin{align*}
\alpha_1 &= NP \\
\alpha_2 &= VP_A \\
\alpha_3 &= \langle D \rangle V_T \\
\alpha_4 &= NP \\
\alpha_5 &= U \langle PP_A \rangle \\
\alpha_6 &= U \langle PP_A \rangle
\end{align*}$$

This replaces (105). For sentences with no $PP_A$, the effect of a transformation from $F_p$ with $\alpha_5^A = \neg U$ is exactly that of $T_p$ of (105). If $PP_A$ does occur, then we can choose a transformation of $F_p$ with $\alpha_5 = PP_A$ and $\alpha_6 = U$, giving a passive of the form of "the man was questioned behind closed doors by the police"; or we can choose a transformation of $F_p$ with $\alpha_5 = U$ and $\alpha_6 = PP_A$, in which case we derive a passive of the form (206).

We must revise $T_{pd}$ (cf. (122)) correspondingly. By "$T_{pd}$" we will henceforth understand the transformation defined as follows:

(210) $T_{pd}$ is determined by $(Q_{pd}^i, t_{pd}^i)$

where $Q_{pd}^i = \{ (I_1 \neg \text{en} V, \text{by} \neg \text{NP}, U \langle PP_A \rangle) \}$

$$t_{pd}^i : I_1 - I_2 - I_3 \rightarrow I_1 - U - I_3 - I_1 - I_3$$
When we choose \( U \) as the third term of \( Q_{pd} \), we have the case covered by (122). Sentences like "the man was questioned behind closed doors" are derived from the corresponding sentence of the form (206). This is the simplest way to cover all cases. Such sentences as "the man was questioned behind closed doors by the police" do not fall within the domain of \( T_{pd} \). Hence (206) appears to be the basic form of the passive for such sentences as (203). (210) now replaces (122).

As the \( T \)-marker schema for passives we now have, instead of (123), the schema (211).

\[
(211) \; \mathcal{A}^{K}_{p} \prec T_{pd} \prec \mathcal{F}^{p}.
\]

Note that (201) is not really of the form \( NP_1 - V - NP_2 \), as we have been rather loosely describing it in our informal exposition. In (201) the only parts that are actually \( NP \)'s are "the police" and "the man behind the counter", but not "the man" (though this is an \( NP \) in other sentences). If this distinction is not kept clear, then \( T_p \) and \( T_{pd} \) will not apply in the intended way to (201). The grammatical sketch of the kernel grammar in \( \text{67.2} \) gives a correct account of this situation. In the kernel grammar, the basic steps in the derivation of (201) are

\[
(212) \begin{align*}
1. \text{Sentence} \\
2. \text{NP-VP}_A-\text{VP}_1 \\
3. \text{NP-VP}_A-\text{V}_T-NP \\
4. \text{NP-VP}_A-\text{V}_T-\mathcal{T}^{N}\mathcal{Z}^{PP} \\
5. \text{the police- } \emptyset - \text{suspect - the man behind the counter}
\end{align*}
\]

Thus "the man behind the counter" is an \( NP \), but "the man" is not, in this sentence. The most that we can say about "the man" is
that it is a $T^\wedge N^\wedge \emptyset$, in terms of §67.2. It is important to remember that the prepositional phrase in (201) is introduced by statement 7; in (202), by statement 23 (extended to include $V^\wedge \text{complement}$); in (203), by (207) (replacing statement 3).

In §67.2 we did not go into the distribution and restrictions on prepositional phrases (and in fact, we could not have, without a more thorough investigation of grammaticalness). But these restrictions are certainly real enough, and it is apparent that a considerable simplification in any such distributional statement is effected by the transformational analysis, under which the restrictions need be stated only for the kernel sentences.

Just as we must revise the passive transformation to account for the element $PP_\alpha$, we must revise the definition of $T_{sep}$ and the mapping $\varphi_{l1}^P$. (177) is replaced by (213), and (199) by (214).

(213) $T_{sep}$ is determined by $(Q_{sep}', T_{sep}')$

where $Q_{sep}' = \{(NP, VP_A, V_{sep}, P, NP, U\langle PP_\alpha \rangle)\}$

$T_{sep}' \leftrightarrow (1, 2, 3, 4, 5, 6)$

(214) $\varphi_{l1}^P$ is determined by $(Q_{sep}'', T_{sep}'')$

where $Q_{sep}'' = \{(NP, VP_A, V, \{\text{Pronoun}, \text{Complement}, NP\}, U\langle PP_\alpha \rangle)\}$

Obviously, these statements can be coalesced.

92.3. We now have a formal explanation for the intuitively felt difference between the sentences "this picture was painted by a real artist" and "this picture was painted by a new technique", which were discussed in §72.1 (these are, respectively, (1) and (2) of §72.1). In the kernel we have the sentences—(215a-b) but not (216).
(a) a real artist painted the picture
(b) the artist painted the picture by a new technique

(216) a new technique painted the picture

The passives corresponding to (215a-b) are:

(217) (a) the picture was painted by a real artist
(b) the picture was painted by the artist by a new technique

Because of the possibility of (217b), we see that (215b) is a case of \[NP-V-NP-PP\], like (203). \[T_{pd}\] (as reformulated in (210)) applied to (217b) gives

(218) the picture was painted by a new technique.

The distinction between (217a) and (218) (1) and (2) chapter VIII, respectively) is thus a real difference of transformational history, i.e., a difference of structure on the level \(T\). Both are instances of \[NP-was-en^V-PP\], in terms of phrase structure, but the \(T\)-marker of (217a) is (219), and the \(T\)-marker of (218) is (220).

(219) \((215a)^{K_1}T_{p}^\circ\Phi^P\)
(220) \((215b)^{K_2}T_{p}^\circ T_{pd}^\circ\Phi^P\)

where \(K_1\) and \(K_2\) are the respective \(\Phi\)-markers. The problems raised in §72.1 concerning sentences (3)-(9) of that section are solved transformationally in much the same way. Similarly, the sentences

(221) John was \{frightened, surprised, bored\} (= (10), §72.1)

automatically receive the intuitively correct analysis as homonyms. Their 'non-verbal' sense arises from the fact that these expressions all belong to the kernel, as instances of the \(en^V\) form of the adjective phrase (cf. statement 17, §67.2), as
we know from the fact that they can be preceded by "very". Their
'verbal' sense derives from their transformational derivation by
\( T_p \) and \( T_{pd} \) from "...frightened John", etc.

In §§24.4, 25.2, and 25.5, we have seen that the remaining
intuitive inadequacies pointed out in §§22.1 and §§22.2 have
been remedied by transformational analysis. This, along with
the many similar cases that we have come across where transformational
analysis gives results that correspond to strong intuitive
judgments (e.g., the priority of actives over passives and
declaratives over questions), and where behavior that appears
irregular on the level \( P \) is shown to be systematic on the level
\( T \) (e.g., the behavior of "be", "have" in questions, negatives,
and so-phrases, discussed in §§20.5, 22.2), indicate that we
are well on our way to resolving, in transformational terms,
the problems that originally suggested the need for a higher
level of syntactic analysis.

22.4. The distinctions made in §29.2 permit certain cases of
constructional homonymy. A given sentence of the form
\( \text{NP-V -NP}_2\text{-PP (=(200))} \) may turn out to be in the overlap of the
pattern \( \text{NP}_1\text{-V-NP}_2 \) (with \( \text{NP}_2 \rightarrow \text{NP}_3\text{-PP} \)), as (201), and the
pattern \( \text{NP}_1\text{-V-NP}_2\text{-PP} \), like (203). For instance,

(222) the police questioned the man in the other room

is subject to either interpretation, since we have passives of
the form (204) and (206), respectively,

(a) the man in the other room was questioned by the police

(b) the man was questioned by the police in the other room

This dual interpretation for (222) certainly has intuitive
support.
The analysis to which we have come above also permits concatenation of different types of PP's. We may have a sentence \( Z \) of the form \( \text{NP}^\text{a}\text{VP} \), where

\[
\text{VP} \rightarrow \text{VP}_\text{a}^\text{a}\text{V}_\text{b}^\text{b}\text{NP}^\text{a}\text{PP}_\text{a}^\text{a},
\]

\[
\text{V}_\text{b}^\text{b} \rightarrow \text{Verb} \rightarrow \text{Complement}
\]

\[
\text{Complement} \rightarrow \text{PP}_\text{a}^\text{a}
\]

Thus \( Z \) is of the form

\[
\text{NP}-\text{VP}_\text{a}^\text{a}-\text{Verb}-\text{PP}_\text{a}^\text{a}-\text{NP}-\text{PP}_\text{a}^\text{a}
\]

\( \Phi^\text{P} \) will carry this into

\[
\Phi^\text{P} \rightarrow \text{NP}-\text{VP}_\text{a}^\text{a}-\text{Verb}-\text{NP}-\text{PP}_\text{a}^\text{a}-\text{PP}_\text{a}^\text{a}
\]

As an instance of (226), we have, e.g.,

\[
\text{they-will-name-him-after} \, \text{his} \, \text{grandfather} \, \text{-after} \, \text{his} \, \text{first} \, \text{birthday}
\]

In (227), the \( \text{V}_\text{b}^\text{b} \) is "name - after his grandfather". (228) seems a good deal less acceptable.

\[
\text{they-will-name-him-after} \, \text{his} \, \text{first} \, \text{birthday} \, \text{-after} \, \text{his} \, \text{grandfather}
\]

The who-transformation of \( \Phi^\text{P} \) applied to (227) (with him replacing the NP "his grandfather") gives (229), and applied to (228) (with the same change) it would give (230).

\[
\text{whom will they name him after after his first birthday}
\]

\[
\text{whom will they name him after after his first birthday after}
\]

(229) is acceptable, but the ungrammaticalness of (230) supports the exclusion of (228).

The revision of \( \Phi^\text{P} \) given above in (214) accounts for the possibility of (227) (hence (229)), and excludes (228) (hence (230)). If (228) is admitted as grammatical, we will have to define \( \Phi^\text{P} \)
as a family of transformations, as in (209).

29.5. In §24.4, in discussing the effects of the passive criterion for constituent analysis, we discussed a case just like those of §29.4. We noted there that all cases of (231) are grammatical, though (232) is not

(a) I knew the boy studying in the library (= (115))
(b) I found the boy studying in the library (= (116))
(c) the boy studying in the library was known (by the teacher) (= (117a))
(d) " " " " " " found (by the teacher) (= (117b))
(e) " " was found studying in the library (by the teacher) (= (117c))

(232) the boy was known studying in the library (by the teacher) (= (118))

It follows that (231a) has a compound object of the form NP-ing VP₁, as we noted in §24.4. Just as (201) with the passive (204) has a compound object NP-ing PP. Since we have the passive (231d) of (231b), we know that (231b) also has this analysis, under one interpretation. But from the passive (231e), we see that (231b) has an alternative interpretation as the image under P₁ of "I - found studying in the library - the boy", an instance of NP-V₁-NP with the verb "found - studying in the library". In this interpretation, it is analogous to (202), which has the passive (205). We see then that (231b) is an instance of constructional homonymity, and that we must add (233) to the construction verb-complement.

(233) v-ing VP₁

This leads us one step closer to the solution of one of the problems that arose in the attempt to present a comprehensive system of phrase structure for English. In §64 we found (i) that
phrases of the form \( \text{NP-} \text{ing} \text{VP}_1 \) could not be admitted into
the grammar of phrase structure (i.e., into the kernel)
without great complication, and (ii) that if they were
admitted, we would not be able to distinguish such obviously
distinct forms as (231a) and (231b) in structural terms. We
now have a solution for (ii). We can make the distinction, if
we can introduce these phrases. But we still have not indicated
how these phrases are to be introduced. We return to this
problem below, in §104.2, §105.5.

The same situation holds for the case of the verbs \( V_\sigma \)
discussed in the last paragraph of §65. We add to the verb-
complement construction the forms

\[
(234) \quad V_\sigma - \langle \text{ing} \rangle \text{VP}_1
\]

Once again, when means for introducing these sentences (e.g.,
"I saw him come", "I saw him coming," etc.) have been developed,
the correct distinctions will be forthcoming.

100.1. Since we managed to delete \( V_\sigma \text{NP}_1 \text{NP}_2 \) from the \( \text{VP}_1 \)-analysis
(in §98), we should expect the analysis of \( V_\sigma \text{NP}_1 \text{NP}_2 \) to follow
fairly smoothly. But these verb phrases pose a somewhat
different problem.

\( V_\sigma \) is formally distinct from \( V_\delta \) on the level \( P \) in that
\( \text{NP}_1 \) and \( \text{NP}_2 \) do not necessarily have the same number after \( V_\sigma \),
as they must after \( V_\delta \). We have

\[
(235) \quad \begin{align*}
(a) & \text{ the teacher gave him several books} \\
(b) & \text{ The teacher asked him several questions}
\end{align*}
\]

but not

\[
(236) \quad \text{they elected him officers}
\]

On the transformational level, other differences appear.
The only passive formed from "they elected him an officer" \((= (235))\) is "he was elected an officer by them", \textit{elect} being a \(V_g\). But from \(V_g^r \text{NP}^r \text{NP}\) we can have two passives. Thus from (235a) we have (235b), namely

(237) *she was given several books by the teacher*

(b) several books were given him by the teacher

and the same is true of (235b).

The \textit{wh}-transformations also apply differently to (235) and (196). From (196) we have, for instance, both (238a–b), as we would expect.

(a) whom did they elect an officer

(238)

(b) what did they elect him

But from (235a) we have only (239b), not (239a). (22)

(a) whom did the teacher give the books

(239)

(b) what did the teacher give him

Instead of (239a), we have

(a) whom did the teacher give the books to

(240)

(b) to whom did the teacher give the books

This suggests that there is a sentence more elementary than (235a), namely

(241) the teacher gave several books to him

and that (235a) is derived from (241) by a transformation \(T_\kappa\).

It seems to be true in general that for sentences of the form \(\text{NP}_1 - V - \text{NP}_2 \left\{ \begin{array}{c} \text{to} \\ \text{for} \end{array} \right\} \text{NP}_3\), there is a related form \(\text{NP}_1 - V - \text{NP}_2 - \text{NP}_3\).

Thus we have "they offered him the job" ("they offered the job to him"), "they found him a job" ("they found a job for him"), etc.

We must then require that the \textit{who}-question-transformation not apply to any \(T_\kappa\)-transform, in order to eliminate (238a).
Consider now the sentence (241). Its passive is

(242) several books were given to him by the teacher

But the passive (243) and the form (244) also appear permissible.

(243) several books were given by the teacher to John
(244) to John, the teacher gave several books.

From the possibility of (243) and (244) it follows that (241), like (203) in §99.1, is an instance of NP-V-NP-PPa.

100.2 One way to describe this situation is in the following series of steps:

1. To the construction Verb-Complement we add the form

(245) \( V_x \overset{\text{PP}_a}{\rightarrow} \)

one instance of which is "give - to him", with "to him" being the PPa.

2. We construct the transformation \( T_{\alpha} \) which carries \( X^{\text{to}} Y \)

into \( X^{\alpha} Y \), for any strings \( X, Y \).

3. We allow for the following strings of transformations:

\[
\begin{align*}
(246) \quad & (a) \left\{ \begin{array}{l}
T_{\alpha} \end{array} \right\} \\
& \left\{ \begin{array}{l}
T_{\beta} \end{array} \right\} \\
& \left\{ \begin{array}{l}
\Phi^P \end{array} \right\}
\end{align*}
\]

Applying (a) to the sentence

(247) the teacher - gave to him - several books

we derive (237b). Applying \( (246b) \) to (247), we derive (242). Applying \( (246c) \) to (247), we derive (235a). Applying \( \Phi^P \)
directly to (247), with no intervening transformation, we derive (241). This leaves (237a) and (243) still unaccounted for.
4. We can account for the remaining sentences by permitting the strings (248) in T-markers

\[ (248) \text{ (a) } \Phi_{11}^P \wedge T_p \wedge \Psi_l^i \]

\[ \text{ (b) } \Phi_{11}^P \wedge T_\alpha \wedge T_p \wedge \Psi_l^i \]

Applying (248a) to (247), we derive first (241), by \( \Phi_{11}^P \), this being a sentence of the form \( \text{NP} \cdot V \cdot \text{NP} \cdot \text{PP} \cdot \alpha \). Applying \( T_p \) as reformulated in (209), we derive (243), with "him" for "John". Applying (248b) to (247) we derive first (241), then "the teacher - gave - several books - him" by \( T_\alpha \).

But this derived sentence has the same form as the kernel sentence

\[ (249) \text{ "they - elected - an officer - him} \]

Hence application of \( T_p \) to "the teacher-gave-several books-him" will give (237a), just as it gives "he was elected an officer by them", from (249).

5. We place the condition on T-markers that the who-question cannot follow \( T_\alpha \), thus eliminating (239a)

Note that if steps 1-3 of this solution are adopted, then it follows that \( \Phi_{11}^P \) must precede the question transformation, so that (240) is generated properly. But we have seen above that the question transformation must apply before \( \Phi_{15}^P \). Hence \( \Phi_{11}^P \) must apply before \( \Phi_{15}^P \). If step 4 is accepted as well, then \( \Phi_{15}^P \) must precede \( \Phi_{13}^P \), since \( T_p \) must precede \( \Phi_{13}^P \).

There are certain potential trouble spots in the derivation of (237a) as outlined in step 4, above. It is necessary that the kernel grammar be carefully formulated so that the proper derived constituent structure is conferred on "the teacher-gave-several books - him" (by analogy to (249)). Later, in fact, we will see
that (249) is itself derived by transformation. Hence to carry through step 4 it will be necessary to extend our conception of derived constituent structure in such a way that constituent structure can be conferred on a transform by virtue of an analogic form which is itself derived. Because of this difficulty, and because of the ad hoc nature of step 5, above, we see that the reduction of \( V_T \cdot NP \cdot NP \) to the basic form \( V_T \cdot NP \) is only partially satisfactory.

Note that if (248) is accepted, then it will not be necessary to formulate the passive transformation as a family of transformations, as in (209). It is possible to construct the passive as a single transformation, and to achieve the necessary variety by applying it either before \( P_1 \) (as before) or after \( P_1 \) (as in (248a)). There are further implications to this course that we have not studied.

101.1. The passive criterion shows us that sentences of the form

(250) \( NP \cdot V_x \cdot \) that'sentence ("everyone knew that the play would be a success")

are also of the form \( NP \cdot V \cdot NP_2 \), with \( NP_2 \cdot \) that'sentence. This follows from the fact that we have such sentences as

(251) that the play would be a success was known by everyone.

These sentences, too, must thus be dropped from the \( VP \) analysis. Since sentence in (250) may be a passive or a transform of some other type, we see in fact that (250) must be dropped from the kernel grammar and reintroduced transformationally.

From this point on, the analysis of that-phrases is just a simpler version of the discussion in §25 of ing-phrases. To (166), 495.8, we add
(252) \( Z_1 \uparrow K_1 \uparrow T_{\text{that}} \uparrow Z_2 \uparrow K_2 \uparrow I \uparrow T_{\text{IN}} \uparrow \Phi^P \)

with condition \( K_A \) extended to include \( T_{\text{that}}(Z_1, K_1) \) as one of the forms of \( W \).

\( T_{\text{that}} \) is based on a deformation that inserts \( \text{that} \) before a sentence. Not every sentence can appear here. For instance, questions are sentences in terms of their derived constituent structure, but they cannot appear after \( \text{that} \). Questions, however, are not represented by \( NP^\uparrow VP \); and closer investigation shows that we can use the representation \( NP^\uparrow VP \) to define the set of elements that can follow \( \text{that} \) (i.e., kernel sentences, passives, sentences with ing-phrase nouns, etc., are included — questions, and, as we will see below, imperatives are excluded). We thus define \( T_{\text{that}} \) as

(253) \( T_{\text{that}} \) is determined by \( (Q_{\text{that}}, e_{\text{that}}) \)

where \( Q_{\text{that}} = (NP^\uparrow VP) \)

\[ e_{\text{that}} \leftrightarrow (\text{that}, U) \]

The transformation \( T_{\text{IN}} \) in (252) is defined as

(254) \( T_{\text{IN}} \) is determined by \( (Q_{\text{IN}}, T_A) \),

where \( Q_{\text{IN}} = \{ (\text{that}^\uparrow NP^\uparrow VP, \#, X^\uparrow V_e, N_{\text{inan}}, Y) \} \)

\[ T_A \leftrightarrow (\sigma, 0, 0, 0, 0, \sigma, 1, 0, 0) \] (as in (153))

in particular

Thus, \( T_{\text{IN}} \) converts a string of the form \( \text{that}^\uparrow NP^\uparrow VP \) into \( \text{that}^\uparrow NP^\uparrow VP \) \(-\text{that}^\uparrow NP^\uparrow VP \) \(-\text{that}^\uparrow NP^\uparrow VP \), where \( N_{\text{inan}} \) is \( \text{it} \) (by the extension of condition \( K_A \)), into the corresponding string \( NP^\uparrow VP_A^\uparrow V_e \). The \( P \)-basis for (250), then, is

(255) the play would be a success, everyone knew it

\( T_{\text{IN}} \) differs from \( T_A \) in (166) mainly in that the replaced \( \text{it} \) is inanimate. The choice of \( \text{it} \) as the bearer of the \( \text{that} \)-clause
is determined by the same factors that operated in the case of
ing-phrases, cf. \( \text{f} 95.1 \). A closer analysis of \( T_{\text{th}} \) would show that
there is an internal effect on the sentence, because of sequence
of tense restrictions (alternatively, these could be given as
added conditions along with \( K_A \)). The similarity\(^ {252-4} \) and
(166-8) naturally stands in favor of both of these transformational
analyses, since considerable coalescence is possible.

We know that the that-clause in the transform is an inanimate
noun.\(^ {23} \)

101.2. Such sentences as (251) are subject to a further
transformation. From (251) we can form

(256) it was known by everyone that the play would be a success.

The transformation \( T_{\text{th-it}} \) that gives (256) from (251) can be
applied more generally when we have any adjective, not just
a passive en\(^ V \), as in (251). Thus we have

(257) it was clear to everyone that the play would be a success.

This, incidentally, lends further support to the decision
originally reached in \( \text{f} 62.3 \) to consider the en\(^ V \) forms to be
adjectives, rather than to include be\(^ en \) (alongside of being)
as an element of the auxiliary verb phrase. We thus have

(258) \( T_{\text{th-it}} \) is determined by \( (Q_{\text{th-it}}, t_{\text{th-it}}) \)

\[ Q_{\text{th-it}} = (\text{that}^ {NP} \text{VP}, \text{VP}_A, \text{be}, \text{AP}, \text{U}) \]

\[ t_{\text{th-it}} \circ \text{that}(\gamma_{\text{th-it}}) \]

where \( \gamma_{\text{th-it}} \) maps \( (0,0,0,0,0,0,0,0,0,0,0,1) \)

\[ \delta_{\text{th-it}} \longrightarrow (\sigma, \text{it}, \text{U}, \text{U}, \text{U}, \text{U}, \text{U}, \text{U}, \text{U}, \text{U}) \]

The transformational analysis of that-clauses enables us to
drop one recursion from the kernel grammar, thus effecting a
shortening of derivations. We will see in the course of the
investigation that all recursions drop from the kernel grammar. 

In the last few sections we have managed (with the reservations of §100.2) to drop out of the VP₁-analysis all forms except be ^Predicate and Vᵀ ^NP , reducing all other forms to special cases of Vᵀ ^NP. We have also seen in §96 and §99.5 that other forms not included in §67.2 also fall under this general pattern. The VP₁-analysis given above as (207) (replacing §67.2, statement 3), thus reduces to (259).

(259) VP₁ → <D₂><VP_B>{be ^Predicate(24)} {Vᵀ <NP>}

This is actually a significant reduction. In §67.2, the forms in (259) appeared as part of a list of many forms of the verb phrase (of which statement 3, §67.2, gives only part). They had no particular systematic importance. But it is intuitively clear that they are in some sense basic. Any English grammar will assign primary importance to the grammatical relations of 'predication' and actor-action. But transformational analysis of the verb phrase shows that these are in fact fundamental relations. The forms of (259) are the only two forms of VP₁ -- as we defined "grammatical relation" in §541, the relations of predication and actor-action are basic. Later, in §105, we will see that of all the compound verbs, only the construction Verb-Preposition remains in the kernel.

In connection with the VP₁-analysis, it is interesting to note that the possibility of analyzing intransitive verbs as transitives with zero objects is ruled out. If this analysis were accepted, the passive transformation would yield such nonsense as
(260) \( \varnothing \) - was - en\^sleep - by \( \lambda \text{John}(\varnothing) \) "was slept by John"

103. We have seen (4.25.5) that "growling of lions" and "reading of good literature" can be derived respectively from "lions growl" and "they read good literature." There are many cases of similar constructions with a noun related to a verb, but not so simply related as are "growling" and "reading" to "growl" and "read". Consider, for instance, the noun phrase subjects of (261).

(a) the sight of men working in the fields made him sad
(b) the flight of the birds signalled the coming of spring
(c) his refusal to come was taken as an insult.

etc. (25)

These can be derived by generalized transformations in the familiar way from

(a) he saw men working in the fields \# it made him sad
(b) the birds flew \# it signalled the coming of spring
(c) he refused to come \# it was taken as an insult

The added element is not ing, but a certain nominalizing morpheme \( \nu^- \) which can be taken as a prime of P. In the morphology, we have such rules as

(a) \( \text{see}^-\nu^- = \text{sight} \)
(b) \( \text{fly}^-\nu^- = \text{flight} \)
(c) \( \text{refuse}^-\nu^- = \text{refusal} \)

etc.

Whether one such nominalizing morpheme will suffice is a matter that requires more elaborate study. Furthermore, the distribution of this morpheme will be limited, since some verbs (e.g., "follow") do not appear with it. It seems evident that a considerable simplification should result from this analysis, since otherwise the heavy distributional restrictions in (261) will have to be separately stated, duplicating much of
the analysis of the verb phrase. The transformational analysis
is much like that of §95.6, a fact which stands in favor of
both of these analyses.

In §61.2 we noted that the level of morphology is not
independent of the level of phrase structure, since only on
this level did the criteria for a correct morphological
analysis appear. If the preceding paragraphs are correct, then
morphology is not independent of the level of transformations
for the very same reasons. The distributional relations between
"sight" and "see", etc., could not, apparently, be recognized
on any lower level. Since these words are in fact in different
classes, they are never substitutable for one another. And
purely distributional morphological analysis, if motivated at
all in this case, might well identify "see" with "seat", "sigh"
with "sight", etc. But purely distributional analysis on the
transformational level does bring out the distributional
relation between these forms, and requires that they be related
on the morphological level.

In chapters IV and V (§§33.3,42), we discussed the problem
of constructional homonymy on the level of syntactic categories
(ordinary homonymy) and noted (§33.3) that among the
homonyms relevant to syntax there were, intuitively, two types.
Thus in the overlap of Noun and Verb we have both "walk" and
/riyd/, but these are clearly different kinds of homonyms. In
terms of the nominalizing morpheme \( \nu \), now required by transformational
analysis, we can offer a purely syntactic explanation for this
intuitively felt distinction. The noun "walk" will be analyzed
as "walk-\( \nu \)" (where walk is the verb) if it is the case that many
contexts of this noun are transforms of the contexts of the verb
"walk". In other words, one of the variants of \( \nu \) is zero (i.e.,
in certain contexts, the morpheme \( \nu \) is carried into the phonemic unit). /riyd/ naturally will not be amenable to such analysis. If detailed investigation supports this conclusion, as appears likely, we will have a transformational explanation for the feeling, noted in the final paragraph, that "walk" is a single word, in two classes, while /riyd/ is basically two words with the same phonemic shape, i.e., a real homonym.

104.1. Sentences with relative clauses were not included in the grammatical sketch of §57.2. In fact, the discussion of §64 showed that they could not be included without seriously disturbing the grammar. But we can now introduce them quite simply by generalized transformations.

Consider the sentence

(264) All the students watched the man who was lecturing

We note first of all that this is a sentence of the form \( NP_1-V-NP_2 \), with \( NP_2 = "the \ man who was lecturing". This analysis is determined by the passive criterion, since we have (265) but not (266).

(265) the man who was lecturing was watched by all the students
(266) the man was watched by all the students who was lecturing

Investigating other who-phrases that can occur in this position, we determine that "who was lecturing" is a transform of a sentence, which can be taken as "he was lecturing". Since we have also

(267) I noticed the man whom I had met yesterday

we see that the transformation that carries a sentence into a who-phrase can operate on a string \( X_1-he-X_2 \), carrying it into \( wh-he-X_1-X_2 \), even when \( X_1=\nu \). Hence we have here exactly the
family of transformations \( F_w \) constructed in §91.4. As was mentioned in §91.7, wh-questions are simply \( F_w \)-transforms of questions, and relative clauses are \( F_w \)-transforms of declaratives.

The generalized transformation \( T_R \) which carries who-phrase 

\[ \# - X_1 - NP - X_1 \]  

into 

\[ X_1 - NP \overset{\text{who-phrase}}{-} X_1 \]  

will produce (264) and (267), respectively, from

\[ \begin{align*}
(a) & \text{ who was lecturing} & \# \text{ the students watched the man} \\
(b) & \text{ whom I had met yesterday} & \# \text{ I noticed the man}
\end{align*} \]

But we have seen above that the resulting \( NP \overset{\text{who-phrase}}{-} \) is itself an \( NP \). Hence \( T_R \) must be formulated as an \( T \)-transformation which 'attaches' the who-phrase to \( NP \) as its root. This generalized transformation can apply to many \( NP \)'s in a sentence. Thus we must construct a family of transformations \( F_R \). Thus we have

\[ \text{(269) } F_R \text{ is determined by } \left(C_R, t_R\right) \]

where:

\[ C_R(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) \text{ if and only if } \]

\[ \begin{align*}
(i) & \alpha_1 = \text{wh} \ldots \\
(ii) & \alpha_2 = \# \\
(iii) & \alpha_3 = X_1 \\
(iv) & \alpha_4 = NP \\
(v) & \alpha_5 = X_1
\end{align*} \]

\[ t_R \leftrightarrow (\text{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}) \]

\( t_R \) thus differs from \( t_A \) of (168) in only one place, and \( NP \overset{\text{wh}}{-} \) a single term of the proper analysis of the transform, is an \( NP \).

As \( T \)-markers we have strings in \( T \) of the form

\[ \text{(270) } Z_1^\wedge K_1^\wedge I_F^\wedge Z_2^\wedge K_2^\wedge I_F^\wedge \wedge F \]

Actually, we have not given anywhere near sufficient detail in the specification of this transformation, and considerable qualification is necessary. For one thing, it must be possible
for the NP of $Z_2$ to which the who-phrase is added to appear in the position of the replaced noun of the who-phrase. E.g., we must exclude such sentences as "the man who were lecturing", or, on a higher level of grammaticalness, "the man whom I prevented," etc. The simplest way to account for all such cases is to add a condition $K_R$ on $T$-markers of the form (270), requiring that the NP of $x_4$, (269), be identical with the NP of $Z_1$ which is replaced by "who" under $P_w$. Thus the $P$-basis of (264) would be

(271) the man was lecturing; the students watched the man

But in 4.21.4, $P_w$ was developed in such a way that only "he", not any NP, can be the root of the term "who", "whom". We have not discussed pronouns, but there are actually good reasons for introducing them transformationally as substituends for NP's of the proper kind. Thus the definition of $P_w$ can be made compatible with the condition $K_R$ by replacing the first "I" in (270) by the transformation that replaces an animate NP by "he".

There are several other conditions and revisions that would be necessary in a really accurate and complete analysis. Not every NP in $Z_2$ can have a relative clause affixed to it, so that certain restrictions must be placed on $x_4$ and $x_{14}$ in (269). Furthermore, a more detailed study of pronouns will probably lead to the conclusion that "him" is derived from "he" by a mapping which may precede all other components of $P$. But this necessitates a slight revision of (270), since $P_w$ must naturally apply after this mapping.

Just as we have what-questions paralleling who-questions (cf. 4.21.4), we have which-phrases and that-phrases added to inanimate nouns. When these are constructed, along the lines
sketched above, we will add still further conditions of compatibility on \( Z_1 \) and \( Z_2 \) of (270). We will not investigate these phrases here, nor will we consider the other occurrences of 'relative clauses' (i.e., \( F_w \)-transforms of declaratives), as in "I know who saw him", etc.

104.2. Extension of this transformational analysis of relative clauses serves to integrate §104 with earlier discussions. From (272) I know the boy who is studying in the library

we can derive, by an additional transformation \( T_3 \), the sentence (273) I know the boy studying in the library

From this instance, it appears that \( T_3 \) converts \( X\text{-who} \) into \( X\text{-I} \), but closer investigation shows that sentences like (273) can occur even when the auxiliary verb \( \text{be}^{\text{ing}} \) cannot appear in the corresponding who-phrase. Thus we have (274) and (275), but not (276).

(274) people owning property should pay higher taxes

(b) everyone knowing the answer may leave

(275)(a) people \( \langle \text{who} \rangle \) own property

(b) everyone \( \langle \text{who} \rangle \) knows the answer

(276)(a) people \( \langle \text{who} \rangle \) are owning property

(b) everyone \( \langle \text{who} \rangle \) is knowing the answer

From this we see that \( \text{ing} \) in (273) is added by \( T_3 \); it is not the \( \text{ing} \) of (272) carried over under the transformation. We thus set up a family of transformations \( F_3 \) such that

(277) \( F_3 \) is determined by \( (\alpha_x, \delta_3) \)

where: \( C_3 (\alpha_1, \alpha_2, \alpha_3) \) if and only if

(i) \( \alpha_1 = X \); (ii) \( \alpha_2 = \text{who} \text{VP}_A \); \( \alpha_3 = I \)
\( \delta_{\beta} \leftrightarrow (u, u, u, u, u, u) \)

A member of \( F_{\beta} \) thus carries a string \( X-\text{who}^{\text{VP1}}-Y \) into \( X-U-\text{ing}^{\text{VP1}}-Y \). We now add "\( F_{\beta} \)" to (270) following "\( F_{\beta} \). In this way we derive all those phrases \( \text{NP}^{\text{ing}^{\text{VP1}}} \) which are noun phrases as determined by the passive criterion (cf. \( \S 94.4 \), \( \S 92.5 \)). This gives us one source for the \( \text{NP}^{\text{ing}^{\text{VP1}}} \) phrases which caused so much trouble in \( \S 64 \). The difficulties raised in \( \S 64 \) are now undercut, since (273) is derived from the two kernel sentences "I know the boy", "the boy is studying in the library", neither of which poses any of the problems of \( \S 64 \). We saw in \( \S 92.5 \) that there is also a second source for \( \text{NP}^{\text{ing}^{\text{VP1}}} \) phrases. In \( \S 105.5 \), below, we arrive at the second source.

104.2. There are a variety of other transformations similar to those of \( F_{\beta} \). Thus relative pronouns can be dropped in many positions, as can that in that-clauses (cf. \( \S 101 \)), etc. One such transformation can be used to derive the noun phrases \( \text{NP}^{\text{PP}} \) discussed in \( \S 92.1 \), e.g., "the man behind the counter" in (278) the police suspect the man behind the counter \( (=201) \).

\( \text{NP}^{\text{PP}} \) can be derived from a noun phrase \( \text{NP}^{\text{who}^{\text{is}^{\text{PP}}}} \). E.g.,

(278) can be derived from

(279) the police suspect the man who is behind the counter by a transformation very similar to \( T_{\beta} \).

Such analysis should effect quite a considerable simplification in the kernel grammar. A more careful grammar than ours would recognize many restrictions on the selection by noun phrases of a modifying \( \text{PP} \). These restrictions need now only be given for the occurrence of \( \text{PP} \) as a predicate in sentences \( \text{NP} - \text{be} - \text{PP} \). They will be carried over automatically to the
NP^0PP position by transformation.

Investigation of the NP^0PP noun phrase construction shows that not all such instances are covered by the transformation from NP – be – PP. For instance, we have (280) but not (281).

(280) (a) the man with the felt hat
     (b) the men of great wealth

(281) (a) the man is with the felt hat
     (b) the men are of great wealth

We do, however, have

(282) (a) the man has the felt hat
     (b) the men have great wealth

and (280) and similar sentences might be derived from these, by a second transformation much like T_s. Thus (280a) will have a P-basis and transformational history somewhat different from that of "the man with John". "the men with friends" would then be a case of constructional homonymity, deriving either from "the men are with friends" or "the men have friends." Other modifying prepositional phrases have still different origins in the kernel. Thus "the forerunners of modern mathematics" might be derived from "modern mathematics has forerunners", etc.

A detailed study of this construction should be interesting.

105.1. The object Noun Phrase of (278) (=201) is now eliminated from the kernel in favor of a sentence NP-be-Predicate. In discussing (278) in §9.1, we compared it structurally with

(283) the police put the man behind bars (=202)

This sentence has been shown to be an instance of the very productive construction

(284) V_T \rightarrow V^*Complement
of which many other instances were found in §96 and §97. But NP^PP in (283) can also be regarded as derived from a sentence of the form NP-be-Predicate, namely (285) the man is behind bars.

In fact, we have NP-put-NP^x-PP^y only for such NP^x and PP^y as occur as NP^x-is-PP^y. Investigation of other instances of (284) shows that these too can be transformationally derived. We have already noted in §97.2 that certain verbs given by (284) must agree in number with their objects as well as their subjects (more precisely, the complement part must agree with the object, and the verb part, with the subject), and we have used this fact to support the formulation of the passive transformation with inversion of NP's. But it is more generally the case in the Verb-Complement construction that the complement and the object of the compound verb have heavy selectional restrictions which suggest transformational analysis. We will see that all cases except those of §96 (e.g., "call up") can be dropped from the kernel with profit. Transformational analysis is suggested wherever the complement and the object have the same selectional relation as appears in some simple kernel sentence Z. In this case, the verb phrase Verb-Complement-object can be derived transformationally from Verb-Z. In particular, if the complement contains a noun phrase NP^c which agrees in number with the object NP^o, we may be able to derive the verb phrase from Verb-Z, where Z is NP^o-is^cNP^c.

We must be careful to assure that the result of such transformation will be a verb phrase Verb^Complement-object. All such transformations of Verb-Z into Verb^Complement-object will thus have to precede
the application of $\Phi^{2}_{1}$, which converts this phrase into verb-object-Complement. If these requirements are not met, the analysis of §26-7, with the simplifications there introduced, will not stand.

To simplify this discussion, we may discuss only one of the subclasses of transitive verbs. Let us assume, for this discussion, that $V^T$ has only the subconstructions $V^T_{1}$ and $V^T_{2}$. The addition of other subclasses will only lead to duplication of the detailed development that we give below. Similarly, in chapter VII and in §26-7 we have not distinguished subclasses of these compound verbs that differ in terms of their selection of subject and object. We consider now only the selectional relation between the complement of the complex verb and the object.

105.2. Consider first verb phrases of the form (286) as discussed in §27.

(286) $V^h_{h} NP^o_{o} Predicate$  

"consider John incompetent"

We note that $NP^o_{o} Predicate$ can occur in this position only if $NP^o_{o}is Predicate$ is a sentence. (26) Thus we can have "kindness is a virtue", "I consider kindness a virtue", "John is a good pianist", "I consider John a good pianist", but not "kindness is a good pianist", "I consider kindness a good pianist", "John is a virtue", "I consider John a virtue", etc. The selectional relation between $NP^o_{o}$ and $Predicate$ (and, in particular, the numerical agreement between $NP^o_{o}$ and $NP$ of the predicate) motivate transformational analysis in the case of (286).

There are several ways to carry out this analysis. Since each member of $V^h_{h}$ can apparently occur with a simple object, we can consider such simple sentences as "he considered it" to be part of the $P$-basis. To ensure the correct derived structure for
transform (286) we can, in the kernel grammar, analyze the
simple occurrences of $V_h$ as compounds $V_h \wedge \emptyset$, where $\emptyset$ is the
complement. This seems to be the simplest overall solution.

We have to do here with a generalized transformation, since
there are two kernel sentences involved. As $T$-markers we have
strings of the form

\[(287) \quad Z_1^\wedge \wedge K_1^\wedge I - Z_2^\wedge K_2^\wedge I - T_{cmp} - \Phi^P\]

where the generalized transformation is analyzed as

\[(288) \quad T_{cmp} \text{ is determined by } (Q_{cmp}, t_{cmp}) \]

\[\text{where: } Q_{cmp} = (NP, VP, \wedge \text{be, Predicate, } #, \text{NP, VP, } V_h, \emptyset, \text{NP}) \]

\[t_{cmp} \longleftrightarrow (\Phi, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)\]

$T_{cmp}$ will thus carry a string of the form (289) into the
corresponding string of the form (290).

\[(289) \quad NP_1^\wedge VP^\wedge \text{be-Predicate-#-NP}_2^\wedge VP^\wedge V_h - \emptyset - NP_3^\wedge \]

\[(290) \quad U - U - U - U - NP_2^\wedge VP^\wedge V_h - \text{Predicate-NP}_1^\wedge \]

where Predicate, replacing the complement $\emptyset$, is a complement in
the transform, so that $V_h^\wedge \text{Predicate}$ is a $V_T$ (since its root $V_h^\wedge \emptyset$
is a $V_T$) with the object $\text{NP}_1$, replacing $\text{NP}_3$. Application of
$\Phi^P_{11}$ to (290) gives a sentence with the verb phrase (286), as
previously. The $P$-basis of "I consider John a good pianist," then, is

\[(291) \quad \text{John is a good pianist; I consider it}\]

As far as $V_h$ is concerned, then, we can drop the restriction
in statement 6, §67.2, as well as the selectional restrictions
which we had not given there. This is an important saving, since,
as we noted in fn.33, chapter VII, this restriction was not
really formulable in terms of the notational devices then
available.
Note that the passive transformation forms sentences of the form NP - is - Predicate (cf. §82.2). Hence T_{cmp} can form verb phrases from passives as well. In fact we do have such sentences as

(292) I consider the issue completely closed

from "the issue is completely closed" # "I consider it" — ultimately, from the kernel sentences "this completely closes the issue", "I consider it".

105.2. In §98 we saw that \( V_e \) and \( V_g \) can be analyzed in the same way as \( V_h \). It is quite clear that the further reduction of §105.2 can be carried over for these elements too. In the case of \( V_e \) and \( V_g \) we also find the selectional relations and agreement in number characteristic of NP-is-Predicate, NP-is-NP, respectively. A further support for the analysis in the case of \( V_g \) is the fact that there are special forms of the noun phrase which occur only in the predicate position and after \( V_g \)-NP, e.g., "president" without the article, as in "he is president: they elected him president."

We can account for \( V_g \) by revising (288), redefining \( Q_{cmp} \) as

\[
Q_{cmp} = \left\{ \begin{array}{l}
(NP, \, VP_A, \, be, \, \{ \begin{array}{l}
(\text{Predicate}) \end{array} \} \), m, NP, \, VP_A, \, \{ V_h \}, \, t, \, \{ V_g \}, \, \{ \theta, \, NP \} \right\}
\]

\( V_e \) is a slightly different case, since \( be \) is not dropped here, and \( to \) is prefixed to it. \( V_e \) appears in

(294) NP - \( V_e \) - NP - to\^be\ Predicate ("they know John to be honest")

The P-basis of this sentence will consist of the kernel sentences "John is honest", "they know it". And we see from (294) that "it" is replaced not by \( N_P \text{^Predicate} \), but by \( NP \text{^to^be^Predicate} \). In other words, another transformation applies to the
first component of the complex string $Z_1 \#^nZ_2$ to which $T_{\text{cmp}}$ is to apply, before the application of $T_{\text{cmp}}$. The $\Xi$-markers by which $\nu_2$ is introduced will thus be of the form

$$(295) \quad Z_1^{\wedge}K_1^{\wedge}T_\gamma^{\wedge}Z_2^{\wedge}K_2^{\wedge}I-T_{\text{cmp}}^{\wedge}\Phi$$

The transformations in (295) are defined as follows:

$$(296) \quad T_\gamma \text{ is determined by } (Q_\gamma, \xi_\gamma)$$

where:

$$Q_\gamma = \{ (NP, VP_{Al}, \langle VP_{A2} \rangle \text{ be Predator}) \}$$

$$\xi_\gamma \leftrightarrow (U, U, U, U, to, U)$$

$$(297) \quad T_{\text{cmp}} \text{ is determined by } (Q_{\text{cmp}}, t_{\text{cmp}})$$

where:

$$Q_{\text{cmp}} = \{(NP, VP_{Al}, to \langle VP_{A2} \rangle \text{ be Predator}) \}$$

$$t_{\text{cmp}} \text{ is as in (288)}$$

Suppose that $Z_1$ in (295) is "John is honest" (=John - S - be honest) and $Z_2$ = "they know it". Then $T_\gamma (Z_1, K_1) = John-S-to \text{ be honest}$. Just as $T_{\text{cmp}}$ carries (289) into (290), $T_{\text{cmp}}$ carries $T_\gamma (Z_1, K_1)^{\wedge}Z_2$ into (294).

Dropping $\nu_2^{\wedge}NP^{\wedge}NP$ from the kernel enables us to eliminate the exception in statement 4, 467-2. For $\nu_2$ and $\nu_2^{\wedge}$ as well as $\nu_2$, the special condition in statement 6 is now unnecessary, and thus can be dropped, since these are the only forms to which it applies. This simplification is now possible because all cases of agreement in number have been reduced to the predicative sentence form $NP-is-NP$.

105.4. The same analysis carries over for (283) (= (202), 422.1) and like cases. We can extend the brackets in (293) to include a class of verbs $V_j$, writing "$V_j^{\wedge}\$ below "$V_2^{\wedge}\$ in (293), and "$PP^{\wedge}\$ below "$NP^{\wedge}\$ in the first bracket of (293). Thus with $\text{put}V_j$, (283) will have the $\Xi$-basis

$$(298) \quad \text{the man is behind bars; they put it }$$

Here we find the same problem as was discussed in \textsection 4104.2. Certain prepositional phrases appearing as complements may not be derived from $\text{NP}_1{\text{-is-PP}}_x$, but rather from $\text{NP}_1{\text{-has-}}\text{NP}_2$ (where $\text{PP}_x$ is $P_x{\text{-NP}}_2$), or perhaps from some other kernel sentence. This is a detailed problem in itself, and we will not go into it here.

In \textsection 499.1 we discussed three types of \text{NP-PP} construction(\textsection 201.3). In \textsection 4104.3 we dropped (201)(\textsection 278) from the kernel, and now we have dropped (202)(\textsection 283). This leaves only (203)("they questioned the man behind closed doors") as a kernel sentence. This is the only case where intuitively the prepositional phrase seems to be added on to the sentence, and not bound up with either the verb or the object.

In \textsection 499.4 we discussed cases of constructional homonymity falling in the overlap of the patterns of which (201) and (203) are instances. Now, comparison of \textsection 4104.3 and \textsection 4105.4 suggests that we should be able to find overlaps of (201) and (202), i.e., sentences of the form $\text{NP}_1{\text{-V-NP}}_2{\text{-PP}}$ with $\text{NP}_2{\text{-PP}}$ as the object, or $\text{V-PP}$ as the verb. An instance of this is (299) John kept the car in the garage.

Since we have "the car was kept in the garage by John", this must be an instance of $\text{NP}_1{\text{-Verb}}$ \text{Complement} $\text{NP}_2$, with "in the garage" being the complement. Since we can have "the car in the garage was kept by John" ("but he gave away the one in the lot"), this is also a case of $\text{NP}_1{\text{-V-NP}}_2$, with $\text{V}=\text{kept}$, and $\text{NP}_2=\text{the car in the garage}$. This case has a certain interest, if the constructional homonymity described here is accepted as a correct analysis. In both cases, the $P$-basis is the pair of kernel sentences
(300) the car is in the garage; John kept it

and the structural ambiguity of (299) arises from the fact that this sentence can be produced from (300) by two transformational routes, either in the manner of §104.2 or §105.4. But this raises some interesting questions about the relation between transformations and meaning. It is fairly clear from all the examples that we have discussed in this chapter that in some sense meaning is preserved under transformation. Naturally we could not hold that transform and transformed string are synonymous, since a transform may add or subtract morphemes. But we might have proposed that transform and pre-image differ in meaning only in the meanings of the morphemes dropped or added. E.g., "John is not here" differs semantically from "John is here" in the meaning of not. But (299), as analyzed above, indicates that this cannot be the case, because we have here a case of two distinct sequences of transformations with exactly the same starting point in unambiguous kernel sentences, and with exactly the same end point, but with different meanings associated with this final transform. While transformations have semantic correlation, it is not obvious just how this is to be described.

105.5. In §99.5 we took up, from a transformational point of view, the problem posed in §64 by the construction NP-ing-VP. It appeared in §99.5 that if these phrases can be introduced into the grammar, they can be properly distinguished. In §104.2 we saw how these phrases can be introduced when they are noun phrases. Now, extending the analysis of §105.1-4, we can account for the occurrence of these phrases as instances of the general construction Verb-Complement.

In §62.1, we distinguished the following subclasses of verbs:
(301) \( V_a \) = want, like, etc. ("I want him to come", "I want to come")
\( V_b \) = persuade, advise, etc. ("I persuaded him to come", not "I persuaded to come")
\( V_c \) = try, decide, etc. ("I tried to come", not "I tried him to come")

(302) \( V_x \) = imagine, prefer, etc. ("I imagined him coming", "I imagined coming")
\( V_y \) = find, catch, etc. ("I found him reading", not "I found reading")
\( V_y \) = avoid, begin, etc. ("I avoid arguing", not "I avoid him arguing")

In §63.2 we suggested incorporating \( V_a \) into \( V_b \) and \( V_c \) as the overlap of these classes, and \( V_x \) into \( V_y \) and \( V_y \) as their overlap. Following this suggestion, we can regard \( V_y^{\text{ing}} V_P \) as another form of the Verb-Complement construction, thus deriving sentences of the form \( NP_1 - V_y^{\text{ing}} V_P , \) which, by \( \Phi_1^P \), is carried into \( NP_1 - V_y - NP_2 - V_P. \) This is the analysis at which we arrived in §92.5 for such sentences as "I found the boy studying in the library". But in §105.3 we saw that the very similar construction \( V_a \) = to \( V_P \) (appearing in "they know John to be honest", etc.) can be introduced transformationally as a form of Verb-Complement (\( V_P \) in this case being limited to the form be \( ^P \text{Predicate} \)). A very similar transformational analysis will provide for the introduction of \( V^{\text{ing}} V_P \) as a form of Verb-Complement, thus giving (by \( \Phi_1^P \)) the second source for the \( NP - V^{\text{ing}} V_P \) phrases. Thus alongside of (295) we have \( T \)-markers

(303) \( Z_1 K_1 T^*_y - Z_2 K_2 I - T^*_c - \Phi \)

The transformations of (303) are analyzed as follows:
(304) \( T_y \) is determined by \((Q'_y, \delta'_y)\)

where \( Q'_y = \{(\text{NP}, \text{VP}_{A1}, \langle \text{have}^{\text{en}} \rangle \text{VP}_1)\} \)

\( \delta'_y \leftrightarrow (U, U, U, U, ing, U) \)

(305) \( T_{\text{comp}}'' \) is determined by \((Q''_{\text{comp}}, t_{\text{comp}})\)

where \( Q''_{\text{comp}} = \{(\text{NP}, \text{VP}_{A1}, \text{ing}\langle \text{have}^{\text{en}} \rangle \text{VP}_1, \#), \text{NP}, \text{VP}_{A1}, V_\beta, \emptyset, \text{NP}\} \)

\( t_{\text{comp}} \) is as in (288), (297)

Suppose that \( Z_1 \) in (303) is "the dog climbs that tree" (=the dog - S - climb that tree) and \( Z_2 \) is "I can imagine it". \( T_y \) carries \( Z_1 \) into the dog - S - ing climb that tree. \( T_{\text{comp}}'' \) then replaces "it" in \( Z_2 = I\text{'can imagine} \emptyset \text{it by} \) "the dog", and \( \emptyset \) by "ing climb that tree", yielding

(306) I-can-imagine-climbing that tree-the dog

which becomes

(307) I can imagine the dog climbing that tree

by \( q^P \). Thus the operation of \( T_{\text{comp}}'' \) is just like that of \( T_{\text{comp}} \) and \( T_{\text{comp}} \) (in fact, the three can obviously be combined into a single transformation, with \( Q_{\text{comp}}', Q''_{\text{comp}}, Q''_{\text{comp}} \) coalesced into a single restricting class). Note that \( \delta'_y \) differs from \( \delta_y \) only in that \( \text{ing} \) replaces \( \emptyset \). \( Q_y \) and \( Q'_y \) are also very similar. Note further that \( Q'_y \) is exactly \( Q_{\text{ing}} \) of (168), and \( \delta'_y, \delta_y \) are very close to \( \delta_{\text{ing}}, \delta_{\text{to}} \) of (168), respectively.

This concludes the discussion of the complex of problems around \( \text{NP} \text{ing} \text{VP}_1 \), originally begun in §64-5. Sentences (307) and (308) now have different transformational histories.

(308) I can recognize the man climbing that tree

(307) is based on the kernel sentences "the dog climbs that tree", "I can imagine it", just as (308) is based on the kernel sentences "the man climbs that tree", "I can recognize it". But the
transformational routes by which these sentences are derived are quite different. (307) is derived by way of (306) which is a sentence of the form

(309) \[ \text{NP}_1 - V_T - \text{NP}_2 \]

with \(\text{NP}_2\), the dog, and \(V_T\) being the compound verb imagine-climbing that tree of the Verb–Complement type. (308) is derived by way of "I can recognize the man who climbs (or: who is climbing) that tree", which is also a sentence of the form (309), but with "the man who climbs that tree" as \(\text{NP}_2\) and recognize as the simple verb.

In §99.5 we noted that

(310) I found the boy studying in the library (= (231b))

is a case of constructional homonymity (similarly, "I can't catch the dog climbing that tree"). The passive criterion shows that it is subject to both the analysis of (307) and that of (306), and intuitively, this is a clear case of structural ambiguity with a correlated meaning difference. But under both interpretations, the kernel sentences from which (310) is derived (i.e., the \(P\)-basis of the alternative \(T\)-markers) are the sentences "the boy studies in the library", "I found the boy". It is the transformational histories that are distinct. In one case, (310) derives from the intermediate sentence "I-found studying in the library - the boy" (analogous to (307)). In the second case, the intermediate sentence is "I-found-the boy who studies in the library" (or "I-found-the boy who is studying in the library", if "the boy is studying in the library" is taken as the kernel sentence of the \(P\)-basis, cf. fn. 30). Hence the meaning difference under the two structural interpretations of (310) cannot be due to the kernel sentences from which (310) is derivable, or to the morphemes
added or deleted by the transformations that produce (310). This
supports the conclusion of the last paragraph of
does not
4105.4 that a transform differs in meaning from its pre-image
only in the meanings of the morphemes deleted or added.

105.6. We have still not discussed

(311) John wanted him to come

and similar sentences involving \( V_\beta \) instead of \( V_\gamma \) (cf. (301-2)).

Clearly (311) can be handled in the same way as (307). Alongside of (303) we add \( T \)-markers

(312) \( Z_1 \wedge K_1 \wedge T_1 \wedge K_2 \wedge I - T_{\text{comp}} \wedge Q \)

with the transformations analyzed as in (313-4).

(313) \( \delta \cdot \) is determined by \((Q_\gamma, \delta_\gamma')\)

where: \( Q_\gamma = \{(NP, VP_{A1}, \langle VP_{A2}, VP_1 \})\} \)

\( \delta_\gamma' \leftarrow \{U, U, U, to, U\} \)

(314) \( T_{\text{comp}} \) is determined by \((Q_{\text{comp}}, t_{\text{comp}})\)

where \( Q'_{\text{comp}} = \{(NP, VP_{A1}, to \langle VP_{A2}, VP_1, \# NP, VP_{A1}, V_b, \emptyset, NP \})\} \)

\( t_{\text{comp}} \) as in (288), (297), (305)

Note that \( \delta_\gamma' \) is exactly \( \delta \) of (296), and \( Q_\gamma' \) is exactly \( Q_{\text{to}} \) of (168). \( T_{\text{comp}} \) can be combined along with \( T_{\text{comp}}, T_1 \), and \( T_{\text{comp}} \), into a single transformation. (311) is derived from the kernel sentences "he comes", "I wanted it", just as (307) was derived by (303).

There is no homonymy here, as there was in the case of \( V_\gamma \) and \( V_\beta \), since to-phrases, unlike ing-phrases, do not appear as noun modifiers.

A peculiarity of this construction is that the transforms (311), etc., are not subject to the passive transformation. I.e., we
cannot have "I was wanted to come". Similarly, the question
transform "whom did John want to come" is at best unnatural.
Actually, it is not $V_b$ as a whole that is subject to this
limitation, but only the subclass $V_a$, which (cf. paragraph below
(302)) has been incorporated into $V_b$. That is, "I was persuaded
to come" and "whom did John persuade to come" are grammatical.
We can provide for this with a restriction in the $T_a$-grammar,
or we can perhaps reformulate (313-4) so that the transform
does not have the form required by the passive and question
transformations. Neither solution seems particularly good.
We return to sentences of the form (311) below, in §107.

105.2. We can now readily introduce the verb class $V_j$ (cf. §62.2,
§99.2) containing "see", "watch", "hear", etc. and giving such
sentences as

(315) I saw him come (ing)

The forms without ing can be given by $T$-markers

(316) $Z_1 \wedge K_1 \wedge I \wedge Z_2 \wedge K_2 \wedge I \wedge T_{\text{cmp}} \wedge \Phi$

where

(317) $T_{\text{cmp}}$ is determined by $(Q_{\text{cmp}}^{iv}, t_{\text{cmp}})$

where $Q_{\text{cmp}}^{iv} = (NP, VP_A, VP_1, #, NP, VP_A, V_j, \emptyset, NP)$

$t_{\text{cmp}}$ is as in (288), etc.

The forms with ing require a preliminary transformation, like
$T_j$, etc. in place of the first $I$ of (316) to introduce the element
ing into the kernel sentence "he comes" (in the case of (315), etc.).
We might use $T_j$ for this purpose, then extending $T_{\text{cmp}}$ (of (305))
to include $V_j$ along with $V_{\beta}$, were it not for the fact that "I saw
John having been coming" is impossible, though "imagine the dog
having climbed the tree" is possible, and is permitted by (303-5).
This fact leads us to introduce these sentences with $\Theta$-markers

\[(318) \ Z_1^\wedge K_1^{\prime \prime} T_\gamma^{\prime \prime \prime} - Z_2^\wedge K_2^{\prime \prime} I - T_\text{cmp}^{\prime \prime \prime} - \Phi^P\]

where $T_\text{cmp}^{\prime \prime \prime}$ is as in (317), and

\[(319) \ T_\gamma^{\prime \prime \prime} \text{ is determined by } (Q_\gamma^{\prime \prime \prime}, S_\gamma^{\prime \prime \prime})\]

where $Q_\gamma^{\prime \prime \prime} = (NP, VP_A, VP_1)$

\[S_\gamma^{\prime \prime \prime} \text{ is as in (304)}\]

$T_\text{cmp}^{\prime \prime \prime}$ does not have to be extended to include the case where $T_\gamma^{\prime \prime \prime}$ applies to the first component of the complex string to which $T_\text{cmp}^{\prime \prime \prime}$ applies, since $\text{ing} \ VP_1$, as produced by $T_\gamma^{\prime \prime \prime}$, is a $VP_1$, as is its root $VP_1$. Actually, we could have formulated $T_\text{cmp}^{\prime \prime \prime}$ in such a way that $T_\gamma$ of (304) could have been utilized here, instead of the new transformation $T_\gamma^{\prime \prime \prime}$.

The derivation of sentences of the form (315) follows the familiar pattern, and requires no special comment.

105.8. Summing up the constructions of 105, we add to $\mu^T$ the $\Theta$-markers

\[(320) \ Z_1^\wedge K_1 \{ I \ T_\gamma \ T_\gamma^{\prime \prime \prime} \} - Z_2^\wedge K_2^{\prime \prime} I - \{ T_\text{cmp}^0 \ T_\text{cmp}^{\prime \prime \prime} \ T_\text{cmp}^{\prime \prime} \ T_\text{cmp}^{\prime \prime \prime} \} - \Phi^P\]

where:

\[(321) \ T_\gamma \text{ is determined by } (Q_\gamma, S_\gamma)\]

\[T_\gamma \ " \ " \ " (Q_\text{ing}, S_\gamma) \quad (Q_\gamma \text{ as in (168)})\]

\[T_\gamma^{\prime \prime \prime} \ " \ " \ " (Q_\text{to}, S_\gamma) \quad (Q_\text{to} \text{ as in (168)})\]

\[T_\gamma^{\prime \prime \prime} \ " \ " \ " (Q_\gamma^{\prime \prime \prime}, S_\gamma^{\prime \prime \prime}).\]
and:
\[ (322) \left\{ \begin{array}{c} \delta_s \\ \delta_y \end{array} \right\} \rightarrow (U, U, U, \{ \text{to} \}, U) \]

\[ \left\{ Q_{3' \gamma} \right\} = (NP, \left\{ \begin{array}{c} VP_{A1}, \left\langle VP_{A2} \right\rangle \text{be Predicate} \end{array} \right\}) \]

\[ \left\{ Q_{3' \prime \prime} \right\} = (VP_A, VP_1) \]

\[ (323) \text{t}_{cmp}^{k} \text{ is determined by } (Q_{cmp}^{k}, t_{cmp}) \quad (k = 0, \ldots, 4) \]

where:
\[
\left\{ \begin{array}{c} Q_{cmp}^0 \\ Q_{cmp}^1 \\ Q_{cmp}^{11} \\ Q_{cmp}^{111} \\ Q_{cmp}^{111} \\
\end{array} \right\} = \left\{ \begin{array}{c} \text{NP, VP, be} \left\langle \begin{array}{c} \text{Predicate} \\ \text{NP} \end{array} \right\rangle \\ Q_{31}^{2} \\ Q_{31}^{3} \\ Q_{31}^{4} \\
\end{array} \right\} \quad \left\{ \begin{array}{c} \# \text{NP, VP, be} \\ V_{3} \\
\end{array} \right\} \quad \left\{ \begin{array}{c} \# \text{NP} \\ V_{3} \\
\end{array} \right\} \quad \left\{ \begin{array}{c} \# \text{NP} \\ V_{3} \\
\end{array} \right\} \quad \left\{ \begin{array}{c} \# \text{NP} \\
\end{array} \right\} \quad \left\{ \begin{array}{c} \# \text{NP} \\
\end{array} \right\} 
\]

\[ t_{cmp} = (\sigma, 0, \sigma, 0, \sigma, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, \sigma, 1) \]

This is quite similar to the set of transformations summarized in \textsection 25.8, (166-8). These analyses can be reformulated so as to bring out further similarities. There are various ways of simplifying (320-3). For one thing, there is no need to set up \textit{T}_{cmp}^{0} - \textit{T}_{cmp}^{4} as five separate transformations. Since the restricting class for a transformation can be defined as a set of sequences \( W_{1}^{(1)}, \ldots, W_{n}^{(1)} \), we can regard (323) as the characterization of the restricting class of a single transformation \textit{T}_{cmp}, and we can rewrite (320)

\[ (324) \]

(320) with the second set of brackets (and the contained terms) replaced by "\textit{T}_{cmp}"

The similarities with previous constructions (eg., (166-8)) can also be more fully exploited.

In \textsection 26-9 we discovered that the complex verb phrases that
had appeared in the grammar of phrase structure all reduced to
the simple verb-object construction, when we set up a sub-
construction Verb-Complement under transitive verbs. This
analysis was forced upon us by the necessity of accounting for
the behavior of these verb phrases under transformations which
had been set up for the simple verb phrases. Now we see that
the construction Verb-Complement can itself largely be eliminated
by generalized transformations in favor of kernel sentences with
simple verb phrases. The motive for this transformational
analysis lies in the heavy selectional restrictions (including,
as a special case, agreement in number) that hold between the
object and the complement, duplicating the selection of
subject and verb in the simple cases. In other words, if
we were to define grammatical relations in terms of selectional
relations, as suggested briefly in §62.2, we would find that
the grammatical relation subject-verb in simple sentences contains
(cf. fn.26) the grammatical relation object-complement in
sentences of the form $NP_1 V_T NP_2$, where $V_T \rightarrow \text{Verb}^\text{Complement}$
(the sentence becoming $NP_1 \text{Verb}-NP_2^\text{-Complement}$ by $\Phi^P_{11}$). The
only instance of the Verb-Complement construction that resists this
analysis is the case of $V_{\text{sep-P}}$, discussed in §96, e.g., "call up",
etc. These might more properly be called cases of $V_{\text{sep-Particle}}$
since many non-prepositions occur as the complement in such
constructions. In §105 we have seen that one of the particles
in this construction is $\emptyset$, which occurs as the complement when the
Verb is any of $V_a$, $V_e$, $V_b$, $V_d$, $V_s$.

106. In the course of this analysis we have found that much of
the recursive part of the grammar of phrase structure in §62.2
has been cut away. It seems reasonable to place the formal requirement that no recursions appear in the kernel grammar. Specifically, we rule out such statements as \(10, 52.2\), and we drop the reductions of \(51.3\) that permit running through the grammar indefinitely many times. As far as I can determine, this formal requirement on \(P\) does not exclude anything that we would like to retain in \(P\); nor does it impose any artificial or clumsy limitation on the actual statement of the grammar corresponding to \(P\), now that transformational analysis presents an alternative way of generating sentences. On the other hand, this requirement almost trivializes the problem of validating those transformations which we would like to set up as elements in \(T\) for the extra-systematic reasons which we have noted throughout this analysis of English structure.

Given this requirement on \(P\), there is no alternative to transformational analysis in many of these cases. The case of the passive transformation can serve to show how effective this criterion can be in avoiding the necessity for detailed and laborious validation based on total simplicity. Given this non-recursion requirement, there is no alternative to transformational analysis in the case ofing-phrases. By the argument of \(27.1\) it then follows that "consider—a fool", etc., must be verbal elements, and from this it follows, as we saw in \(27.2\), that the passive transformation must be constructed with inversion of noun phrases. In \(27.2\), we had to appeal to overall simplicity of the grammar in putting this argument forward, since the transformational analysis ofing-phrases was partially supported by the fact that passives had been deleted from the kernel.

Naturally, much more study is needed to verify this, but it seems at this point that this requirement on \(P\) meets the conditions
discussed in §39. That is, it is a simple and natural requirement that, by and large, makes transformational analysis necessary in just those cases where it leads to intuitively satisfactory results.

There are also purely systematic motivations for this formal requirement on the level \( P \). It follows from the non-recursion requirement that the kernel must be finite. In §51,2, in developing the general relation between the algebra \( P \) and actual grammars, we were forced to consider the problem of recursive production of sentences by the grammar, since we know that the set \( \mathcal{A}^w \) of grammatical strings of words must be infinite. This led to the artificiality of running through the grammar indefinitely many times (a procedure which, as we saw in chapter VII, may lead to considerable complication in the formulation of the grammar). It also produced a serious theoretical gap in our program of devising a mechanical evaluation procedure for grammars. In the last paragraph of §51, we noted that it is necessary to prove that a given grammar is a reduced form of some system \( \mathcal{P} \). This might not be an easy task, in particular cases. In fact, it may even be the case that there is no general mechanical procedure for determining by inspection of the grammar that it is a reduced form of some system \( \mathcal{P} \), if the set of generated strings is infinite. But we can determine in a mechanical way whether or not a given finite set of derivations leads (in the manner discussed in §51) to an underlying algebra satisfying the axiom system for \( \mathcal{P} \). Hence if the kernel is finite, we do have a mechanical way of determining whether a given grammar is a reduced form of some system \( \mathcal{P} \), since the kernel (by definition) is just the set of strings generated by the grammar of phrase structure.
Now that the higher level of transformational analysis has been established, it is no longer necessary to require that generation by the grammar of phrase structure be infinite. As the level $T$ has been formulated, the process of transformational derivation is recursive, since the product of a $T$-marker can itself appear in the $P$-basis of a $T$-marker (cf. condition 4, §37.4). E.g., from a sentence we can form a that-clause which replaces a noun in a second sentence, giving a more complex sentence from which we can form a that-clause, etc. Similarly, the family of generalized transformations that plays the role of the conjunction rule will indefinitely construct longer and more complex sentences.

In §35.1 (cf. also §§28.2, 45, 453), we sketched the general lines of a definition of grammaticalness, noting that each linguistic level provides a certain descriptive apparatus in terms of which a given set of sentences can be characterized. New sentences are automatically added to this set when we utilize this descriptive machinery to give the simplest characterization of the given sentences. Applying the methods of chapter IV to a linguistic corpus, we construct a finite set $Gr(W)$ containing the highest degree grammatical sentences of less than or equal to some fixed length. We present a system of phrase structure for some subset of $Gr(W)$, producing, perhaps, a finite extension of $Gr(W)$ to a kernel $K$. $K$ is the set of strings of words corresponding (under $P^*$) to the set $Gr(P)$ of products of restricted $\rho$-derivations. We then construct a set of $T$-markers that generate the rest of $Gr(W)$ from the kernel. Allowing these constructed transformations to run freely, applying to transforms, we generate the infinite set $A^W$ of grammatical strings of words. In §45, we suggested that not
all of the corpus need be included in $\text{Gr}(W)$. Similarly, we may be able to construct the systems $P$ and $T$ in a much more simple way if a limited part of $\text{Gr}(W)$ is not regenerated. This is a schematic picture, which must be filled in with detailed construction. It may be that along these lines we will be able to develop an adequate explication of the notion of 'grammatical sentence' in the infinite sense, and an explanation for the general process of projection by which speakers extend their limited linguistic experience to new and immediately acceptable forms.

107.4. Statement 10. §67.2 (now reformulated as (169), §259.2) is the only instance in §67.2 of a recursive statement. To meet the non-recursion requirement of §106, this statement must be eliminated in favor of a transformational analysis. However, there are independent reasons, quite apart from this non-recursion requirement for making this move.

In §105.6 we found that such sentences as "John wanted him to come" (=311) are introduced by transformation. If we investigate these sentences in more detail, we discover that there are certain restrictions on the occurrence of pronouns. Alongside of (311) we have (325) but not (326).

(325)(a) I wanted him to try
(b) I wanted you to try
(c) I wanted to try

(326) I wanted me to try

The only way to avoid a special restricting statement on the level $P$ is to add a mapping $\Phi^P_{1X}$ that carries (326) into (325c).

(327) $\Phi^P_{1X}$ carries $I \text{- want } I \text{- to } \text{try}$ into $I \text{- want-to } \text{try}$

We must determine how extensive is the range of application of $\Phi^P_{1X}$. First of all, it is clear that the analogous restriction
holds for "you". The case of "he" is more difficult. We have both (328a-b).

(328)(a) he wanted him to try
(b) he wanted to try

The simplest way to handle this situation appears to be to set up two distinct elements he and he* corresponding to the element he of _W_, he being an element just like I and you (which accounts for (328b), and he* being an ordinary proper noun (which accounts for (328a), just as we have "he wanted John to come"). The establishment of this pair of homonyms on the level _W_ is further supported by its usefulness for other purposes, as we will see below. We thus replace (327) by the more general characterization

(329) \(\Phi^P_{1x} \text{ carries } \{\frac{I}{\text{you}}, \frac{I}{\text{he}}\} - \text{want-} \{\frac{I}{\text{you}}, \frac{I}{\text{he}}\} - \text{to try into } \{\frac{I}{\text{you}}, \frac{I}{\text{he}}\} - \text{want-to try} \)

As elsewhere in the discussion of pronouns, we will leave out here the plural pronouns and the question of the relations of the pronouns to the auxiliary verbs. These problems cause some further complication, but no change in the structure of the argument or the relative evaluation of alternative solutions.

We also have

(330)(a) John wanted him to try
(b) John wanted to try
(c) John wanted me to try

The simplest way to account for this is to revise \(\Phi^P_{1x}\), replacing (329) by

(331) \(\Phi^P_{1x} \text{ carries } \{\frac{I}{\text{you}}, \frac{I}{\text{NP}_x}\} - \text{want-} \{\frac{I}{\text{you}}, \frac{I}{\text{he}}\} - \text{to try into } \{\frac{I}{\text{you}}, \frac{I}{\text{NP}_x}\} - \text{want-to try} \)
applied to "John wanted him* to try" gives (330a), i.e., it does not apply. Applied to "John wanted him to try", \( \Phi^{P}_{1} \) gives (330b). (325a) is derived from "I wanted him* to try", to which \( \Phi^{P}_{1} \) is inapplicable.

Next we note that \( \Phi^{P}_{1} \) applies generally for the class \( V_{a} \) (cf. (301), 105.5) and we note further that in place of "try" in (331), we may have any instance of to \( \langle VP_{a2} \rangle VP_{1} \). We thus replace (331) by

\[
(332) \Phi^{P}_{1}: \{ \begin{array}{l}
\text{you} \\
\text{he}
\end{array} \} \quad \text{becomes } \emptyset \text{ in env. } \{ \begin{array}{l}
\text{you} \\
\text{NP}_{1}
\end{array} \} \quad \text{to } \langle VP_{a2} \rangle VP_{1}.
\]

107.2. Suppose that \( VP_{1} \) is itself of the form \( V_{a} \text{NP} \text{to } \langle VP_{a2} \rangle VP_{1} \) as in the sentence

(333) I expect you to want him to try

This would be derived in several steps by (312-4), 105.6, from the kernel sentences

(334) (a) he tries
   (b) you want it
   (c) I expect it

From the first two we derive, in the usual way,

(335) you want him to try, \( \text{"you-want-to try-him"} \)

an instance of the same pattern as, e.g., (330a). But we can now take (335) as \( Z_{1} \) in (312), and (334c) as \( Z_{2} \), thus deriving

(333) from (335), (334c) just as we derive (335) from (334a), (334c).

Thus (333) will be an instance of the pattern \( NP_{1} V_{m} NP_{2} \), with \( NP_{1} \) being I, \( V_{m} \) being the compound verb \( \text{expect} \text{to want} \text{to try} \) (an instance of \text{Verb-Complement}), and \( NP_{2} \) being \text{you}; and (333) is derived from "I-expect-to want to try-you" by \( \Phi^{P}_{1} \). Investigating various other cases of the same type, we find that we can have
(336) but not (337).

(336)(a) I expect you to want to try
(b) I expect you to want me to try
(c) I expect to want to try
(d) I expect to want you to try

etc.

(337)(a) I expect you to want you to try
(b) I expect me to want to try
(c) I expect to want me to try
(d) I expect me to want you to try

etc.

Obviously, $\Phi_{1 \chi}$ applies exactly as in (332) to each 'stage' in the transformational history of sentences like (333). That is, it applies to the intermediate sentence (335), and then to (333) as a whole. There is no ambiguity in the way it applies to (333) as a whole, since there is only one way to analyze this sentence as an instance of $\text{NP-VP}_{1} - V_{0} - \text{NP-to}<\text{VP}_{2}>\text{VP}_{1}$, no matter what the analysis of $\text{VP}_{1}$ may be. This suggests that we regard $\Phi_{1 \chi}$ not as a mapping (since, by condition 2, 487.5, a mapping cannot apply more than once in the derivation of a string), but as a transformation $T_{\chi}$ that always follows the transformation $T_{\chi}'$ (cf. (314)) that carries (334a), (334b) into (335) and (335), (334c) into (333).

But $T_{\chi}$ must precede the mapping $\Phi_{1 \chi}$ that carries Verb-Complement-NP into Verb-NP-Complement. We must therefore rephrase (332) slightly in formulating $T_{\chi}$.

(338) $T_{\chi}$ is determined by $(Q_{\chi}, \epsilon_{\chi})$
where:

$Q_{\chi} = \{(\text{YOU})_{\text{NP}_{\chi}} \text{VP}_{1} \text{to} \text{X}, \{(\text{YOU})_{\text{he}}\} \}$

$\epsilon_{\chi} \leftarrow (\text{U}, \text{U}, \text{U}, \text{V}, \text{X})$

(338) is thus (332) revised to apply before $\Phi_{\chi}$ and given in
the proper transformational form. It is the transformation that carries, e.g., "I-want-to come - I" into "I - want - to come - ∅", which is carried by $\mathcal{T}^p_{1_1}$ into "I-want-∅-to come", and by $\mathcal{Q}^p_{1_0}$ into "I-want-to come".

(312) gives the $\mathcal{T}$-markers in which $\mathcal{T}_{\text{cmp}}^{***}$ occurs. We must therefore rewrite (312), replacing "$\mathcal{T}_{\text{cmp}}^{***}$" in (312) by "$\mathcal{T}_{\text{cmp}}^{***}$ $\mathcal{T}^c_{\text{cmp}}$". But in (324) we dropped $\mathcal{T}_{\text{cmp}}^{***}$ and the set of related transformations in favor of a single transformation $\mathcal{T}_{\text{cmp}}$. Thus $\mathcal{T}^c_{\text{cmp}}$ must be added in (324) after $\mathcal{T}_{\text{cmp}}$. This causes no problem when $\mathcal{T}_{\text{cmp}}$ is taken as some transformation of (320) other than $\mathcal{T}_{\text{cmp}}^{***}$. In this case, $\mathcal{T}^c_{\text{cmp}}$ will simply not be applicable. Note that $\mathcal{V}^a_{\text{cmp}}$ (in the restricting class of $\mathcal{T}^c_{\text{cmp}}$) is a subclass of $\mathcal{V}^a_{\text{cmp}}$ (in the restricting class of $\mathcal{T}_{\text{cmp}}^{***}$). We thus replace (320) not by (324), but by

(339) (320) with the second set of brackets (and the contained terms) replaced by "$\mathcal{T}_{\text{cmp}}^{***}$ $\mathcal{T}^c_{\text{cmp}}$

Investigating in detail the history of (336a), we find the following stages:

(340) I. From the kernel sentences "you try", "you want ∅ it", we derive by (320), fourth line, the reduced correlate (33) "you-want to try-you"

II. By $\mathcal{T}^c_{\text{cmp}}$, which now must follow $\mathcal{T}_{\text{cmp}}^{***}$ in (320) (as revised in (339), we derive "you-want to try-∅" (which would be carried by $\mathcal{Q}^p_{1_1}$ into "you-want-∅-to try")

III. From the reduced correlate (33) "you-want to try-∅" and the kernel sentence "I expect ∅ it", we derive by (320), fourth line, the sentence "I-expect to want to try ∅-you"
to which $T_5$ does not apply. But the sentence "I expect to want to try $\emptyset$-you" contains the Verb-Complement construction twice. Hence in mapping this sentence by $(\Phi_3^p, \Phi_1)$ must apply twice, once to want-to $\text{\^try}$-\$\emptyset$, giving want-$\emptyset$-to $\text{\^try}$, and once to expect-to want-to $\text{\^try}$-\$\emptyset$, giving expect-you-to want-to $\text{\^try}$-\$\emptyset$. Applying the remaining mappings we derive (336a).

In a similar way, the other instances of (336) are derived, while those of (337) are excluded. Investigation of sentences with "he" shows that these follow automatically in the correct way, now that we have the two elements "he" and "he*". The extension to longer and longer strings $V_a \ldots V_a \ldots$ etc., allows automatically so that the recursive formulation of (169) (originally, statement 10, §67.2) can be dropped in the case of to-phrases. Of course, statement 10 did not account for the full range of sentences of the form (336). To give a rule which produces (336), and excludes (337), directly in terms of phrase structure would be quite difficult. But we see that the correct forms result automatically from the transformational analysis for the simple sentences (325), etc. This complex of sentences is another example of behavior which is simple and systematic from the point of view of transformational structure, though it appears quite complex in terms of lower levels.

In §87.2, in connection with Condition 5, we discussed the problem of repeated application of mappings $(\Phi_3)$ and the conditions under which it arises. In step III of (340) we have an instance of the general problem discussed in §87.2. If we construe the $T$-marker in this analysis as (339), then the reduced correlate will exclude all mappings, and $\Phi_3$ in particular, will apply only
at the conclusion of the derivation of complex sentences, as in (340), necessitating repeated application. A much neater solution in this case would be to rewrite (339) as

\[(341) \quad (320), \text{ with } \left\{ \cdots \right\} \Phi^P \text{ replaced by } \left( T_{\text{cmp}} T_{\Phi} \Phi^P \Psi_1 \right)^a \]

where \(\left\{ \cdots \right\}\) stands for the second set of brackets in (320).

Now the reduced correlate will exclude only the mappings after \(\Phi^P\) (cf. Def. 54, §87.5). But if we accept condition 5, §87.5, we will not be able to reapply (341), substituting a reduced correlate from (341) for \(Z_1\) (a term of the \(\Phi\)-basis) in (341), since in this case \(\Phi^P\) will appear twice, once in the reduced correlate, and once in the \(\Phi\)-marker itself, thus violating II, condition 5. If we drop condition 5, and accept (341), then in place of (340) we will have

(342) I. From the kernel sentences "you try," "you want \(\emptyset\) it," we derive by (320), fourth line, the reduced correlate "you-want to try-you".

II. By \(T_{\Phi}\), which now must follow \(T_{\text{cmp}}\) in (320) (as revised now in (341)), we derive "you-want to try-\(\emptyset\)".

III. By \(\Phi^P_{\Phi}\) we derive "you-want-\(\emptyset\)-to try". This is the reduced correlate from (341).

IV. From the reduced correlate "you-want-\(\emptyset\)-to try" and the kernel sentence "I expect \(\emptyset\) it," we derive by \(T_{\text{cmp}}\) (in this case \(T_{\text{cmp}}\) of (320)) the sentence "I-expect to want \(\emptyset\) to try-you" to which \(T_{\Phi}\) is inapplicable. Applying \(\Phi^P_{\Phi}\) to this, we derive "I-expect-you-to want \(\emptyset\) to try" which becomes (336a) by \(\Psi_1\).

This approach, with \(\Phi^P\) 'sealed in' to the transformation is considerably neater in application, and will considerably simplify
the definition of $\Phi_{11}^p$, since at least for this case, $\Phi_{11}^p$ need
apply only once to each stage of the development of (335a), etc.,
rather than indefinitely often at the end of this development.

107.2. To sum up the discussion of 107.1-2, we have made the
following points.

1. There are two elements he and he*, with he* a proper
noun, and he a pronoun just like I, you.

2. (320) of §105.8 is replaced finally by (343) or (344),
depending on whether or not Condition 5, §107.5 is retained.

$$\begin{align*}
\left\{ \begin{array}{c}
I \\
T\gamma \\
T\gamma' \\
T\gamma''
\end{array} \right. \\
\left\{ \begin{array}{c}
\overset{\sim}{Z}_1 \overset{\sim}{K}_1 \\
\overset{\sim}{T}_\gamma \\
\overset{\sim}{T}_\gamma' \\
\overset{\sim}{T}_\gamma''
\end{array} \right. \\
\overset{\sim}{I} \overset{\sim}{T}_{cmp} \overset{\sim}{T}_\gamma \overset{\sim}{\Phi^p}
\end{align*}$$

(344) (343) with "$\Phi^p$" replaced by "$\Phi_{11}^p \overset{\sim}{\psi}_i$"

T_{cmp} is as in (324), §105.8, and $T_\gamma$ is as in (338).

It follows that the simpler sentences (325), (328), (330),
etc., as well as the more complex forms (336), etc., are
correctly generated in a simple and uniform way, while (326),
(337), etc. are rejected. Thus a description which would be
quite complex in terms of lower levels, can be given with
extreme simplicity in transformational terms, with more
complex sentences produced from simpler ones. In particular,
"I want to try" is derived from "I try", "I want it" just as
"I want him to try" is derived from "he tries", "I want it"; and
in the very same way, "I expect to want to try" is derived from
"I want to try", "I expect it", and "I expect him to want to try"
from "he wants to try", "I expect it", ultimately from the kernel sentences "he tries", "he wants it", "I expect it". In all cases, there seems to be good intuitive and semantic support for the resulting analysis. The setting up of two elements he and he* also has support in the referential use of these words. Whereas I and you have an unambiguous reference in sentences like "John said that I would come", "John said that you would come", in "John said that he would come" the reference is ambiguous. In our terms, if "he" in this sentence, derived from the syntactic element he (the pronoun), the reference is to John, as it is in (330b), where he becomes θ; if it is derived from the syntactic element he* (the proper noun), the reference is to a second person, as in (330a). While the results of our grammatical analysis naturally tell us nothing about reference, this syntactic discussion does provide the means for an adequate description of reference in some semantic description of English.

The analysis at which we have arrived is supported systematically by the following considerations.

1. It eliminates the need for special restrictions on the occurrence of pronouns in the context NP_e V_a -- to V_P_1, as well as in longer sentences of the form (336). These sentences are correctly generated in a simple way. (169) of 325.2 (the revision of statement 10, 367.2) would not correctly account for these forms without complex emendation. This problem did not arise in chapter VII, we because we could not handle sentences of the form (325a), etc., at that point.
2. This analysis eliminates (325c) as a special sentence type, reducing it to (325a-b). This will lead to the elimination of the element $VP_2$, with consequent simplification of several statements of the kernel grammar, and elimination of statement 10 ((169) of §25.2).

3. It is not necessary to repeat the fact that to occurs with $VP_{A2}$ (and ing with have/en, as we will see directly), since $VP_2$ is eliminated in favor of the standard forms $NP-V-NP-to-VP_1$, where this fact is anyway stated. This eliminates a difficulty noted in §25.2.

4. In the final paragraph of §63 we pointed out that one difficulty in analyzing "want to", etc., as auxiliaries is that, under conjunction, the "to" belongs with the following verb, so that in $V_1-to-V_2$ constructions, the constituent break falls after $V_1$. It now follows automatically that the break is at this point.

5. Sentences such as

(345) he tried not to fail (=87))

are now automatically constructed in the correct way from "he ed not fail" (which is mapped into "he did not fail") and "he tried it". Cf. §92.3. Note that the intuitive shortcomings of our earlier analysis noted in §92.3 are now eliminated by this transformational analysis of $V-to-V$ constructions.

6. This analysis eliminates a recursive statement in the kernel grammar, namely, statement 10, §67.2 (since rephrased as (169), §25.2).

In view of §106, the final consideration is alone a
sufficient validation for the transformational analysis we have adopted. But I think it is important to note that just as in each of the earlier cases that we have considered, there are strong independent reasons for eliminating the recursive statement. This adds weight to the conclusion of §106 that the non-recursion requirement should be regarded as a formal condition on grammars (hence a condition of 'simplicity', in the broad sense of chapter III and §80); i.e., the conclusion that the kernel should be finite, and that the process of generation of new and longer sentences is transformational.

Note incidentally that we can now arrive at an intuitively adequate explanation for the difference between (345) and (346).

(346) he did not try to fail (from he\textsuperscript{ed} not \textsuperscript{try to fail})

(346) is derived by $T_{\text{not}}$ (cf. §92 -- i.e., by negation) from "he tried to fail", which in turn is derived transformationally from the kernel sentences

(a) he fails

(b) he tried it

(345) is derived, as we have seen, from the kernel sentences (347), where (347a) is subject to the earlier transformation $T_{\text{not}}$ that carries it into "he\textsuperscript{ed} not\textsuperscript{fail}".

Both (345) and (346) thus originate from (347) (i.e., (347a-b) form their $F$-basis). They differ in that in forming (345), (347a) is negated, while in forming (346), (347b) is negated.

107.4. This gives in outline the reasoning involved in the elimination of $VP_B$ and some of the consequences of this step. It
remains to carry out this elimination in detail. In chapter VII, we limited the discussion to certain classes of verbs that avoided the pitfalls uncovered there. In the establishment of a kernel in chapters VIII and IX we have found a rationale for this limitation. But we must now extend the discussion to include verbs of the types that cannot be adequately described in terms of the level of phrase structure.

In §62.1, chapter VII, we set up three types of verbs that occur with to-phrases (cf. (301), §105.2). \( V_a = \{ \text{want, like}, \ldots \} \) occurs with to-phrase or NP-to-phrase (e.g., "I want to come", "I want him to come"). \( V_b = \{ \text{persuade, advise}, \ldots \} \) occurs only with NP-to-phrase (e.g., "I persuaded him to come", not "I persuaded to come"). \( V_c = \{ \text{try, decide}, \ldots \} \) occurs only with to-phrase (e.g., "I tried to come", not "I tried him to come"). In §62.2, we dropped \( V_a \), considering it as the overlap of \( V_b \) and \( V_c \). We then excluded \( V_b \) from consideration, because of the difficulties that appeared in §64.5. In §107 we have reversed the order of precedence and have given \( V_a \) as the basic subclass of verbs, and we must now state the special restrictions that mark \( V_b \) and \( V_c \). Furthermore, we must give the analogous constructions for \( V_x, V_y, V_z \) (cf. (302)), which parallel \( V_a, V_b, V_c \), respectively, for the case of ing-phrases.

\( V_x \) is readily introduced. Since it shares all the features of \( V_a \) discussed above, we need only revise the definition of \( T_c \), replacing (338) by
(348) $T_0$ is determined by $(Q_0, \xi)$

where: $Q_0 = \left\{ \left\{ \begin{array}{c} I \to \text{VP} \\ \text{you} \end{array} \right\}, \left\{ \begin{array}{c} V\to \text{VP} \\ \text{you} \end{array} \right\} \right\}$

$\xi \leftrightarrow (U, U, \sigma, 2)$

This formulation permits the construction of longer strings $V_\alpha \to V_\alpha \to \to \to \to \cdots$, etc., as did the recursive definition of $\text{VP}_B$, since transformations can be freely compounded.

We can regard $V_\alpha$ and $V_\gamma$ as subclasses of $V_\alpha$ and $V_\gamma$, respectively, with a special restriction on their occurrence in $T$-markers (in the $P$-basis). We thus add to the grammar of the level $T$ the condition

(349) Condition $\text{INF}_1$: If $Z_2$ is a string of the form

\[
\left\{ \begin{array}{c} \text{you} \\ \text{NP}_X \\ \text{you} \end{array} \right\} \text{VP}_A \left[ \begin{array}{c} V_\gamma \\ V_\gamma \end{array} \right] \cdots
\]

then $Z_1$ is a string of the form: $\left\{ \begin{array}{c} \text{I} \\ \text{you} \end{array} \right\} \cdots$

In other words, we require that a verb of $V_\alpha$ or $V_\gamma$ be followed by "I" if its subject is "I", by "you" if its subject is "you", and by "he" if its subject is $\text{NP}_X$ as in (338).

Condition $\text{INF}_1$ is a condition on $T$-markers of the form (343). Alternatively, we could dispense with (349) and revise (348) to the same effect.

This leaves only $V_\beta$ and $V_\gamma$ to be described. The obvious suggestion is to supply for these elements a condition differing...
from Condition INF₁ only in that \(^{\left[ \begin{bmatrix} y_s \\ y_b \end{bmatrix} \right]}\) is replaced by
\(^{\left[ \begin{bmatrix} y_b \\ y_\beta \end{bmatrix} \right]}\), and "Z₁ is a string of the form ..." is replaced by
"Z₁ is not a string of the form ..." This would be a condition
that \( y_b \) and \( y_\beta \) are never followed by "I" if their subject
is "I", etc. But this approach is not quite correct. Before
suggesting an analysis for \( y_b \), \( y_\beta \), we turn to the consideration
of another transformation.

107.5. Consider the following sentences:

(a) I persuaded him to try
(b) I persuaded you to try
(c) I persuaded myself to try
(d) he persuaded him to try
(e) he persuaded himself to try
(f) John persuaded him to try
(g) John persuaded himself to try
(h) John persuaded me to try

These sentences are exactly parallel to
and (330) of 4107.1. persuade is a \( y_b \), and the sentences of
(350) differ from the sentences of (325), (328), (330) only in
that where the pronoun \( X \) would be dropped after want (a \( y_s \)),
it becomes \( X\)-self after \( y_b \).

Consider now the more complex forms discussed in 4107.2.
If in (336) we replace the \( y_s \)'s "expect" and "want" by the \( y_b \)'s
"persuade" and "force", and if we replace the deleted pronoun
\( X \) of (336) by \( X\)-self, then we derive exactly the set (351) of
grammatical sentences with \( y_b \)'s.
(351) (a) I persuade you to force yourself to try
    (b) I persuade you to force me to try
    (c) I persuade myself to force myself to try
    (d) I persuade myself to force you to try
       etc.

We see then that we have a transformation $T_{\text{self}}$ which applies to $V_b$ exactly as $T_\phi$ applies to $V_a$. The effect of $T_{\text{self}}$ is to replace a pronoun $X$ by $X$-self after $V_b$, whereas $T_\phi$ would have replaced it by $\emptyset$ (i.e., essentially, deleted it) after $V_a$. Investigation of $V_b$ shows exactly the same phenomenon. Thus we have

\[(352) \quad T_{\text{self}} \text{ is determined by } (Q_{\text{self}}, \delta_{\text{self}})\]

where:
\[Q_{\text{self}} = \{\left\{\frac{I}{\text{you}}\right\}, \frac{V_P}{\text{to}}, \frac{V_{\beta} \text{ ing}}{\text{he}}, \left\{\frac{I}{\text{you}}\right\}\} \]

where $X$ is any string and $NP_X$ is the subclass of nouns excluding $I$ and $\text{you}$, as in (333)

\[\delta_{\text{self}} \longmapsto (U, U, U, \text{self})\]

This explains why $V_b$ and $V_{\beta}$ cannot be introduced directly as subclasses of $V_a$ and $V_{\alpha}$ respectively, subject to a condition which is the contrary of (349). In fact the constructions $I-V_b$-to-phrase-$I$, etc., do occur, but $T_{\text{self}}$ rather than $T_\phi$ applies to them. A simple way to give just as much information about $T_{\text{self}}$ as we have developed so far would be simply to revise the $T$-marker-schema (343) as

\[(353) \quad Z_1^{K_1} \{\text{as in (343)}\} \left\{\begin{array}{l}
Z_2^{K_2} I - T_{\text{cmp}} - T_\phi - T_{\text{self}} - F_P
\end{array}\right\}\]

Since $T_\phi$ by definition applies only to $V_a$ and $V_{\alpha}$, and $T_{\text{self}}$...
only to $V_\beta$ and $V_\delta$, this will give the correct forms in all cases. But $Z_1$ and $Z_2$ in (353) can be transforms, even transforms produced by (353) itself. Hence just as we can derive stepwise a string $V_a \rightarrow V_d \rightarrow \ldots$ of any length, we can also derive strings in which $V_p \rightarrow X$-self, etc., are interspersed freely. This permits, e.g.,

(354) John visualized me forcing myself to expect him to come which derives ultimately from the kernel strings "he comes", "I expect it", "I force it", "John visualized it". Thus quite a complicated network of sentences is generated by continued application of very simple transformations.

Further examination of the distribution of "self", however, reveals that $T_{\text{self}}$, as we have stated it, is only a special case of a quite general transformation. Note that "his", "you", etc. in (351) are each the object of the complex verb "persuade - to try" (an instance of Verb - Complement). But it is true in general that the object of a verb undergoes the transformation $T_{\text{self}}$, even in kernel sentences. Thus we have

(355) he saw himself, etc.

The transformation $T_{\text{self}}$ must be reformulated, then, to hold more generally of NP-$V_T$-NP, and it must be given as a mapping, since it is obligatory even for kernel sentences. We replace (352) by (356), defining the component $\Phi^P_{10}$ of $\Phi^P$

(356) $\Phi^P_{10}$ is determined by $(Q_{\text{self}}, \delta_{\text{self}})$
where: \( Q_{\text{self}} = \{ \left\{ I \right\}_{\text{you}} \} \cup \left\{ I \right\}_{\text{you}} \}

\(\delta_{\text{self}} \rightarrow (U, U, U, \text{self})\)

The application of \( T^P_{10} \) to transitive verbs of the form \( V_T \), as above, is thus just a special case of its application to all transitive verbs. In particular, it also applies to \( V_{\alpha} \). Thus alongside of (350c) we have

(357) I can't imagine myself acting that way (imagine a \( V_{\alpha} \))

Whether it applies to \( V_{\alpha} \) or not depends on a decision as to the grammaticality of such sentences as

(358) I want myself to act that way (37)

If such sentences are considered grammatical, then no special statement is needed. If not, then a qualification must be added stating that \( T^P_{10} \) does not apply to \( V_{\alpha} \). This will not be the first instance of a special transformation restriction on \( V_{\alpha} \). We noted above (in \( 105.6 \)) that the passive transformation does not apply to it. If the exceptions in the case of \( V_{\alpha} \) are in fact not merely sporadic, this may be an indication that a deeper analysis should be sought. But it seems that a much more intensive study of grammaticality (and, no doubt, finer criteria) should precede any such investigation.

In any event, we see that for \( V_{\alpha} \), and perhaps for \( V_{\alpha} \) as well, \( T^P_{10} \) is not obligatory as was stated in \( 107.2-3 \). It may not apply, in which case \( T^P_{10} \) will automatically give (357), (358), etc. We thus might consider reformulating (343) as
(359) $Z_1 \overset{K_1}{\leftarrow} \{ \text{as in (343)} \} - Z_2 \overset{K_2}{\leftarrow} I - \bar{\Phi}_{\text{cmp}} \langle T_8 \rangle \bar{T}^P$

But (359) poses certain problems relating once more to condition 2, §87.5. Suppose that we accept Condition 2, and with it, (343). Cf. §107.2-3. Consider now the sentence

(360) I expect you to force yourself to try

From the kernel sentences "you try", "you force $\emptyset$ it", we derive, as the reduced correlate from (359),

(361) you – force to try – you

since $T_8$ is inapplicable here, force being a $V_b$. Application of $\bar{T}^P$ to (361), as prescribed by (359), would give the correct form

(362) you force yourself to try

But in forming (360) we take as $Z_1$ in (359) the reduced correlate (361), not the fully mapped form (362) (cf. Condition 4, §87.4). And as $Z_2$ of (359) we select "I expect $\emptyset$ it". With these choices of $Z_1$ and $Z_2$ in (359) we derive the reduced correlate

(363) I – expect to force to try you – you

But $\bar{T}^P_{10}$ does not apply to (363), since the pronominal object "you" is distinct from the subject "I". Hence application of $\bar{T}^P$ to (363) will yield

(364) I expect you to force you to try

instead of (360). This shows that if we accept Condition 2, we must set up both $T_{\text{self}}$ and $\bar{T}^P_{10}$ as distinct and separate elements, and we must replace (359) by
(365) (359) with "$\Phi^P$" replaced by $\Phi^P_{\text{self}}$

Now the derivation of (360) proceeds smoothly. Since $\Phi^P_{\text{self}}$ figures in the reduced correlate (which drops only the final component of the mapping $\Phi^P$, in this case, $\Phi^P$ itself), we have, instead of (361),

(366) you-foce to try - yourself

And from this, (360) follows by reapplication of (365) with (366) as $Z_1$.

But this is an unfortunate solution, since it means that we cannot regard $T_{\text{self}}$ as simply a special case of $\Phi^P_{10}$, but must give it separate status.

If we drop Condition 5 and accept (344) (cf. §107.2-3), then we simply "seal in" $\Phi^P_{10}$ along with $\Phi^P_{11}$. Thus instead of (365) we have

(367) $Z_1^\lambda K_1^\lambda$ \{ as in (343) \} - $Z_2^\lambda K_2^\lambda I - T_{\text{cmp}} < T_5$ \psi_1^\psi_1^\psi_1$

(\text{where } \psi_1 \text{ is the compound } \Phi^P_{11} (\Phi^P_{10})). \text{ With this, } T_{\text{self}} \text{ is dropped as a separate element.}

There is a related problem which does not, apparently, further affect the discussion pertaining to condition 5. Consider the sentence

(368) John wanted to better himself

This comes from the kernel sentence "he bettered himself", "John wanted $\varnothing$ it". But the first of these is a sentence to which $\Phi^P_{10}$ has already applied. Thus whether we accept (365) or (367), it is necessary to reformulate the $T_\sigma$-marker so that
\( \Phi_{10}^P \) applies independently to \((Z_1, K_1)\) and \((Z_2, K_2)\). Thus instead of (365), we have

\[
(369) \quad Z_1^K K_1^\Phi_{10}^P = \left\{ \begin{array}{l} \text{as in (343)} \end{array} \right\} - Z_2^K K_2^\Phi_{10}^P - T_{\text{cmp}} -(T_\delta) - T_{\text{self}} - \Psi_0
\]

and instead of (367), we have

\[
(370) \quad Z_1^K K_1^\Phi_{10}^P = \left\{ \begin{array}{l} \text{as in (343)} \end{array} \right\} - Z_2^K K_2^\Phi_{10}^P - T_{\text{cmp}} - T_\delta - \Psi_1 - \Psi_1
\]

(369) still meets Condition 2, and (370) now violates it in two respects.

107.6. To recapitulate, we have either (369) or (370) as a \( T \)-marker schema, depending on whether or not Condition 2, §87.5, is adopted. \( V_\alpha \) contains a subclass \( V_C \) (and \( V_\alpha \) a subclass \( V_\gamma \)) appearing only in \( T \)-markers meeting Condition INF_1 ((349)). That is, if the verb of \( Z_2 \) in the \( T \)-marker is \( V_C \) or \( V_\gamma \), then the subjects of \( Z_1 \) and \( Z_2 \) must agree in person.

Furthermore, we add

\[
(371) \quad \text{Condition INF}_2: \quad T_\delta \text{ must appear in the } \alpha \text{-marker if } Z_2 \text{ is an NP} \left[ V_C \right] \left[ V_\gamma \right] \ldots
\]

and we see that perhaps "\( V_C \)" in the brackets should be replaced by "\( V_\alpha \)", depending on a decision as to the grammaticalness of such sentences as (358). In other cases, \( T_\delta \) may or may not appear in the \( \alpha \)-marker.

If the verb of \( Z_2 \) in (369) or (370) is a \( V_\beta \) or a \( V_\beta \), then \( T_\delta \) is inapplicable by definition even if it appears in the \( T \)-marker. In this case \( \Phi_{10}^P \) (or \( T_{\text{self}}, \) in (369)) will therefore apply, giving such sentences as "I persuaded myself to try". In this way we derive the sentences (350).
If the verb of \( z_2 \) is a \( V_a \) or a \( V_{\alpha} \) and \( T_p \) does appear in the \( T \)-marker, then we derive such sentences as "I want to try", "I imagined trying", "I want him to try", etc., and in general, (325), (328), (330), etc.

If the verb of \( z_2 \) is a \( V_a \) or a \( V_{\alpha} \) and \( T_p \) does not appear in the \( T \)-marker, then \( T_{10} \) (or \( T_{self} \)) will apply, giving (357), (358), etc.

A sequence of such markers can be used to generate more complex sentences, since transforms can appear in the \( P \)-basis of a \( T \)-marker. Since such a sequence can be chosen in any order, we can derive such complex sentences as (336), (351), and (354), and by continuing the process, even more complex varieties without limit.

With this we can drop statement 10, \( \text{§67.2} \), from the kernel grammar, and with it the element \( V_{PB} \), thus incidentally eliminating a recursive statement. But we have done much more than this. We have described a great number of sentence forms that were not incorporated into the description of phrase structure in \( \text{§67.2} \), and which could not have been incorporated without considerable complication and special statements. We have seen that these sentences are all constructible from elementary kernel sentences by a simple set of operations which may be freely applied and reapplied.

In addition, we have reduced the sentence form \( NP - \text{verb} - \text{to} - \text{verb phrase} \) (similarly, \( NP - \text{verb} - \text{ing} - \text{verb phrase} \)) to a special case of the construction \( NP - \text{verb} - NP - \text{to} - \text{verb phrase} \) (or \( NP - \text{verb} - NP - \text{ing} - \text{verb phrase} \)), thus dropping a special restriction on the occurrence of pronouns as objects. We also have the incidental systematic gains noted in \( \text{§107.2} \). Finally, as we noted in \( \text{§107.2} \), the
kernel sentences from which a given complex sentence is derived do give what intuitively is the 'content', in some sense, of this complex sentence.

107.2. The investigation of "self" and its distribution will, I think, turn out to be of some importance for the study of transformational structure. The occurrence in a sentence of X-self, for some pronoun X, indicates a special relation between this element and some noun or some other pronoun in the sentence. To give a general rule concerning all such cases directly for all sentences may be quite difficult. But we may be able to show that whenever this relation exists between two positions in a complex sentence Z, then the elements filling these places have some fixed relation in the kernel sentences from which Z is derived (e.g., they may be subject and object). If so, the distribution of self may be statable simply in terms of this kernel sentence relation. The simplification thus introduced can be an important support for transformational analysis, in particular cases. This is just the course we have followed in the preceding discussion.

There are many other instances where this approach can be used to support transformations that we have constructed. There are also cases where problems arise. For instance, the relative order of $F^P_{10}$ and the passive transformation $T_p$ (cf. 494) is unclear. $T_p$, it will be recalled, preceded all mappings in the formulation we presented above. There is good reason to retain this order even with $F^P_{10}$ added. Thus we cannot have

(372)(a) himself was seen by John in the mirror

(b) dinner was eaten by himself (from "John ate dinner by himself", by $T_p$ and $T_{pd}$ (cf. 4794.2, 95.2)
On the other hand, we have seen that $\Phi^P$ applies in forming the reduced correlate of $T$-markers of the form (369), and $T_p$ certainly applies to sentences derived from such $T$-markers. If Condition 5, $\S 87.2$, is dropped, this and several other problems are avoided. On the other hand, we have no good alternative to condition 5. The further investigation of "self" gives rise to other problems of this nature, and it seems best to put off this study until a specific investigation into the complex of problems surrounding Condition 5 is undertaken.

107.8. For similar reasons, we will not present a precise transformational statement of the components $\Phi^P$ of the mapping $\Phi^P$. There is no particular problem in reformulating the characterization of these components in transformational terms, but, as we have seen, the nature of this formulation depends directly on a decision as to the status of Condition 5. In particular, if Condition 5 is relaxed sufficiently, it will be possible to avoid the unwelcome necessity of stating the mappings as infinite families of transformations.

Although, for these reasons, we will not undertake to formulate the mappings in an exact way, it is important to recognize that there is no fundamental difficulty in doing so. In chapter VII, $\S 67.3$, however, there was a fundamental difficulty, since there was no clear sense at that point to the notion of derived constituent structure under mappings,
to the process of compounding of elementary components of a
mapping, etc. Nor was it clear in what manner and by
what precise mechanism a mapping $\Phi_1^P$ can refer to the
'derivational history' of a sentence. Thus although specific
problems remain unresolved, the general problem of giving an
effect, simple, and systematic analysis of the mapping $\Phi^P$
which relates phrase structure and word structure, is eliminated
by the theory of transformations. In addition, many of the
specific difficulties in constructing mappings for English
have been eliminated -- cf., e.g., §83.5 (iii).

108.1. In accordance with the formal condition posed in §106,
it is necessary to eliminate all statements of the tentative
kernel grammar of §67.2 that require running through the grammar
more than once. Statement 17, the analysis of the adjective
phrase $AP$, still has this property. This statement introduces
the possessive noun phrase $NP^S_1$ as one of the forms of $AP$, but
this element leads to a recursion, since one of the forms of
$NP$ may be ..$NP^S_1$. In the formulation of statement 17 we
discovered independent indications that $NP^S_1$ is a difficult
element. In §67.1 we established a simple convention that
enabled us to avoid specifying for each substatement of the
grammar, whether or not it is obligatory. This convention effects
a considerable simplification in a natural manner, but it
caused difficulties in statements 16 and 17. Statement 16 has
since been dropped, and in statement 17, it is the analysis of
the element $NP^S_1$ that is at fault.

$NP^S_1$ has already been eliminated transformationally
from the context -- $ing^V_P_1$. $NP^S_1^V_P_1$ has been analyzed above
(cf. §95.4) as a transform of $NP_1 \overset{VP_A}{\to} VP_1$ (e.g., "John's flying" comes from "John flies"). Similarly, we might analyze

$NP_1 \overset{\wedge}{SE_1} \overset{\wedge}{NP_2}$ (e.g., "John's book")

as a transform of some sentence form

$NP_1 \overset{\wedge}{VP_A} \overset{\wedge}{V_X} \overset{\wedge}{NP_2}$

The best choice for $V_X$ will have to be made in terms of the selectional relations between $NP_1$ and $NP_2$ in (373) and (374). The element which comes closest to giving complete selectional identity is apparently "have". It seems that for many noun phrases of the form (373), there is a sentence of the form (374) with $V_X$ = have. In fact, certain phrases fail this condition. E.g., we have "John's behavior", "John's action", but not "John has behavior", "John has (an) action". But there is another source for phrases (373), where $NP_2$ is analyzable, in the manner of §103, as verb-nominalizer. In this case, (373) is derived from $NP_1$-verb. But we do in fact have "John behaves", "John acts", so that this analysis is available in the case of "John's behavior", "John's action". This leads us to the conclusion that such words as "behavior" and "action" are not simple nouns, but are constructed from verbs by a nominalizing morpheme. There is much more to say about this, but it appears to be another instance of an intuitively satisfactory conclusion reached on formal, transformational grounds. It may well be the case that still further sources exist for the construction (373). For instance, $NP_1$-is-A may be a further source, accounting for such phrases as "the country's safety", "his sincerity", etc. This matter should
be thoroughly investigated, since it may provide a good source for insight into morphology. We will consider here only those instances of (373) for which some of the form of the sentence \((374)\) with \(V_x\) taken as "have" provides a transformational origin.

This transformational analysis, aside from eliminating a recursion in the kernel grammar, has several incidental desirable consequences. One artificial feature of the analysis of the construction \(NP^S_1\) as an \(AP\) was that it was necessary to assume a \(\emptyset\) article, so that "\(\emptyset\) John's book" was analogous to "a good book". This artificiality is avoided now. A much more significant simplification arising from this transformational analysis would appear had we considered sequence-of-adjective rules in \(467.2\). \(NP^S_1\) must be the first 'adjective' in a sequence. Thus we have (375) but not (376).

(375) my old book
(376) old my book

This is an automatic consequence of the suggested transformational analysis, but it would have required a special statement in the grammatical statement of \(467.2\). The essential simplification would appear from the consideration of selectional restrictions, had we gone into sufficient detail in \(467.2\). Thus we have "John's toothache", "John has a toothache", but not "victory's toothache", "victory has a toothache", etc.. Even though the transformational analysis of (373) is necessitated by the non-recursion requirement of \(4106\), it is interesting and important to note that a careful and detailed argument in terms of simplicity would have led to the same conclusion.
Note that in forming (373) from (374) the article of \( NP_2 \) in (374) is dropped. We thus construct the transformation \( T_{S_1} \) such that

\[
(377) \ T_{S_1} \text{ is determined by } (Q_{S_1}, \delta_{S_1})
\]

where:
\[
Q_{S_1} = \{ (NP, VP_{A'}^\text{have}, T, X) \} \quad (X \text{ any string})
\]

\[
\delta_{S_1} \maps (U, S_1, \sigma, U, U, U, U, U)
\]

\( T_{S_1} \) thus carries \( NP - VP_{A'}^\text{have} - T - X \) into \( NP^\text{S_1} - U - U - X = NP^\text{S_1} - X \); e.g., it carries "John has an old car" (=John\-'s\-have\(\text{-an} - \old\text{-car}_1\)) into "John\-'s\-old\text{-car}_1".

The phrase (373) must now be substituted by a generalized transformation for some segment of a sentence. In general a phrase of the form (373) can appear in a sentence in a given position only if the \( NP_2 \) of this phrase can also appear in this position. Hence we have to do here with a family of transformations \( F_{pos} \) such that

\[
(378) \ F_{pos} \text{ is determined by } (C_{pos}, t_{pos})
\]

where
\[
C_{pos}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) \text{ if and only if }
\]

\[
(i) \ \alpha_1 = NP^\text{S_1} \upharpoonright X_i \quad (X_i \text{ a string in } F)
\]

\[
(ii) \ \alpha_2 = \#
\]

\[
(iii) \ \alpha_3 = X_i
\]

\[
(iv) \ \alpha_4 = T_i X_i
\]

\[
(v) \ \alpha_5 = X_i
\]

\[
t_{pos} \maps (\alpha, \sigma, U, U, U, U, U, U, U, U, U) \quad (\text{i.e., } t_{pos} \text{ is a substitution})
\]

Thus a transformation \( T_{pos} \) from the family \( F_{pos} \)
carries a string of the form (379) into the corresponding string of the form (380).
For example, $T_{pos}$ will carry (381) into (382).

(381) John's old car -#- I saw - the old car - in the lot
(382) I saw - John's old car - in the lot

We thus construct $T$-markers of the form

(383) $z_1^X_1^S_1^T_1^N^F_1^P$ - $z_2^X_2^S_2^T_2^N^F_2^P$

Where $z_1$="John has an old car" and $z_2$="I saw the old car in the lot", the string derived by (383) will be (382), and "John's old car" will be an NP in the transform.

The similarity between (378) and (168) can be exploited in stating them together. Thus, e.g., $t_{pos}$ of (378) is exactly $t_A$ of (168). Certain conditions will have to be added to (383) to ensure correct and exhaustive generation, and as we noted above, other sources for the NP construction must be considered, but we will carry the analysis no further at this point.

108.3. This analysis leaves out one 'adjectival' context of $NP^S_1$ which was in fact included in the grammatical sketch of 167.2, namely, in such sentences as

(384) $NP$ - be - $NP^S_1$ (e.g., "it is John's")

But further investigation shows that this is an instance of a pattern with a much larger distribution not shared by

adjective phrases. Thus we have

(385) (a) John's is nicer than mine
    (b) I took John's
    (c) I bought it at John's, etc.
The only reasonable way to handle such instances as (384–5) is by an 'elliptical' transformation \( T_{S_1d} \) having much the same relation to \( T_{S_1} \) that \( T_{pd} \) has to \( T_p \) (cf. §99.2). We define the transformation \( T_{S_1d} \) such that

\[
(386) \quad T_{S_1d} \text{ is determined by } (Q_{S_1d}, \delta_{S_1d})
\]

where:

\[
Q_{S_1d} = \{(x_1^{NP}, S_1, N_0^S, x_1^d)\}
\]

\[
\delta_{S_1d} \leftarrow (U, U, U, U, U)
\]

We can now revise (383), replacing it by

\[
(387) \quad (383) \text{ with } "F_{pos} (T_{S_1d} \Phi P" \text{ replacing } "F_{pos} \Phi P".
\]

If \( Z_1 = "\text{John has a car}\)" and \( Z_2 = "\text{I saw the car in the lot}"\), then (387) will yield "I saw John's car in the lot" if \( T_{S_1d} \) does not appear in the \( T \)-marker, and it will yield "I saw John's in the lot" if \( T_{S_1d} \) does appear in the \( T \)-marker.

\( T_{S_1d} \) deletes the noun that follows the possessive phrase \( NP^{NP} S_1 \). Its underlying elementary transformation \( \delta_{S_1d} \) is precisely the underlying transformation \( \delta_{pd} \) of (210), §99.2. We will see below that \( T_{S_1d} \) (and similarly, \( T_{pd} \) of §99.2) is just one of a larger class of elliptical transformations based on this elementary transformation.

Actually we should have a family of transformations in place of \( T_{S_1d} \) in (387), and no doubt certain conditions should be given on these \( T \)-markers. The sentences (385) are generated in a simple way by this analysis, though they would require a special statement in §67.2. (384) did indeed appear in the grammatical sketch of §67.2 as a form of \( NP - be - AP \), but the
analysis at which we have arrived here seems intuitively much more adequate. It is intuitively evident that "it is John's" is not a sentence of the same form (it\textsuperscript{is}AP) as "it is old", but is rather elliptical in the same way as is "I took John's", etc., and this is just the analysis to which we have been led in transformational terms.

109.1. The analysis of the adjective phrase provides still another instance of a recursive statement. We did not consider this fact in §67.2, but statement 17 should contain a recursive indication that a sequence of adjectives of any length can precede a noun. This statement must be transformationally eliminated. There are also a variety of selectional restrictions on noun and modifying adjective which we did not consider in the sketch of §67.2. Thus we have (388) but not (389)

\[(388)\begin{align*} 
(a) & \text{ a talkative man} \\
(b) & \text{ a flagrant violation} \\
(c) & \text{ an abundant harvest} 
\end{align*}\]

\[(389)\begin{align*} 
(a) & \text{ a } \{\text{flagrant}\} \text{ man} \\
(b) & \text{ a } \{\text{talkative}\} \text{ violation} \\
(c) & \text{ a } \{\text{flagrant}\} \text{ harvest} 
\end{align*}\]

Actually, the distinction between singular and plural is sufficient to establish the fact of selectional relation between noun and modifying adjective, since certain adjectives (e.g., "numerous", "mutually exclusive") occur only with plurals or collectives. Both the fact of selectional relation, and the fact that an indefinitely long sequence of adjectives can modify a noun indicate the need for a transformational elimination of the adjective – noun construction.
For any phrase article-adjective-noun \((T^A N)\) there is a sentence

\[(390) T^N \text{-is-} A\]

Thus phrases \(T^A N\) as in (388) can be dropped from the kernel in favor of sentences of the form (390). Then the selectional relation between noun and modifying adjective need only be stated once, for the construction (390), and we can drop from the grammar the now-inadmissible recursive statement that allows an indefinitely long sequence of adjectives modifying a noun.

We note at once that this transformational analysis is necessarily unidirectional. That is, we must derive

\[(391) \ldots T^A \ldots N \ldots\]

from (390) as the kernel form, not vice versa. If (390) is chosen as the kernel form, then sequences \(T^A A^A N, T^A A^A A^A N, \ldots\) can be generated by repetition of the transformation that gives (391) from (390), thus eliminating a recursion in the kernel grammar. But if (390) is derived from kernel sentences of the form (391), it will be necessary to eliminate this recursion in some other way. Furthermore, the transformational derivation of (391) from (390) follows the familiar pattern of a generalized transformation which substitutes a phrase derived from a sentence for some segment of a second sentence, while the analysis of (390) as a transform of (391) would require a transformation of a new kind. Similarity of distinct analyses leads to a higher valued grammar, since we have defined simplicity in terms of the possibilities of coalescence.
of distinct statements.

A phrase $T^\wedge A^N$ as in (391) is produced by a transformation $T_{adj}$ such that

(392) $T_{adj}$ is determined by $(Q_{adj}, t_{adj})$

where: $Q_{adj} = \{ (T, N[S], VP_A^\wedge be, AP) \}$

$t_{adj} \leftrightarrow (0, 0, 4, 0, \overline{4}, 0, \overline{4}, 0)$ (thus $t_{adj}$ is a substitution)

$T_{adj}$ thus carries (393) into (394).

(393) $T - N[S] - VP_A^\wedge be - AP$ (e.g., "the-boy-is-tall")

(394) $T - AP^\wedge N[S]$ (e.g., "the-tall boy")

There is one difficulty here. We want the term $AP^\wedge N[S]$ of the proper analysis of the transform to have the same constituent structure as its root $N[S]$, so that it will be possible to reapply the transformation, giving strings of adjectives. This would be an instance like case (iii), 483.5. But as we formulated Def.32, 483.5, this will be the case only if the root is represented by a prime (cf. (vii), Def.32, 483.5). But in the kernel grammar of 467.2 we had no prime representing $N[S]$. It is necessary, therefore, either to relax Def.32 or to introduce into the kernel grammar an intermediate prime representing $N[S]$, thus a prime Noun representing, e.g., "boy", "boys", or a prime Noun representing "boy" and a prime Noun representing "boys". For a variety of reasons, the latter alternative seems the best. We thus replace statement 7, 467.2, by
This revision actually permits a slight simplification of (392) and a somewhat more satisfactory statement of several other transformations, but we will not trouble to make the necessary alterations. In any event, we see that in the transform (394), $AP^N\emptyset$ is a $Noun_g$ and $AP^N'S$ is a $Noun_p$.

Reformulating $Q_{adj}$ of (392) as

$$Q_{adj} = \{(T, \left\{ \begin{array}{c} Noun_g \\ Noun_p \end{array} \right\}, VP_A^\wedge be, AP)\}$$

we see that $T_{adj}$ carries (397) into (398), since $AP_1^N\emptyset$ is a $Noun_g$.

(397) $T - AP_1^N\emptyset - VP_A^\wedge be - AP_2$ "the young boy is tall"

(398) $T - AP_2^\wedge AP_1^N\emptyset$ "the tall young boy"

Sequence of adjective restrictions can be given by restricting the order in which such sentences are transformationally derived, i.e., by conditions on $T$-markers in the grammatical statement corresponding to the level $T$.

We must now construct a generalized transformation which will substitute such phrases as (394), (398) for some segment of a second string, in the familiar manner. The simplest approach seems to be to construct the $T$-markers

$$Z_1^\wedge K_1^\wedge T_{adj} - Z_2^\wedge K_2^\wedge I - FA_w^\wedge \bar{P}^P$$

where
(400) \( F_{\text{ADJ}} \) is determined by \((C_{\text{ADJ}}, t_{\text{ADJ}})\)

where \(C_{\text{ADJ}}(\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5)\) if and only if

\[
\begin{align*}
\alpha^1 &= T^\text{AP}X_i \\
\alpha^2 &= \# \\
\alpha^3 &= X_i \\
\alpha^4 &= T^\text{X}X_i \\
\alpha^5 &= X_k
\end{align*}
\]

\(t_{\text{ADJ}} = t_{\text{pos}}(\text{of (378)}) = t_A(\text{of (168)})\)

Suppose that we have a \(T\)-marker of the form (399) with \(Z_1 = \text{"the boy is tall"},\)

\(Z_2 = \text{"I noticed the boy"}. \) Then application of \(T_{\text{adj}}\) to \(Z_1\) gives "the tall boy", and application of a transformation \(T_{\text{ADJ}}\) from the family \(F_{\text{ADJ}}\) to the complex string

(401) the tall boy - \# - I noticed - the boy - U

gives "I noticed the tall boy".

It is necessary to place certain restrictive conditions on \(T\)-markers of the form (399) as matters now stand. For instance, \(T_{\text{adj}}\) cannot apply to sentences with pronominal subjects, giving, e.g., "the tall he" from "he is tall". But this is due to a deficiency in our analysis of pronouns. Actually there is good reason to exclude pronouns from the kernel and to introduce them transformationally as substituends for noun phrases. Thus pronouns, if treated as nouns, not noun phrases, would require special statements to the effect that they occur without article (or with \(\emptyset\) article) and with no modifiers (\(\text{AP}'s\) or \(\text{PP}'s\)). (40) If pronouns are treated in this way, then no
special statement is needed to exclude them from this
of adjectives.
transformational analysis. Transformational introduction of
pronouns will also make it possible to simplify certain of
the transformational analyses given above. E.g., Condition \( K_A \)
of (139), \( \mathcal{L} \) can be dropped if the \( T \)-marker (137) is
reformulated so that the introduction of the pronoun must
precede the application of \( F_A \). This is an important subject
that should be further investigated.

With this we drop \( AP \) from the kernel grammar except in
the predicate position. This leads to an incidental simplification
in the kernel grammar, since in several statements of \( \mathcal{G} \)
(e.g., statements \( 9^\ast, 14 \)), it was necessary to refer to \( AP \)
as a part of a restricting context even though this element
was not really relevant. The major gains, however, are the
elimination of a recursive statement, and avoidance of the
necessity of duplicating the statement of selectional relation
between noun and modifying adjective, as noted at the outset
of this section.

109.2. It is important to determine the range of applicability
of the transformation \( T_{adj} \) that forms adjective-noun constructions
from sentences of the form noun-is-adjective. In statement 17,
\( \mathcal{G} \), we analyzed the adjective phrase \( AP \) into six kinds of
elements.

\[
(402)(a) \quad \text{NP}^\prime \text{S}_1 \quad ("John's")
\]
\[
(b) \quad \langle D \rangle \quad \text{A} \quad ("<\text{very}> \text{ old}"
\]
\[
(c) \quad \langle D \rangle \quad \text{ing} V \_1 \quad ("<\text{very}> \text{ interesting}
\]
\[
(d) \quad \langle D \rangle \quad \text{en} V \_k \quad ("<\text{very}> \text{ tired}
\]
\[
(e) \quad \langle D \_2 \rangle \quad \text{ing} V \_1 \quad ("<\text{loudly}> \text{ barking}
\]
\[
(f) \quad \langle D \_2 \rangle \quad \text{en} V \_t \quad ("<\text{completely}> \text{ forgiven}
\]
Form (a) has been dropped from the kernel above in §108, and (f) has been eliminated as a passive in §94. We discussed the derived constituent structure of passives in §83.2, noting in particular that $\text{en}^\wedge \text{v}_\text{T}$ is an adjective even if deleted from the kernel, by virtue of its resemblance in form to $\text{en}^\wedge \text{v}_\text{A}$ which remains in the kernel as an adjective. Hence $\text{T}_{\text{adj}}$ as we have defined it will apply to transforms of the form

$$(403) \quad \text{T - Noun - V}_\text{P}^\wedge \text{be} - \text{en}^\wedge \text{v}_\text{T} \quad \text{("the-man-was-murdered")}$$

which are derived from kernel sentences by $\text{T}_\text{p}$, $\text{T}_\text{pd}$ (cf. §94, 92.2). And in fact we do have such forms as

$$(404) \quad \text{the murdered man}$$

We know that this is an instance of (402f), not (402d) (and hence derived by $\text{T}_{\text{adj}}$ from (403), ultimately, from "...murdered the man"), since we do not have "the very murdered man" (just as we do not have "the man was very murdered").

The crucial factor in the determination that passives are $\text{AP}$'s was the term by term resemblance of $\text{en}^\wedge \text{v}_\text{A}$ to the kernel adjective $\text{en}^\wedge \text{v}_\text{A}$. But this consideration (turning on absolute category co-membership) does not hold for such $\text{v}_\text{T}$'s as "consider a fool", etc. There is no $\text{AP}$ of the form $\text{en}^\wedge \text{v}_\text{T}^\wedge \text{Noun}$, differing from "considered a fool" (as in "he was considered a fool") word by word in members of the same absolute category and belonging to the kernel. That is, it will not automatically be the case in the kernel grammar that $\zeta(\text{AP, en}^\wedge \text{consider}^\wedge \text{a'fool})$ as it is automatically the case that $\zeta(\text{AP, en}^\wedge \text{murder})$, as we noted in §83.2. Hence Def. 30, §83.2
will not apply in this case, assigning "considered a fool" to AP.

Hence $T_{\text{adj}}$ as we have formulated it above, will not apply to such strings as

(405) $T_{\text{Noun}} - VP_{A}^{\text{be}} - en^{\text{V}_{A}^{\text{NP}}}$ ("John was considered a fool")

which, just like (403), are derived by $T_{p}$, $T_{pd}$. And in fact we do not have such noun phrases as

(406) the considered a fool person

We note the following difficulty, however. In §83.2 we considered a more general condition, namely, Def. 31, under which constituent structure is assigned. And Def. 31 does in fact apply to "considered a fool", assigning it to AP. We conclude, then, that Def. 31 must be weakened in some respect.

The discussion of "consider a fool" can be carried over for all other instances of the construction verb-complement that were deleted from the kernel in §105. It will be recalled that the only instances of this construction not deleted from the kernel are those of the form $V_{sep}^{\text{Particle}}$ (cf. §96, and the final paragraph of §105). All other cases of verb-complement fail to qualify as AP's, and thus do not appear as noun modifiers under the transformation $T_{\text{adj}}$.

With respect to the construction $V_{sep}^{\text{Particle}}$, however, the situation is less clear. (402d) does appear to have certain instances of this form. If such sentences as

(407) he was very tired out

are admitted as grammatical, then $en^{V_{sep}}^{\text{Particle}}$ will be represented in (402d) as $en^{V_{k}}$ (hence as AP), and $en^{V_{sep}}^{\text{Particle}}$
will qualify as a subconstruction of AP, hence subject to T\textit{adj} if this transformation stands unaltered as in (392), (396). In this case, noun phrases of the form

\[(408) \text{ } T - \textit{en} \text{ } \text{\textsuperscript{\textit{v sep}}} \text{\textsuperscript{\textit{particle}}} - \text{Noun} \]

will be generated. Certain cases of (408) do seem to occur fairly freely. Thus we have

\[(409)(a) \text{ a broken down house} \\
(b) \text{ a carefully carried out plan} \\
\text{etc.} \]

On the other hand, such phrases as

\[(410)(a) \text{ the called up people} \\
(b) \text{ a carried out plan} \]

are hardly acceptable. A much more careful analysis of these phrases appears necessary before we can determine the correct analysis.

Consider now case (402e). It is necessary to include \(\langle \textit{D}_2 \rangle \textit{ing}^{\textit{\textit{v}_I}}\) as a form of AP because of such phrases as "a barking dog", "a sleeping child", etc. Now that cases of \(\text{\textit{T}}^{\textit{\textit{AP}}}\text{\textit{Noun}}\) are derived by \(\text{T}_{\text{adj}}\) from \(\text{T-Noun-is-AP}\), these phrases will be derived from

\[(411)(a) \text{ a dog is barking} \\
(b) \text{ a child is sleeping} \]

But the sentences of the form (411) exist even if case (402e) is not included as a form of AP in the predicate position. (411a-b) are sentences of the form \(\text{NP-VP}_A-\text{\textit{v}_I}\), with \(\text{VP}_A\) taking the form \(\text{\textit{S be}^\textit{\textit{ing}}}\). Thus we can drop (402e) from the analysis of AP by revising \(\text{T}_{\text{adj}}\) so that it applies to (411), thus by replacing (396) by
(412) $Q_{adj} = \{ (T, \left\{ \frac{\text{Noun}}{\text{Noun}} \right\}, V^A \text{be}, \left[ \frac{\text{AP}}{\text{ing}} \langle \text{be} \rangle V_I \right] ) \}_2$

But in fact even this elaboration is not necessary if we make an assumption about the absolute analysis (cf. §32 and the analysis of passives in §83.2), which appears to be a reasonable one. Just as we exploited the resemblance between (402f) and (402d) to show that passives $\text{en}^T V_T$ are AP's, even though deleted from the kernel, we can exploit the resemblance between (402e) and (402c) to show that $\text{ing}^T V_I$ is an AP, even if not given explicitly in the analysis of AP.

Suppose that we revise the transformational analysis of T-AP-Noun constructions, replacing the T-marker (399) by

(413) $Z_1^T F_1^P - T_{adj} - Z_2^T F_2^P - F_{ADJ}$

The essential change is that both $T_{adj}$ and $F_{ADJ}$ now apply after, not before $F^P$, and in particular, after the component $F_1^P$ of $F^P$ that carries $\text{ing}^T V$ into $V'\text{ing}$. Note that (413) is no more complex than (399), and that the replacement of (399) by (413) leaves our earlier analyses intact. Hence (413) might just as well have been taken in the first place as the T-marker schema for this transformational analysis.

In §83.5, (iii), we briefly discussed $F_1^P$, noting that it carries $\text{ing}^T V$ into $U-V'\text{ing}$, with $V'\text{ing}$ being a term of the proper analysis of the transform. $F_1^P$ applied to a sentence containing (402c) as predicate gives

(414) $\text{NP} - V^A \text{be} - V_I' \text{ing}$

and applied to $\text{NP} - S' \text{be} \text{ing} - V_I$ (e.g., to (411)), it gives
(415) \( NP-S^\text{be} - V_1^\text{ing} \)

with the proper analysis in each case as given by the dashes. We note that \( V_1^\text{ing} \) in (414) is an AP, since its root \( \text{ing}\overline{V}_1 \) is an AP; hence \( \xi(\text{AP, } V_1^\text{ing}) \). (41) It is in fact the case that \( V_1 \) is a subclass of transitive verbs. If however, it is the case that

(416) transitive and intransitive verbs are not distinguished on the absolute level

then Axiom 10, \( \xi 56 \) guarantees that \( \xi(\text{AP, } V_1^\text{ing}) \) as well. Hence from Def. 30, \( \xi 83.2 \), it follows that \( V_1^\text{ing} \) in (415) is an AP.

This argument is perfectly parallel to the argument of \( \xi 83.2 \) concerning the adjectival status of the passive. The latter, however, made a much weaker assumption about absolute categories than (416), since in the case of the passive, it was only necessary to assume that \( V_1 \) is not distinguished on the absolute level from other transitive verbs. But even the assumption (416) seems to be a reasonable one.

Suppose now that we replace (396) not by (412) but by

(417) \( Q_{\text{adj}} = \{(T, \left\{ \frac{\text{Noun}^g}{\text{Noun}^p} \right\}, \text{C^\text{be}, AP})\} \)

where \( C \), it will be recalled, is the element of the auxiliary verb that becomes either the singular or plural element of the verb phrase. (417) differs from (396) in one important respect, namely, in that the status of \text{be} is ambiguous in (417) — it may be either the main verb or part of the auxiliary. But (417) is no more complex than (396), and in fact, it could
just as well have been given above in place of (396). But now T_{adj} with the restricting class (417), applies to (411) in just the same way as to "the book is interesting."

If we assume (416), we replace (396) by (417) and we replace (399) by (413). Neither of these changes adds to the complexity of the grammar; hence either change can be made independently of this discussion. If we reject (416), we replace (396) by the somewhat more complex (412). Either way, then, it is unnecessary to include (402e) as a special form of the adjective phrase. The appearance of \textit{ing}^{\text{\textit{V}}_{1}} in the predicate position is guaranteed anyway, because of such forms as (411) (from NP-VP_{A}-V_{I}), and one way or another, all occurrences of "barking", "sleeping", etc. are accounted for exactly as in the case of the kernel adjectives "old", "interesting", etc. The fact that only \textit{D}_{2} (i.e., not "very", "rather", etc.) can appear with "barking", "sleeping" (i.e., we do not have "the very sleeping child", though we have "the very interesting book") is accounted for by the fact that these adjectives are derived from the verbal sentences NP-VP_{A}-V_{I}, where (cf. statement 2, \textsection 67.2) only \textit{D}_{2} can occur. Thus we do not have "the child - is ing - very sleep" \rightarrow "the child is very sleeping". We have already seen that the limitation on the occurrence of adverbs in the case of the passives "murdered", "accused", etc. can be explained in exactly the same way.

The fact that adjectives of the form <D_{2}> \textit{ing}^{\text{\textit{V}}_{1}} are dropped from the analysis of the adjective phrase has an important consequence for the intuitive adequacy of the resulting grammar. In \textsection 72.2, in reviewing the inadequacies of the grammar which did not go beyond phrase structure, we noted
that such sentences as (411) turn out to be cases of constructional homonymy. "the dog is barking" is an instance of the pattern $\text{NP-VP}_A-V_I$ (like "the dog barks"). But it is also an instance of $\text{NP-is-AP}$ (like "the dog is mangy"), since $\text{NP-is-AP}$ is an existing sentence form, and "barking" is in fact an AP because of sentences like "the barking dog...", etc. But there is no corresponding structural ambiguity, as there is in other cases of constructional homonymy -- the analysis as an instance of $\text{NP-is-AP}$ has no intuitive support. This intuitive inadequacy suggested some flaw in the conception of linguistic structure that led to the dual analysis. But we see that under transformational analysis, there is no constructional homonymy in these cases. "barking" is no longer an instance of AP in the kernel. Hence "the dog is and only one P-marker barking" has only one analysis -- as $\text{NP-VP}_A-V_I$.

The fact that $<b_2>\text{ing}V_I$ occurs as an adjective phrase in other contexts is accounted for by the formal similarity between the occurrence of this phrase as a verb, and the occurrence of adjectives of the form (402c) in the predicate position, $T_{adj}$ applying to both of these cases. With this we see, incidentally, that all of the difficulties noted in $\eta_2$ have disappeared under transformational analysis.

This leaves only (402b-c-d) as instances of AP in the grammar of phrase structure for the kernel. This seems to be a satisfactory result. It is clearly the case that intuitively, "the sleeping child" and "the murdered man" have more 'verbal' force, in some sense, than, respectively, "the interesting book" and "the tired man". There is no explanation for this on lower levels, since these pairs do not differ in any significant way in morphological or constituent
structure. But there is a transformational explanation, as we have just seen.

109.2. Adjective phrases also occur as predicates of 'stative verbs' such as "seem", "become", "act", "look", etc. (cf. fn. 24). Thus one of the sentence forms of the kernel will be

\[(418) \text{NP} - \text{V\text{stat}} - \text{AP}\]

In the kernel, only cases (402b, c, d) remain as forms of the adjective phrase, and all of these can appear in (418). Thus we have

\[(419)\]

(a) he seems old
(b) this seems interesting
(c) he seems tired

(402a, e, f) are now introduced transformationally. But the transformational analysis of these adjectival elements as we have developed it in §108-9, above, does not provide for the introduction of these elements in the context (418). This analysis introduces these adjectival elements only as noun modifiers. It is therefore quite interesting to note that these elements (402a, e, f) in fact do not occur with stative verbs in the context (418). We cannot have

\[(420)\]

(a) it seems John's
(b) it seems barking
(c) he seems forgiven

On the level of phrase structure, the exclusion of cases (402a, e, f) would have to be given as a special restriction on the selection of AP in (418), but we see that the correct distributional statement follows automatically from the simplest transformational characterization constructed for the other instances of AP. Again we have an instance where
transformational analysis shows an apparently capricious exception to be the result of underlying regularity and structural simplicity.

This discussion does not exhaust the distribution of adjectives. As is the case throughout these applications of transformational analysis, this is only an introductory survey. 110.1. Wherever we have a transformation based on a deformation

\[(421) \delta \longleftrightarrow (U, U, U, U, U, U)\]

or the like, the result of this transformation will be an 'elliptical' sentence. If the restricting class is

\[(422) Q = (X_1, X_2, X_3)\]

the element \(X_2\) is dropped by the transformation, and a sentence \(Z\) of the form \(X_1 \uparrow X_2 \uparrow X_3\) is carried into the corresponding sentence \(Z'\) of the form \(X_1 \uparrow X_3\). In such a case we can expect to find that there is a feeling that \(X_2\) is 'understood' in \(Z'\). We have come across two instances of such elliptical transformations. In \(\S 24.5\) and \(\S 99.2\) we constructed \(T_{pd}\) which carries (422) into (423).

\[(422) NP^{VP_A \uparrow be \uparrow en \uparrow V} - by^NP - \langle PP \alpha \rangle U\] ("the picture was painted by the artist-by a new technique")

\[(423) NP^{VP_A \uparrow be \uparrow en \uparrow V} - \langle PP \alpha \rangle U\] ("the picture was painted-by a new technique")

The second instance is the transformation \(T_{S_1 d}\) constructed in \(\S 108.3\), carrying (424) into (425).

\[(424) \ldots NP^3_{S_1} - \text{Noun} - \ldots\] ("I liked John's - book - better")

\[(425) \ldots NP^3_{S_1} - \ldots\] ("I liked John's - better")
Dashes in both cases indicate the terms $\theta_1$, $\theta_2$, $\theta_3$ of the proper analysis.

There are other cases where considerations of simplicity compel us to set up an elliptical transformation. Consider for instance such sentences as

(426) (a) John will
(b) John is
(c) John has
(d) John has been
(e) John does
etc.,

the answers to "who will go", "who is in the other room", etc.

Intuitively, these appear to be 'truncated' sentences, with a verb phrase 'understood'. But we will see directly that formal analysis leads to the same conclusion, thus providing grounds in formal linguistic structure for this intuition.

Consider the possibility of including (426) in the kernel. We note first that the final elements "will", "be", "have", "do" in (426) cannot be regarded as main verbs which happen to be intransitive. This solution is ruled out by such considerations as the following:

1. If "will" (or any other member of $M$, e.g., "must", "can") is regarded as a main verb, it is necessary to add a special statement to the effect that auxiliary verbs cannot occur freely with $M$ as they can with all other main verbs. E.g., we do not have "John is musting".

2. As main verbs, "be", "have", and "do" can occur with the auxiliary "is ing", as in "John is being nice about it", "John is having breakfast", "John is doing his homework". But
we do not have "John is being", "John is having", "John is doing" as forms of (426). Hence special restrictions must be stated for the 'intransitive' use of "be", "have", and "do".

3. As main verbs, "have" and "do" occur with the negative element preceding, as in "he doesn't have a chance", "he doesn't do his work". But in (426) we have only "John hasn't", "John doesn't", as is normal for auxiliaries.

4. "do" normally takes stress in sentence final position as a main verb (e.g., "what sort of work does John do"). But in (426e) it is unstressed.

Thus if these elements are regarded as main intransitive verbs, rather than as auxiliaries, a variety of special statements will be necessary. In other words, the distributional features of "will", "is", etc., in (426) are precisely those of auxiliaries, and these elements must therefore be regarded as such.

Recognizing that the verbs in (426) are auxiliaries, the only way to retain these sentences in the kernel is to give the analysis of the Sentence not as Sentence → NP'VP₂<VP₁ (as in statements 1, 2, §67.2), but as

(427) Sentence → NP'VP₂<VP₁>

so that VP₁ may or may not occur, (426) resulting when it does not occur. This would indeed be a possible analysis, and in fact the simplest analysis, were it not for the fact that, as we have seen, the elements en, ing are best associated with the auxiliary verb, not the main verb. Thus (427) gives not (426), but

(428) (a) John will
(b) John is ing
(c) John has en, etc.
when \( VP_1 \) does not occur. This solution is ruled out, then, because the analysis of auxiliary verbs would have to be reformulated with consequent complexities on both the level \( P \) and \( T \).

It is therefore necessary to delete (426) from the kernel, and provide a transformational analysis with an elliptical transformation. Suppose we set up \( T_\xi \) such that

\[
(429) \quad T_\xi \text{ is determined by } (Q_\xi, \delta_\xi)
\]

where \( Q_\xi = (NP, VP_A, VP_1) \)

\( \delta_\xi \) is as in (421)

We require that \( T_\xi \) apply after \( \Psi_5^P \), the mapping that carries \( \text{ing}^*V \) into \( V^\text{ing} \), etc. We thus construct the \( T \)-markers

\[
(430) \quad \Psi_5^T = T_\xi - \Psi_5^T
\]

\( \Psi_5 \) carries "John - S will - come" into "John-will - come", "John - is ing - come" into "John - is - come ing", "John - has en - come" into "John - has - come en", etc. (cf. 4467-2, 43.2(iii)), where "will", "is", "has", etc. remain as the \( VP_A \), and "come", "come ing", "come en" (which become "come", "coming", "come," respectively, by the morphological rules \( \Phi^M \)) constitute the element \( VP_1 \). \( T_\xi \) then carries these strings into (426a-d).

With (426e), however, we face a certain problem. \( \Psi_5 \) carries "John - S come" into "John - come S", and \( T_\xi \) will carry this into "John". Thus \( T_\xi \) must precede the mapping which carries \( S^\text{verb} \) into \( \text{verb}^S \), \( S^\text{verb} \) into \( \text{verb}^S \), and \( \text{ed}^\text{verb} \) into \( \text{verb}^\text{ed} \). But this is again the mapping \( \Phi_1^P \). The simplest solution is to split \( \Phi_1^P \) into two mappings \( \Phi_1^P_{15} \) and \( \Phi_1^P_{16} \), where
\(\Phi^P_{15}\) applies to the affixes en and ink, and \(\Phi^P_{16}\) applies in exactly the same way to \(\mathcal{S}, \text{ed}\). (The component of \(\Phi^P\) which introduces "do" as the bearer of a displaced affix is now renamed "\(\Phi^P_{17}\)", since it must follow the new \(\Phi^P_{16}\).) Now, retaining (430), cases (426a-d) are still handled correctly, but \(\Phi^P_{15}\) leaves "John S come" unchanged. \(\Phi^P_{16}\) then carries "John S come" into "John S", and \(\Phi^P_{17}\), which introduces "do" as the bearer of a displaced affix, carries "John S" into "John do S", ultimately, (426e).

The negatives "John won't", "John isn't", etc., are automatically produced in the correct way with no further changes, since, as we noted in 322.1, if \(\ldots\) is a \(V^P_A\), then \(\ldots \text{not} \ldots\) is a \(V^P_A\) as well.

We thus have formal grounds for construing the sentences of (426) as elliptical constructions with the verb phrase 'understood'. Note that it is not always the case when we have sentences \(Z=X_1^\wedge X_2^\wedge X_3\) and \(Z'=X_1^\wedge X_3\), that \(Z'\) is to be derived by an elliptical transformation from \(Z\). For instance, we have seen that \(X^\wedge \text{not}^\wedge Y\) is best derived by a transformation adding \text{not} to \(X^\wedge Y\), not vice versa, and in the case of sentence pairs like "he immediately accepted", "he accepted", the best analysis is evidently to include both in the kernel, giving the analysis of this construction as \(\mathcal{NP}(D)>V^P\). In these three kinds of instance, then, formal analysis leads to different descriptions. "John will" is derived from "John will Verb Phrase" by an elliptical transformation; "John will not try" is derived from "John will try" by a deformation adding \text{not}; and "John accepted" and "John immediately accepted" are both
(Similarly, "they come" and "they will come" are both kernel sentences.) These are the analyses that result from the attempt to construct the simplest possible grammar. But it is quite clear that intuitively, "John will try", "John accepted", "they come", etc., are not derived elliptically from "John will not try", "John immediately accepted", "they will come", respectively, in the same sense in which "John will" is derived from "John - will - Verb Phrase". This can be recorded, then, as another instance of an intuitive distinction with formal grounds.

The suggested analysis (427) which, if accepted, would have excluded (426) from the class of elliptical constructions, was ruled out because of the fact that the simplest analysis of the auxiliary verb associates ing, en with VA with VP1 rather than VP1. This analysis of VA (given originally in §61.2) in itself had little intuitive correspondence to recommend it, though it did enable us to give a very simple description of a fairly complex construction. But we see that this analysis does lead to further analyses that are marked by strong intuitive correspondence. In fact, almost every transformation established hitherto depends to some extent on this analysis of VA. It is too much to expect a point by point correspondence between linguistic intuition and the results of applying a given linguistic theory, but just as this theory is supported when in application it leads to intuitively satisfactory analyses, it is also supported when an analysis which itself fails to have intuitive correspondence serves as the cornerstone for other analyses that do have such correspondence. This is the case with the analysis of VA which was systematically motivated by considerations of simplicity.
110.2. Imperatives are still another form of elliptical sentences. In imperatives, the noun phrase subject and the auxiliary verb are dropped, leaving only \( VP_1 \). Thus we have such imperatives as

\[
(431) \text{give me the record}
\]

But only a restricted set of \( VP_1 \)'s can occur as imperatives. Investigation of the various possibilities serves to determine the restricting class for the transformation \( T_{\text{imp}} \). Since we have "give me the record to\( ^\text{morrow} \)" but not "give me the record yesterday", we see that the choice of \( VP_A \) in the transformed string is not free. For instance, \( VP_A \) can be "will" (since we have "you will give me the record tomorrow") but not "you will give me the record yesterday") but it cannot be "have en" (since we have "you have given me the record yesterday", but not "you have given me the record tomorrow"). It is simplest to restrict \( VP_A \) to the element \( M \) containing "will", "can", "must", etc. \( (44) \)

Since we have "look at yourself", but not "look at myself", etc., we see that the noun phrase subject of the string which is carried into an imperative must be \( \text{you} \), and we see that \( T_{\text{imp}} \) must apply after the mapping \( F_{10}^P \) which carries \( X \) into \( X\)-self (where \( X \) is a pronoun) in the context \( X\)-verb \( \) (cf. \( \chi 107.5 \)). We might therefore construct the \( T \)-markers

\[
(432) \quad Z^X K^P F_{10}^P - T_{\text{imp}} - Y_0^i
\]

where \( T_{\text{imp}} \) is the transformation such that

\[
(433) \quad T_{\text{imp}} \text{ is determined by } (Q_{\text{imp}}, \delta_{\text{imp}})
\]

where \( Q_{\text{imp}} = (you^M, VP_1) \)

\[
\delta_{\text{imp}} \leftrightarrow (U, U, U, U)
\]
Timp thus carries "you - M - VP₁" into "VP₁"; as it carries "you - will - give me the record tomorrow" into "give me the record tomorrow".

It is still necessary to account for such imperatives as

(434)(a) do come to visit us
(b) don't come to visit us

We note that do in (434a) always has heavy stress. In §23 we set up an element Ac of the same class as not which has the effect of assigning heavy stress to the vowel of the preceding morpheme. (434a-b) could thus be derived from

(435)(a) do Ac-you-come to visit us
(b) do n't - you - come to visit us

by deleting you. But in §23 we saw that (435a-b) in turn are derived from "you - Ø - come to visit us" by application of either the Ac-transformation or the not-transformation, followed by the question transformation Ṭₚ (giving "Ø \( \begin{bmatrix} \text{Ac} \\ \text{not} \end{bmatrix} \) - you - come to visit us"), with the element do introduced by \( \Phi _{17}^{P} \) as a bearer of the displaced affix Ø. We can thus reformulate Ṭₚ to apply after the question transformation, so that it can apply to (435). We will see below that this alteration is required for independent reasons. We thus replace (433) by

(436) Ṭₚ is determined by \((Q_{\text{imp}}' ) , \delta_{\text{imp}}'\)

where \(Q_{\text{imp}}' = \{ (U, Ø \{ \text{Ac} \}) , (M, U) \}

\( \delta_{\text{imp}}' \rightarrow (\tau, U, U, U, U, U, U) \)

Timp thus carries \( U - Ø \{ \text{Ac} \}_{\text{imp}} \) - you - VP₁ into \( Ø \{ \text{Ac} \}_{\text{imp}} \) - VP₁, to which application of \( \Phi _{17}^{P} \) gives (434); and it carries \( M-U\text{-you}-VP₁ \) into \( VP₁ \), as before, giving (431), etc. We must revise the
T-markers (43a) so that $T_{\text{imp}}$ follows $\Phi^{P}_{12}$, since the question transformation must follow $\Phi^{P}_{12}$, and $T_{\text{imp}}$ now applies only to questions. We thus replace (432) by

$$Z^{*}K^{*}Y_{3} - T_{\text{imp}} - Y_{3}$$

Also to be covered by this analysis are such sentences as

(438a) you get it
(438b) you be the first volunteer
(438c) don't you get it (i.e., "let him get it")

with heavy stress in all cases on "you". This peculiarity suggests that these sentences be handled uniformly. These sentences are also formed by dropping the auxiliary, and (438c) at least is most simply formed from the question. Hence a uniform treatment is possible only if all are derived from questions. (Note that a further advantage in deriving imperatives from questions is that this treatment excludes them from the context.)

These sentences can be formed by a transformation $T_{\text{IMP}}$ such that

(439) $T_{\text{IMP}}$ is determined by ($Q_{\text{IMP}}$, $\delta_{\text{IMP}}$

where $Q_{\text{IMP}} = \{([U, \emptyset^{n(t)}], \text{you}, VP_{1})\}$

$$\delta_{\text{IMP}} \longmapsto (Q, U, U, U, Ac, U, U)$$

As a $T$-marker schema we have

(440) $Z^{*}K^{*}Y_{3} - T_{\text{IMP}} - Y_{3}$

$T_{\text{IMP}}$ carries $\emptyset^{n(t)}$ - you - VP1 (formed by a negative and a question transformation from the kernel sentence you - C - VP1) into $\emptyset^{n(t)}$ - you $^{Ac}$ - VP1, which, by $\Phi^{P}_{17}$ (introducing "do" as a bearer of the displaced $\emptyset$) and the morphological analysis of Ac (which adds heavy stress to the preceding morpheme -- cf. (96)), becomes (438c). It also carries $M$ - you - VP1 into you $^{Ac}$ - VP1,
which goes into (438a–b).

\( T_{\text{imp}} \) of (439) is almost identical in its definition with \( T_{\text{imp}} \) of (436), and the definitions of these transformations can thus be stated together. The resulting simplification gives a further reason for defining \( T_{\text{imp}} \) so that it applies after the question transformation. It appears then that imperatives are derived from questions by the elliptical transformation \( T_{\text{imp}} \) (with an alternative form \( T_{\text{IMP}} \) differing from \( T_{\text{imp}} \) in that it adds heavy stress to the subject instead of deleting it). Though this is a fairly simple analysis of this set of sentences, there are enough asymmetries to suggest that there is some better analysis.

111.1. In \( \mathbf{h69} \), chapter VII, we saw that there was no way to introduce the conjunction rule on the level of phrase structure without excessive complication, or a drastic revision of the form of grammars. Given the non-recursion requirement of \( \mathbf{h105} \), there is of course no hope at all of introducing a rule for conjunction into the kernel grammar. But it is clear from \( \mathbf{h86-7} \) that this rule can be formulated as a family of generalized transformations.

Fundamentally, the rule for conjunction asserts that any constituent \( C \) of a sentence \( Z \) can be replaced by \( C' \text{and} C' \), where \( C \) and \( C' \) are both \( \text{Pr}_\iota \)'s, for some prime \( \text{Pr}_\iota \). But this replacement may be carried out only if \( C' \) also appears in the context of \( C \) as \( \text{Pr}_\iota \). In other words, the conjunction rule will effect the transformation

\[
(441) \quad X^C Y \neq X, C' \text{and} C' \rightarrow X - C' \text{and} C' - Y
\]

where for some \( \text{Pr}_\iota \), \( C \) is a \( \text{Pr}_\iota \) of \( X^C Y \) and \( C' \) is a \( \text{Pr}_\iota \) of \( X^C Y \).
The elementary transformation underlying the family $F_{\text{AND}}$ will thus be a generalized transformation that carries $x_1 \cdot x_2 \cdot x_3 \cdot \# \cdot x_4 \cdot x_5 \cdot x_6$ into $u \cdot u \cdot u \cdot x_4 \cdot x_5 \cdot \# \cdot \text{and}$. $x_5 \cdot x_6$, and $F_{\text{AND}}$ will be defined as follows:

\[ (442) \quad F_{\text{AND}} \text{ is determined by } (C_{\text{AND}}, t_{\text{AND}}) \]

where $C_{\text{AND}}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)$ if and only if

\[ \begin{align*}
\alpha_1 &= \bar{x} = \alpha_5 \\
\alpha_2 &= \text{Pr}_1 = \alpha_6 \\
\alpha_3 &= \bar{y} = \alpha_7 
\end{align*} \]

where $x, y$ are strings, and $\text{Pr}_1$ is a prime

\[ t_{\text{AND}} = s_{\text{AND}}(\delta_{\text{AND}}) \]

where $\delta_{\text{AND}} \leftarrow (u, u, u, u, u, u, u, u, u, u, u, \text{and}, u, u, u)$

\[ s_{\text{AND}} \leftarrow (v, v, 0, v, 0, 0, v, 0, 0, 0, 0, 0, 2, 0, 0, 0) \]

$\delta_{\text{AND}}$ is a deformation adding and, and $s_{\text{AND}}$ is a generalized substitution (cf. Def. 26, §82.4) deleting the first four terms, and adjoining the second term to the sixth. It would be possible to extend the definition of substitutions (and to extend Th. 5, §82.4) so that $t_{\text{AND}}$ can be more simply defined as the substitution with the defining sequence

\[ (443) \quad (v, 0, v, 0, v, 0, v, 0, 0, 0, (2, \text{and}), 0, 0, 0) \]

where this has the obvious meaning.

Since from "John comes" and "John goes" we can form "John comes and goes" by conjunction, if follows that $F_{\text{AND}}$ must apply after $\Phi_4^P$ (cf. §110.1), which forms Verb's from
\(S^{\text{Verb}}\). We will see below (\text{III.3}) that \(F^{\text{AND}}\) must precede \(\Phi^{P}_{17}\), the component of \(\Phi^{P}\) that introduces "do". We therefore add to the set of \(T^{\text{e}}\)-markers all strings

\[(444) \ Z_{1}^{\wedge} K_{1}^{\wedge} \Psi_{6} - Z_{2}^{\wedge} K_{2}^{\wedge} \Psi'_{6} - F^{\text{AND}} - \Psi'_{6}\]

Where \(X-Y=U\) in (442), a transformation \(T^{\text{and}}\) of \(F^{\text{AND}}\) will carry a pair of sentences \(Z_{1}, Z_{2}\) into \(Z_{1}^{\wedge}\) and \(Z_{2}^{\wedge}\). In other cases, \(T^{\text{and}}\) will conjoin properly included constituents. There are a good many qualifications and reservations that must be added to (442), (444). For one thing, \(T^{\text{and}}\) cannot apply to certain constituents (e.g., the article \(T\) -- we cannot have "the and a boy", etc.). There are many other cases in which the internal analyses of \(S\) and \(S'\) (i.e., \(\alpha_{2}\) and \(\alpha_{5}\) of (442)) are relevant. Thus prepositional phrases cannot be freely conjoined when their initial prepositions are not identical (and if we have a high enough degree of grammaticalness, we will find that there are even certain restrictions when the initial prepositions are identical), and there are restrictions of tense (i.e., of choice of \(VP^{A}\)) when verb phrases are conjoined. Sometimes the transformational history of \(S\) and \(S'\) must be taken into account. For instance, we noted in \(\text{IV.7.1}\) that sentences of different types cannot be freely conjoined, and now we consider sentences to be of different types when they have distinct transformational histories. Similarly, we note that "recognized", "applauded", "sad" and "tired" are all adjectives, but that only the latter two are still in the kernel. Since we cannot have "very recognized", "very applauded", we know that these adjectives are introduced by the
passive transformation in the manner described above. And, although we have (445), the sentences (446) are not grammatical.

(445)(a) John was sad and tired
    (b) John was recognized and applauded

(446)(a) John was sad and recognized
    (b) John was tired and applauded
    etc.

It is also interesting to note that though "was applauded" and "was tired" have the same constituent structure, conjunction shows that the passive "was applauded" retains its 'verbal force', since it can be conjoined with a verb phrase. Thus we have (447) but not (448).

(447) the speaker arrived and was applauded
(448) " " " " " tired

The reason for this presumably lies in the verbal origin of "was applauded" under transformation. There are various ways in which this fact, which distinguishes "applauded" from "tired", might be used to simplify the analysis. In general, we see that various conditions must be added to (444) to account for these and other restrictions.

The nature of conjunction is an important matter that deserves a special and detailed investigation. To go into this properly would far exceed the bounds of this study. Though we cannot give the conjunction rule in a precise form at this point, it is important to note that this is apparently a result not of a theoretical shortcoming, as in chapter VII, but simply of lack of data. The specific problems concerning conjunction which were raised in §69 have been overcome, and it appears that the present theoretical framework should be adequate to give a
satisfactory account of conjunction.

The constructions of this section do not exhaust all cases of conjunction, unless certain subsidiary transformations are given. Thus we have such cases as

(449) (a) they elected John president and Bill vice-president
(b) John was elected president, and Bill, vice-president
(c) he let the dog in and the cat out
(d) Bill bought candy and Jim, pretzels

Cases (b) and (d) suggest a zero proverb. (c) and (a) might be handled by transformations that simply rearrange constituent structure (these would be a special class of L-transformations), but it is perhaps more interesting to consider (at least in the case of (a)) the possibility that the original constituent structure of the constituent kernel sentences is somehow taken into account. Further pursuit of this approach, however, leads into areas where our analysis of derived constituent structure appears to be inadequate.

Whether or not all cases of conjunction can with profit be handled by a single family of generalized transformations, only a more detailed study will tell. However this may be, it does not affect the validity of the conjunction criterion for constituent analysis as this has been employed throughout our discussion of English syntax in chapters VII and IX. It can scarcely be doubted that the conjunction rule is tremendously simplified if constituents are chosen in such a way that (442) can be formulated directly in terms of constituents, whether it turns out to be the case that (442), (444) covers all cases of conjunction, or only a large majority. And this is the only sense in which conjunction has been used as a criterion
for constituent analysis in our study.

111.2. We have not yet accounted for the fact that the subject $NP \land NP$ takes a plural verb. Even if $F_{AND}$ is made to precede $\mathcal{F}_1^P$ (which carries $C$ into $0$ and $S$, the number elements of the verb phrase), it is necessary to add a special further transformation to effect this change of the verb from singular to plural, when $NP \land NP$ is its subject. This can be given either as a component of the mapping $\mathcal{F}^P$ or as a transformation added to (444) after $F_{AND}$.

There are other transformations that can be added to (444) following conjunction. Given "John met Bill" and "John liked Bill", we can form, in the normal way,

(450) John met Bill and liked Bill

But the more normal form would be

(451) John met Bill and liked him

In general, it is the case that if conjunction forms a sentence $... C \land C' ...$, where $C$ and $C'$ contain the same animate noun in the same structural position (in some sense to be specified precisely in the definition of the restricting class of the transformation we are now considering), then the animate noun in $C'$ is replaced by "he". Once we have determined the structural requirement, there is no problem in giving this as an added transformation following $F_{AND}$ in (444), either obligatory or not, depending on the decision as to the grammaticalness of (450). Clearly this phenomenon extends beyond animate nouns. With inanimate nouns, "it" plays the role of "he" in quite a parallel fashion (e.g., "I found the
book and sent it"). But the element "it" has an even broader range than this. It can sometimes replace verb phrases (or perhaps, nominalized verb phrases). Thus in (452) I visited John and enjoyed it thoroughly it is clear intuitively that "it" replaces "visiting John", and it should be possible to bring transformational analysis to bear on this question. There are other cases of parallel structures (e.g., "whenever..., then..."), and investigation of pronoun distribution seems likely to offer a good deal of insight into the internal structure of the components of such constructions, in the manner we have just indicated.

111.2. The transformations $T_{so}$ and $T_{too}$ discussed in $\xi 90.4$ and $\xi 92.2$ above are transformations following conjunction and having much the same status as the pronominal transformation that forms (450). As we have seen, "so" is introduced as a pro-element standing for the verb phrase in such sentences as "John saw him and so did I", "John will be there and so will I", etc.

$T_{so}$, incidentally, offers a criterion for determining the analysis of such elements as "have to"(45), "ought to", "use to"(45). We might consider these as some sort of auxiliary verbs, or we might consider them to be instances of $V$-$to$-$V$ constructions like "try to", "want to", etc. The question can be resolved by noting that $T_{so}$ differentiates auxiliaries from all main verbs, including those of the $V$-$to$-$V$ construction (cf. $\xi 107$ for the final analysis of this construction). Thus we have
(453) (a) John is coming and so is Bill  
(b) John comes and so does Bill  
(c) John tries to come and so does Bill  
(d) John will come and so will Bill  

For "haf to" we have  

(454) John has to come and so does Bill  
like (453c), not (453d) or (453a). Thus "haf to" as a case of V-to like "try to" and not an auxiliary, like "will". Similarly,  
(455) John used to come and so did Bill  
shows that "use to" is also such a case. The fact that "does" cannot appear in place of "did" in (455) indicates that "use to" occurs only in the past tense, i.e., that the choice of \( \text{VP}_A \) with the verb "use" in "use to" is restricted to "ed".  

"ought to", on the other hand, is an auxiliary verb, not an instance of V-to like "try to", since we do not have  
(456) John ought to come and so does Bill.  
"ought to" must be in the class \( \mathbb{M} \), since we have "ought to be coming", "ought to have come", etc. But it is limited in distribution. Since we have (457) and not (458), we might consider it to be a variant of "should".  
(457) I ought to come and so should he  
(458) I ought to come and so ought (to) he
This classification of these elements coincides with that given by other criteria. Thus we have "I will haf to", but not "I will use to", "I will ought to". However, the restrictions on distribution of these elements require further study.

112. One interesting and highly productive verbal construction that we have not yet considered is the construction Verb-Preposition where the preposition is not 'separable' as in the cases discussed in 326. As instances of this type we have

(459)(a) everyone laughed at the clown
     (b) John thought of a good answer
     (c) the staff went over the list
     etc. (46)

Investigation of this construction reveals many peculiarities and many conflicting criteria by which different and overlapping classifications can be made (e.g., stress on the preposition, occurrence of an adverb or parenthetical expression between verb and preposition, etc.). As a only the barest introduction to this complex construction, we can investigate the place of its major constituent break by considering its behavior under certain transformations.

The conjunction transformation perhaps favors the analysis of (459a) as NP - V I -PP, since we have

(460) everyone laughed at the clown and at the elephants

On the other hand, (461) also appears to be acceptable, at least with certain verbs

(461) everyone laughed at and mocked the clown.

It is certainly more acceptable than any corresponding form
(462) John worked at and Verb-ed the office
for the sentence

(463) John worked at the office.

With other instances of (459) conjunction is indecisive.

But the sentences of (459) do not undergo certain transformations characteristic of \( NP - V_I - PP \) constructions. For example, there is a transformation that converts \( NP - V_I - PP \) into \( PP, NP - V_I, \)
(noted in \( \eta 92,1 \)) converting (463) into

(464) at the office, John worked.

But this transformation does not carry (459a) into

(465) at the clown, everyone laughed.

Furthermore, certain transformations that apply only to \( NP - V_T - NP \)
do apply to (459). Thus, under the passive transformation, we have

(466)(a) the clown was laughed at (by everyone)
     (b) an answer was thought of (by John)
     (c) the list was gone over (by the staff)

but not

(467) the office was worked at (by John), etc.

This indicates that the sentences of (459) should be analyzed as containing a compound transitive verb \( V_P \). The question transformation also distinguishes (459) from (463) and other instances of \( NP - V_I - PP \). Thus we have (468) but not (469).

(468)(a) whom did everyone laugh at (47)
     (b) what did John think of
     (c) what did the staff go over

(469) what did John work at (48)
This is not very convincing evidence however, since many clear cases of $\text{NP-} V_I - \text{PP}$ are also subject to this transformation. We have "what did John drive away in", from "John drove away in a Cadillac", where "in a Cadillac" must be a PP because of "in a Cadillac, John drove away", "in what did John drive away", and because of the impossibility of "the Cadillac was driven away in". In fact, it is in general the case that the question transformation applies to the noun in $\text{PP}$.

In any event, these various considerations give some reason to analyze (459) as $\text{NP-} V_T - \text{NP}$, with "laugh at", "think of", "go over", etc., as the transitive verbs. This would seem to match intuition. It will also permit considerable constructional homonymity, since now $\text{NP-} V - P - \text{NP}$ can be derived from $\text{NP-} V_T - \text{NP}$ (where $V_T \rightarrow V'P$) or from $\text{NP-} V_I - \text{PP}$, (where $P \rightarrow P^\prime \text{NP}$). There is intuitive support for this homonymity in a great many instances. Thus in (470), the sentences in the left hand column have passives, hence are of the form $\text{NP-} V_T - \text{NP}$, while those of the right hand column, not having passives, are instances of $\text{NP-} V_I - \text{PP}$, and cases where either analysis is possible are easily constructible on these models.

(470)  

<table>
<thead>
<tr>
<th>$\text{NP-} V_T - \text{NP}$</th>
<th>$\text{NP-} V_I - \text{PP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>they ran after John</td>
<td>they ran after lunch</td>
</tr>
<tr>
<td>they hoped for peace</td>
<td>they hoped for three years</td>
</tr>
<tr>
<td>they counted on my support</td>
<td>they counted on their fingers</td>
</tr>
</tbody>
</table>

The grounds for this intuitively correct analysis are not very firm, as presented above. But it appears that transformational analysis may open up suggestive approaches to the analysis of these forms.
The grounds for the validation of any transformational analysis are to be found in considerations of simplicity, given a specification of the form of grammars, and a set of formal requirements that the structure described by the grammar must meet. It is necessary to keep this in mind in considering the relevance to this process of validation of any exceptions to the transformational rules. In one sense, there are no exceptions, since the grammar is defined to be the simplest description, meeting fixed formal conditions, for exactly the set of grammatical sentences. If a certain transformational (or other) formulation fails to include certain grammatical sentences, or includes certain ungrammatical ones, then conditions must be added to right the discrepancy. An 'exception' to a transformation (i.e., a grammatical form not included, or an included ungrammatical form) will count against the validity of the proposed transformational analysis only if the exception would not have required special statement had the transformational analysis not been adopted. The decision as to whether a sentence belongs to the kernel turns on the relative complexity of the alternative descriptions. If a certain special statement is required under both alternatives, then this statement simply plays no role in the decision between the alternative analyses.

There are many exceptions to the transformations that we have set up. In part, this is due to the insufficiency of our analysis, which makes no attempt to discuss the grammatical structure of English in the required detail. But even in a much more detailed and careful study than this, it will no doubt be the case that if the criteria of grammaticalness are strict enough, there will be many ragged edges in the
analysis where special and inelegant formulations will be required. In this discussion of English, these exceptions have not been cited, in part, because of the necessarily sketchy nature of our analysis, and in part because they are irrelevant in the sense of the preceding paragraph. But this is not to say that these exceptions should not be studied. Apparent exceptions may be an important clue, as we have frequently seen above, to deeper underlying regularities.

We might mention several incidental exceptions (whether real or apparent, only future investigation can determine) to the transformations we have constructed. As instances of actives with no corresponding passive we have

\[(471) \text{(a) This costs a lot of money} \]
\n\[\text{(b) This weighs three pounds} \]
\[\text{(c) John travelled three days} \]
\[\text{(d) Mary married John} \]
\[\text{(e) Misery loves company} \]
\[\text{(f) He got his punishment} \]
\[\text{(g) He had an accident} \]
\[\text{(h) No one foresaw any improvement} \]
\[\text{(i) He didn't like either of them} \]
\[\text{(j) He only likes certain people} \]
\[\text{(k) The artist redecorated it completely} \]

From the last four cases it is evident that further analysis of negation and of adverbs is necessary. Suppose that we add a mapping that carries (472) into (473).

\[(472) \text{no}^\text{Noun} \ldots \text{no}^\text{Noun} \]
\[(473) \text{no}^\text{Noun} \ldots \text{any}^\text{Noun} \]

This mapping will account for the fact that case (h) occurs, but not "no one foresees no improvement". And it will also automatically give "no improvement was foreseen by anyone" as the passive of case (h), thus eliminating one discrepancy.
in what appears to be a simple and intuitively correct manner. Further study is needed of all of the 'quantificational' words ('all', 'some', 'no', 'any', etc.).

Case (k) is not handled correctly by our analysis, since its passive should be "it was completely redecorated by the artist". It is thus quite analogous to "John looked it up", which has the passive "it was looked up by John", as we noted above in \[\text{286}\]. There are also many other indications that certain adverbs can be treated as analogous in their distribution to separable complements.

Consider now cases (a), (b), and (c). We have often noted that the passive transformation offers an effective criterion for the analysis of a sentence as \[\text{NP} \rightarrow \text{VT} \rightarrow \text{NP}\]. When we find a passive that will not be produced unless a certain sentence \(Z\) is analyzed as \[\text{NP} \rightarrow \text{VT} \rightarrow \text{NP}\], we can use this fact to support this analysis of \(Z\). Conversely, when we find a sentence \(Z\) which might be analyzed as \[\text{NP} \rightarrow \text{VT} \rightarrow \text{NP}\], but for which the corresponding passive is not grammatical, we can use this fact as a support for not analyzing \(Z\) as \[\text{NP} \rightarrow \text{VT} \rightarrow \text{NP}\]. In the cases (h)-(k) there are passives (e.g., "no improvement was foreseen by anyone", "neither of them was liked by him", etc.) which require analysis, and this fact leads us to a further analysis of quantificational words, negatives, and adverbs, as we noted above. But in cases (a)-(c), there are no passives unaccounted for which could somehow be produced from these sentences, so we may simply refuse to assign the structure \[\text{NP} \rightarrow \text{VT} \rightarrow \text{NP}\] to these sentences as their constituent analysis. Thus we might analyze these sentences as instances of a construction \[\text{NP} \rightarrow \text{VT} \rightarrow \text{Mod}\], where \text{Mod} is a certain type of modifying phrase that occurs after certain intransitive
verbs and has a heavily restricted internal structure, with numerals as adjectives, etc. This analysis would make the passive transformation inapplicable in these cases. On lower levels, there is no way to avoid the conclusion that (a)-(c) are cases of transitive verb constructions with objects. But when we employ transformational considerations to determine the simplest analysis of grammatical sentences, we see that this intuitively absurd conclusion is in fact ruled out. This seems to be another case where transformational analysis provides grounds for a strong and unmistakeable intuition about linguistic form.

Case (d) is a peculiar instance. Although it has no passive, the sentence "the preacher married John" does.

Case (e) does not really belong here if we can carry through the program suggested in \(429\). There we suggested grounds for ruling out this sentence as not fully grammatical, and in transformational analysis we are concerned only with fully grammatical sentences. We might take the failure of the passive transformation as a further corroboration for the independently established semi-grammaticality of (e).

Cases (f) and (g) have a different sort of interest. It may be significant that the three verbs that are also auxiliaries, i.e., "have", "get", "be",(51) are impossible or unnatural in the passive, except for certain semi-calculated expressions such as "a good time was had by all".

As passives with no corresponding active, we find, for example,

\[(474)\]

(a) the game was rained out
(b) John was gone
(c) John isn't finished yet
(d) John was really had that time
etc.
The first three cases suggest that some fairly productive sentence form has been left out of our analysis.

There are also instances of declaratives with no corresponding questions (475) and questions with no corresponding declaratives (476).

(475) (a) John hardly ever comes
    (b) John almost never comes
    (c) He completely forgot (→ did he forget completely)

(476) (a) Did your friend leave yet
    (b) Did he ever finish
    (c) What did he want to visit them for
    (d) Didn’t anyone foresee any improvement
    (e) Is anyone liked by everybody

These cases also suggest the need for a more far-reaching analysis of adverbs, negation, and quantificational words. There is no doubt, on quite independent grounds, that these troublesome words require a great deal more study. Case (d) suggests the need for an extension of the mapping (472-3) discussed above. Case (476c) suggests that "what...for" be considered a variant of "why".

These lists can be extended considerably, revealing great gaps in our analysis of English structure. But as far as I have been able to determine, these gaps and exceptions do not negate the major conclusions arrived at in the preceding sections.

114. Sketching briefly the major structural features of English revealed in this analysis, we have a kernel of simple declarative sentences with the two fundamental grammatical relations of subject-predicate and actor-action. The former is subdivided into predicative sentences with be (NP-be-Predicate) and stative verb constructions...
with "seem", "act", "become", etc. (52) The actor-action construction is subdivided into transitive constructions (with the grammatical relation transitive verb - object) and intransitive constructions. There is no very good reason to consider be to be a verb at all. It shares many of the features of the auxiliary, and we might say that predicate constructions have no main verbs. Transitive verbs are divided into two main structural types, with complement and without complement. The verbs with separable complements are limited to the type "call up", "throw cut", etc. The auxiliary verb phrase is retained in full in the kernel. It is probably best to drop adverbs, or at least some classes of adverbs, but this we have barely touched on. The related "not" and the related element "Ac" are dropped from the kernel, however. Only one type of prepositional phrase remains in the kernel, namely, the 'independent' PP's such as "by a new technique", "in the morning", etc., that are added to the verb phrase. Adjectives are limited to the three types A (e.g., "old", "small", etc.), \(\text{ing}V_1\) (e.g., "surprising"), \(\text{en}V_2\) (e.g., "tired"), and all can be modified by "very", which remains as a kernel element. Adjectives, however, occur only in the predicate position, never modifying nouns. Noun phrases are all of the type article-noun. Pronouns are probably to be dropped from the kernel.

The transformations that produce sentences from simple sentences are the interrogative, elliptical, and passive transformations. A natural classification of simple sentences, based on transformations, will divide them into declaratives and interrogatives. The declaratives include the kernel
sentences and the passives, whose derived constituent structure is that of kernel sentences. The basic interrogative sentences are the yes-or-no questions ("did you come", etc.), with inversion. If we apply the wh-transformations, with a second inversion, to these questions, we derive a second class of interrogatives, including many types ("who was here", "whom did he see", "what plane did you take", etc.) The wh-transformation, applied to declaratives, gives relative clauses. Such sentences as "who saw him", "whom did he see" have the same transformational history, despite the fact that only the second of these contains an inversion. There are several types of elliptical sentences. The verb phrase can be dropped from declaratives, and the final by-phrase from passives. The subject you can be dropped from yes-or-no questions giving imperatives, by a somewhat more complicated (and more dubious) elliptical transformation.

Other transformations form noun phrases, adjective phrases, and verb phrases. ing-phrases appear as abstract nouns, and the subjects of the sentences from which they are transformationally derived may be retained as possessive 'adjectives' (as in "John's refusing to come"). Other transformationally produced noun phrases reproduce the subject-verb relation ("growling of lions", etc.) or the verb-object relation ("reading of good literature", etc.). Similar 'nominalizing' transformations give "John's refusal to come", "the sight of men working", etc. that-clauses also appear as nouns. We might go on to derive many compound nouns transformationally. Various noun modifiers are introduced, including relative clauses, prepositional phrases, ing-phrases,
verbal adjectives from the passive ("forgotten", etc.) or from intransitives ("barking", etc.). Adjective phrases appear in noun-modifying position as transforms of NP-is-AP sentences.

The most interesting transformational development of the verb phrase is the tremendous extension of the verb-complement construction, where the complement is separable, and the noun phrase object of the compound verb (e.g., "John" in "I consider John incompetent", where "consider-incompetent" is the verb-complement) is the subject of the complement in the underlying simple sentence. This is a highly productive and ramified construction, with many different types of instances. In particular, the V-to-V (and V-ing-V) constructions, as in "I want to come", "I like reading", etc., are special cases of it, with the object replaced by ∅. These can be elaborated by simple means into very complex forms (especially, when the transformation of pronoun to pronoun-self is brought into consideration.

Conjunction is the generalized transformation that has by far the broadest scope of all, applying to almost any constituent, and to little else. Further investigation would reveal that parenthetical expressions must also be introduced by transformations. Thus the criteria for constituent analysis stated in §52 reduce to two types, those based on intonational structure (which we have not investigated) and those based on transformational structure. (That these should not be sharply distinguished is pointed out in fn.49). In constructing transformations we discovered many other 'criteria' of the transformational type. It appears that the basic reasons for assigning a given constituent structure to a sentence are to be found in the behavior of this sentence under transformation.
This is a crude and incomplete analysis, as a glance at any text in English will quickly show. There are many other phenomena that suggest transformational analysis, and the kernel also requires a considerable amount of further study. But I think that many of the main features of English structure have been touched on in this sketch. In particular, we have seen that all the difficulties and inadequacies sketched in \( \S 2-4 \) have disappeared with this transformational analysis; we are apparently able to extend the bounds of grammatical description to cover all grammatical sentences without the intolerable complexities and intuitive absurdities that resulted when the devices available on lower levels were extended beyond what in chapter VII appeared to be arbitrary and overly narrow limits.
Footnotes - Chapter IX

(1)(p.509) In fact, the major conclusions follow from a direct comparison of the alternative solutions, with no weighting.

(2)(p.514) Note that in actually formulating $\Phi^P_{15}$ as a grammatical transformation we must be careful to indicate that the element $K$ (cf. definition of $\Phi^P_{15}$ in §67.2) and the following verb are not permuted when $K$ belongs to the preceding noun phrase, as in this case. This is easily done in terms of the notions of derived constituent structure. We simply require that the element $K$, if it is $S$ or $Q$, must be a $C$, and this is easily stated in the restricting class for $\Phi^P_{15}$. This refinement is necessary whether or not we accept the transformational analysis for interrogatives, if these sentences are treated in the grammar.

(3)(p.516) We may add a further specification of $W^4_{12}$ to guarantee that the selectional relation of the verb of the first conjunct of (12) and the noun of the second is maintained. But this is irrelevant to the point at issue here.

(4)(p.523) There are many cases where the forms (33) and (34) are strained and unnatural. But the important thing is that if the grammar is detailed and sensitive enough to distinguish the natural cases like (33) and (34) from the unnatural ones, then it is no simpler to add the natural ones to the grammar by special rules than to strike off the unnatural ones. Thus this further consideration does not affect our conclusions here.

(5)(p.527) In fact, such a careful and detailed study of English structure as Nida's Outline of English Syntax does not even include a discussion of questions. But clearly the opposite procedure of sketching English syntax including only questions, excluding declaratives, would be unthinkable.

(6)(p.529) Cf. Harris, Methods for a discussion of the morphemic treatment of $wh$ and $th$. In this discussion we omit consideration of the other personal pronouns. This is simply a matter of added detail, and need not concern us. E.g., we will have to add, as a morphological rule (a part of $R^M$) that $he'S$ → $they$, $him'S$ → $them$, etc.

(7)(p.536) This may require making a distinction in the kernel grammar between there in "I saw him there" and in "there is a
possibility that he will be present", since we do not have "where is possibility that he will be present", etc. Or a restriction on (56) might cover this.

(8)(p.539) If such sentences as "can I not see it" are also considered grammatical, then we must set up two transformations, one based on the latter defining sequence, to account for this sentence, and one based on the former, to account for (75). We have not troubled to make the distinction between not and n't. We will disregard this distinction, and rule out "can I not see it", etc.

(9)(p.540) Condition 5, § 85.4, is thus the motivating force behind this choice. There is actually a good reason for $F_N$ to apply before $\Phi_{13}^P$, the mapping that converts $C$ into $\emptyset$ or $S$.
If $F_N$ applies after $\Phi_{13}^P$, then not will be affixed to $S$ or $\emptyset$.
Hence it will be the case that $S$-not is an $S$, and $\emptyset$-not is a $\emptyset$ (though this does not change the fact that $S$-not is a $S$ and $\emptyset$-not is a $\emptyset$ -- this following from the fact that $S$ is a $C$ and $\emptyset$ is a $\emptyset$).

But $\Phi_{13}^P$ converts, e.g., $S^v$ into $V^vS$, so that if $S$-not is an $S$, then $\Phi_{13}^P$ will carry $S$-not into $V^vS$-not, giving such impossible sentences as "John livesn't here", etc., unless some special condition is added. If not is inserted before $\Phi_{13}^P$, then $C$-not is a $C$, and when $C \rightarrow S$ (by $\Phi_{13}^P$), $S$-not is still a $C$, but is not an $S$, so that $\Phi_{13}^P$ does not convert $S$-not into $V^vS$-not.

Note that if $F_N$ applies before $\Phi_{12}^P$, then "M" in (44ii-II) must be replaced by "$<C'E$" so as to make (44) available for both $T^c$ and $T^q$. Hence "2$^c$K$^v_2F_N^v$" might be a better choice for a $\in$-marker than (77). But the further transforms of negative sentences may lead to a violation of Condition 5, if this change is made. A good deal of further investigation is needed here.

(10)(p.543) In §61.5, we discovered two sources for sentences of the form $NP_{is}^{c\in}ing\ V_1^P$.

(11) Except under contrastive stress, which can occur with almost any word. Exactly what is the status of contrastive stress is unclear, but we will proceed as if the problem of determining this has been solved, and sentences containing it have been set aside. It is worth noting, in this connection, that when
sentences which are only partially grammatical do occur, they often contain contrastive stress. Hence one of the functions of this element is to mark the breaking of a grammatical rule.

(12) (p. 546) Cf. the study of Hebrew morphophonemics in the appendix to chapter IV for a similar phenomenon.

(13) (9.546) The third case, that of Tₜₙ₂, is inconclusive here, since given "he does the crossword puzzle and so do I", we cannot determine whether the do after so is the main verb or the bearer of the affix introduced by Tₚ₁. Note that the revision of (44) which is sketched above for the case where do is treated as an auxiliary in the kernel, does not affect this situation.

(14) (p. 547) This is the simplest way to give the transformational analysis for Ac. An alternative would be to give only (44ii-1,III) as the definition of the restricting class for Tₐc. This would limit the appearance of Ac to (92). As we have given Tₐc, the element Ac also appears in questions (e.g., "does he come") and in the other positions of not in declaratives (e.g., "he has been here"). These cases could be treated as instances of contrastive stress, since they also occur without stress in this position. But the given simpler and more general statement seems reasonable, since "he has not come" is not grammatical, just as (99) is not.

(15) (p. 549) There are other possible variations with more or differently placed adverbs, but we have not given these other sentence forms in the sketch of the kernel grammar in §62.2. Any complication needed to state adequately the restricting class for Tₚ would be compounded in an alternative non-transformational analysis, since it would appear once in the description of the active, and once in the description of the passive. But it is necessary to study these variations in detail to see what effect they have on the transformational analysis of passives -- i.e., do they necessitate certain special rules, etc. I have not investigated this question.

(16) (p. 552) Cf. chapter IV for discussion of the question of degree of grammaticalness. Recall that the purpose of a grammar, as we have formulated in in chapter IV, is to state the highest degree
grammatical sentences, not those that "can occur" (whatever
this may mean). Given a grammatical statement for first
degree grammatical sentences (like (109) but not (110)), and given
the system $C$ of syntactic categories, as in §§25,26, we will
be able to determine that (110) are partially grammatical sentences,
hence more grammatical than "John of from", etc. In fact, they
may even be 'absolutely grammatical' in the sense of the basic
dichotomy discussed in §22. But none of this has any bearing
on the argument of this section.

(17)(p.565) Note that we are not actually presenting $T$-markers,
but rather $T$-marker schemata. A $T$-marker contains fixed strings.
Hence to turn these schemata into $T$-markers, it is
necessary to choose fixed strings and interpretations for the
variables "Z", "K", etc., and to select a given transformation
from the families listed in the schemata. Cf. §§88. We will make
a sharp distinction only when failure to do so can lead to
ambiguity. Otherwise we will refer to schemata as $T$-markers
as well.

(18)(p.583) The converse is not the case. Certain 'compound
verbs' of the form $V^P$ do not have separable prepositions as
do "bring in", "call up", etc. Cf. §§12.

(19)(p.594) Of course this is nowhere near a sufficient characteriza-
tion of the conditions under which the passive transformation
would be necessary. If such a set of conditions can be given
at all, it will be extremely complex. We are reducing the
problem here to its bare essentials, and assuming that there
are no further instances of sentences that serve as counter-
examples to the passive transformation. This is a fair statement


(21)(p.598) More correctly, there will be such a subdivision for
each subclass of $V_T$. We will not go into the details of this
statement, since much of it will be introduced transformationally
below. For the same reason, we will not go into certain
reformulations in the rules of numerical agreement (statements 4,6,
§67.2) which are necessitated by these revisions.

(22)(p.609) It should be recalled that we are making no attempt to
account for the considerable dialectal variation in this and other cases that are under discussion.

(23) (p. 614) Actually, as we have constructed the kernel grammar, in both (168) and (254) the generalized transformations should have $\mathcal{G}^\mathcal{N}^\mathcal{C}$ as the replaced terms, not just $\mathcal{N}$, and we should thus say that the that-phrase is a $\mathcal{G}^\mathcal{N}_{\text{inan}}^\mathcal{C}$. But a more far-reaching analysis of the kernel, and an explicit analysis of pronouns, either transformationally or as part of the kernel, might alter this in detail. Cf. (197), § 103.1.

(24) (p. 615) This pattern should be extended to cover the 'stative verbs' "seem", "become", "act", etc. These differ from one another somewhat, and they differ from "be" in several important formal respects (e.g., more limited occurrence with following NP, non-occurrence with following PP, etc.). Since these verbs are not also auxiliaries, as is "be", they do not share the features of the auxiliaries manifested by "be" and discussed in § 90.5, 92.2.


(26) (p. 625) The converse is not the case. We did not give the restrictions on the construction (286) in the kernel grammar in § 67.2 (except for the restriction to agreement in number), but there clearly are restrictions. E.g., Predicate can not ordinarily be PP after $\mathcal{V}_\mathcal{R}$. In a non-transformational treatment, these further conditions will be added to the $\mathcal{P}$-grammar; in a transformational analysis, they will be given as a condition on the $\mathcal{P}$-basis of the $\mathcal{T}$-markers involved.

(27) (p. 628) The split between $\mathcal{VP}_A$ and $\mathcal{VP}_B$ here, as in other to-phrases that we have discussed, permits "I consider John to be trying his best", "I know him to have been honest," etc.

(28) (p. 628) The latter sentence is quite unnatural. We might remedy this by extending the transformation so as to take, e.g., "they put it here", as the root. Or we may consider this to be a case of the type discussed in § 87.6, with "it" as a dummy carrier.

(29) (p. 629) If "in the garage, John kept the car" and "the car was kept by John in the garage" are accepted as grammatical sentences, an alternative analysis of (299) will be as $\mathcal{NP-V-NP-PP}_A$, i.e., as a sentence of the same form as (203). We assume here that these sentences are rejected as not fully grammatical.
(29) (p. 632) This permits "I can't imagine the dog having climbed that tree", etc. Cf. fn. 27. This is the distinction between ing-phrases and to-phrases that we have frequently noted above.

(30) (p. 633) However, under the interpretation of (310) as analogous to (308), the \( P \)-basis can be taken as "the boy is studying in the library", "I found the boy". But under the alternative interpretation, this \( P \)-basis is not available as we have formulated \( T_{P} \) in (304). If this distinction stands, with further and more detailed study, then the argument in the text in this paragraph must be carried somewhat further, but the main point still holds. The difference in meaning under these two interpretations cannot be attributed to the \( P \)-basis, but only to the intermediate sentences that underlie (310) in the two transformational developments.

(31) (p. 637) Actually, \( Q_{i} = \{ (NP, VP_{A1}, \langle VP_{A2} \rangle, be^{\text{Predicate}}) \} \) and \( Q_{\text{cmp}}^{i} \) begins \( (NP, VP_{A1}, to\langle VP_{A2} \rangle, be^{\text{Predicate}},...) \). But to\(^{\text{be}}\text{Predicate}\) is a \(^{\text{be}}\text{Predicate}\), and to\(^{\text{VP}_{A2}}\text{be}^{\text{Predicate}}\) is a \(^{\text{VP}_{A2}}\text{be}^{\text{Predicate}}\), in their derived constituent structure (cf. Def. 33, §83.5). Hence we can give the beginning of \( Q_{\text{cmp}}^{i} \) exactly as \( Q_{i} \). The same is true of \( Q_{\text{cmp}}^{ii}, Q_{\text{cmp}}^{iii}, Q_{\text{cmp}}^{iv} \).

(32) (p. 639) Actually, we have not provided the technical means for the introduction of such statements as 10 into the grammar.

(33) (p. 646) The "reduced correlate", it will be recalled, is the string which results from a component of \( \mathcal{F} \) which concludes the \( T \)-marker is applied. Note that further transformations are always applied to the reduced correlate, and not to the transform fully mapped out into a string of words. Cf. §87.4 for details.

(34) (p. 647) Note that this formulation is not quite accurate. There is no question of literal reapplication of mappings. **Condition 5** precludes this. Rather, the question is one of formulating the mappings in a complex way as a family of transformations, perhaps an infinite family, each member of which applies to certain instances. Such formulation is possible (if we permit the condition that defines a family to be a condition on infinitely many elements) but it is inelegant, and should be avoidable. Cf. §87.5.
(35) (p.653) The necessity for treating \( V_a \) and \( V_p \) as distinct subclasses has already been noted in §105.5.

(p.657)

(36) A pronoun in a prepositional phrase that agrees either with the subject or object also undergoes this transformation, even in kernel sentences. Thus we have "I bought a book for myself", "they found John by himself", etc. The construction of such sentences poses a variety of problems that we have not gone into. There may be reasons for dropping these sentences from the kernel altogether. For the purposes of this discussion we will omit such constructions (as well as other occurrences of "self", as, e.g., "he saw the President himself"), and concentrate on the case of the direct object.

(37) (p.658) We have "he wanted himself to be elected", "he imagined himself to be president", etc., with the verb "to be", but this is not compelling, because these sentences can be regarded as containing the verb phrase \( V_\theta \text{to} \text{ Predicate} \), with \text{imagine, want} as \( V_\theta \)'s.

(38) (p.672) We have sequences \( T^A \cdots A \text{ and } A \) and \( T^N \text{is} A \cdots A \text{ and } A \), but these must be considered separately, under the and-transformation.

(39) (p.674) Note that "\(<\text{PP}>\)" has already been dropped from statement 7 (cf. §104.2), and "\(<\text{AP}>\)" is dropped by the transformational analysis now under discussion.

(40) (p.675) Pronouns are also distinct from other nouns in that they are normally unstressed. The characterization of pronouns in terms of lack of stress suggests interesting possibilities. There are certain nouns that also are normally unstressed in syntactic positions that generally require heavy stress, e.g., "people". Thus we have "hard work matures people", "adversity strengthens people", etc., as compared with "hard work matures the mind", "adversity strengthens the character", etc. If pronouns are transformationally introduced, such words as "people" might well be too.

Note that proper nouns share the distributional limitations of pronouns which are under discussion in the text. Proper nouns pose a variety of difficulties on every level which suggest that they may have some special 'extra-grammatical' status, but this is
a question that we have not investigated.

(41) (p. 681) Note that we are using here the fact discussed in §3.2 that \( f \) is recovered in part from \( f^T \), which in turn is defined in terms of derived constituent structure. We are also using the fact that the components \( \Phi^P_{11} \) of \( \Phi^P \) are themselves transformations. (Cf. §3.4).

(42) (p. 682) Note that we have once more (as in §9.4, 92.2) made use of the fact that be can be either a main verb or part of the auxiliary, as a means of simplifying the statement of transformations.

(43) (p. 684) With the verb "look", the situation is different. Case (402f), but not (402a,e), occurs in "it looks --". But this fact does not affect the point being made here, since this is a special and particular exception whether or not transformational analysis is applied, and we are concerned with the relative evaluation of grammatical description with and without such analysis. Cf. also §103.

(44) (p. 691) The auxiliary phrase must contain the element \( C \) which carries number, but we will see below that \( T_{imp} \) must apply after \( \Phi^P_{11} \) which carries \( C \cdot M \) into \( M \) (cf. §62.2), so that \( y \cdot M = M \) is in fact one of the forms of the \( \Phi^P_2 \) at the point where \( T_{imp} \) applies.

(45) (p. 700) as in "I have to go". This element is distinct from "have to", and an absolute contrast can be found as "these are the spark plugs we have to (have to) replace the old ones (with)". Similarly "used to" is distinct from "used to", as can be seen by replacing these forms for "have to", "have to" in the same sentence.


(47) (p. 703) But also "at whom did everyone laugh", favoring the analysis into \( NP - V_1 - PP \). Cf. §91.2.

(48) (p. 703) But this is a colloquial usage, as the question from "John worked at being a success", etc. But there are clearer cases, e.g., "John left at noon" -- "what did John leave at".

(49) (p. 704) This is another instance where extension of the
theory to include such phenomena as pitch, stress and juncture would prove valuable. Though such sentences as "the planes fought over the hills", etc., can be said so as to be ambiguous as to constituent interpretation, they can also be said in an unambiguous manner with intonation features that characterize the left hand column of (47), or with features that characterize the right hand column. These intermittently present intonational features could be transformationally introduced in a more elaborate theory, and the possibilities of one or another intonational transformation could be used as criteria for constituent analysis, in the same way as the segmental transformations we have been discussing. Similarly, the considerations rejected in §60.5 could be reintroduced as criteria of analysis in this way. But as we have mentioned several times, it is not obvious how to extend this theory to include such phenomena, though clearly, such an extension is necessary.

(50) (p.705) This does not rule out a formulation of linguistic theory in which the simplicity of grammar is a factor in determining the grammatical sentences. But the grammatical sentences must be partially determined by other considerations, if we are to avoid triviality, and the statement in the text holds for the limits of this partially determined set.

(51) (p.708) Though we have in fact seen little reason for calling "be" a verb at all.

(52) (p.710) It may be preferable to classify these as instances of the actor-action construction, or as a third independent type. We have touched on these forms too hurriedly for our analysis to have much significance.
Chapter X - Summary

115.1. We have sketched some of the fundamental features of a theory of linguistic form, and have investigated some of its empirical consequences. We will now survey briefly the main lines of development of this theory and some of the considerations that lie behind the particular form that it has taken.

This research has been concerned with linguistic form, with the manner in which utterances are constructed. Basically, this has been an attempt to build an abstract theory of linguistic structure within a framework that admits of operational interpretation, and to show how such a theory can lead to a mechanical (and practical) procedure in terms of which, given a corpus of linguistic material, various proposed grammars can be compared, and the best of them selected. This is not a clear statement of a proposed investigation, because the notion of 'a grammar' is not antecedently clear. But a field of investigation cannot be clearly delimited in advance of the theory dealing with this subject matter. In fact, it is one of the functions of a theory to give a precise delimitation of a field of investigation. A set of notions are held to belong to the same domain of inquiry because of the possibility of giving a single clear, systematic, and integrated theory that covers these notions. Before we have constructed a linguistic theory we can only have certain vaguely formulated questions to guide the development of the theory. A simple and natural theory, once established, determines the precise formulation of the questions that originally motivated it, and leads to the
formulation and resolution of new problems that could not previously have been posed. We are interested in developing a theory that will enable us to clarify and resolve such problems as the following:

(1) Is linguistic behavior 'systematic'? I.e., is there, for each language, a simple set of formal properties that characterize the utterances that speakers produce? If so, can we give a cross-culturally valid account of the kinds of formal properties that determine linguistic structure — can we develop a general and abstract theory of linguistic structure of which each language is a particular exemplification? Can we construct in a completely abstract manner (i.e., with no specific reference to any given language) a general method for determining this set of formal properties (or evaluating a proposed formulation of them) in the case of each particular language?

(2) A speaker of a language has observed a certain limited set of utterances in his language. On the basis of this finite linguistic experience he can produce an indefinite number of new utterances which are immediately acceptable to other members of his speech community. He can also distinguish a certain set of 'grammatical' utterances among utterances that he has never heard and would never produce. Can we reconstruct this ability in a general way? I.e., can we construct a formal model, a definition of 'grammatical sentence' in terms of 'observed sentence', thus, in one sense, providing an explanation for this ability? Can we show that there are deep underlying regularities in observed utterances that lead to these results?
(3) On the basis of a finite linguistic experience, a speaker has developed a mass of conceptions and feelings about his language, a mass of 'intuitions about linguistic form'. Furthermore, different speakers with accidentally different linguistic histories agree by and large in many of these intuitions. E.g., any English speaker knows that:

(a) "bill" and "beat" have the same initial linguistic element, though "bill" and "pill" (despite initial articulatory similarity) do not.

(b) The element /eɪneɪm/ has two distinct interpretations ("a name", "an aim"). Many other strings of sounds are ambiguous (e.g., "shoe" can refer to footwear or times), but in quite a different sense.

(c) "see" has a special relation to "sight" that it does not have to "see". This is a relation similar to that between "refuse" and "refusal", etc.

(d) "they are flying planes" has two interpretations (i.e., "those specks on the horizon are . . .", "my friends are . . ."). Many other (or all) sentences are ambiguous (e.g., "take off the shoe"), but in quite a different sense.

(e) "did you see the book" and "who saw the book" are instances of the same sentence type (interrogatives), as opposed to "you saw the book". "who saw the book" and "what did you see" are instances of the same sentence subtype (a particular subclass of interrogatives), as opposed to "did you see the book".

Can we construct a theory of linguistic form that will reveal a formal basis—provide an explanation for these and thousands of other intuitions about linguistic form?
By "the grammar of a language" we mean that theory of a given language which attempts to deal with problems of this sort wholly in terms of observation of the formal properties of linguistic behavior. And by "the general theory of linguistic form" we mean the abstract theory of which each grammar is an instance, and in terms of which each proposed grammar can be evaluated. It might be supposed that there is no such field of investigation as grammar, in this sense, and no such theory as general linguistic theory. That is, it might be supposed that in order to deal with such topics it is necessary to bring into consideration the meaning and reference of expressions, the particular history of reinforcement of the participating individuals, the sociocultural context of speech, etc. We have held, on the contrary, that a great many aspects of linguistic behavior do fall under grammar in the sense that we have just presented. That is, following one of the central tendencies of contemporary descriptive linguistics, we have tried to show that within the framework of 'distributional analysis', we can develop a simple and internally motivated linguistic theory that will shed considerable light on the range of problems that has been hinted at above. Furthermore, we have argued that broadening the base of this theory to include meaning, etc., does not appear to advance our inquiry into this set of problems. We conclude that a theory of linguistic form constructed within a 'distributional' basis(1) does delimit an interesting and significant area of linguistic behavior, and that it can be an adequate instrument for the exploration and investigation of this area.

115.2. A language is an enormously complex system. Linguistic theory attempts to reduce this immense complexity to manageable
proportions by the construction of a set of *linguistic levels*, each of which makes a certain descriptive apparatus available for the characterization of linguistic structure. Each linguistic level thus provides a certain 'point of view' from which to analyze linguistic behavior. A grammar reconstructs the total complexity of a language stepwise, separating out the contribution of each linguistic level. The adequacy of a linguistic theory containing a given set of abstractly formulated levels can be tested by determining whether the grammars resulting from rigorous application of this theory meet certain formal conditions of simplicity (conditions of optimality)

(4) lead to intuitively satisfactory analyses — i.e., offer explanations on formal grounds for the 'linguistic intuition' of the native speaker.

But such an investigation of the adequacy of a proposed general theory is of course possible only under the condition that 'intuition' and similarly obscure terms do not appear in the theory itself. If they do, we cannot determine which grammars result from application of the theory.

Our main conclusion has been that familiar linguistic theory has only a limited adequacy on the syntactic level — i.e., that it is attempting to do too much with too little theoretical equipment. The specific deficiencies of familiar linguistic theory are explored by making this theory (or one form of it) explicit and investigating the empirical consequences of rigorous application of the devices available in this theory. It appears that this theory is adequate up to a point. That is,
if the set of sentences covered by the grammatical description is artificially restricted in a manner which is intuitively significant, but which cannot be characterized in any independent and systematically significant way within this theory, then the resulting grammars have desirable formal properties and the resulting analyses can provide formal grounds for 'linguistic intuition' in an adequate and a satisfactory way. But any attempt to carry the description beyond these limits leads to grammars which are intolerably complex (in storable respects) and to counter-intuitive analyses. We have argued that the remedy for these specific deficiencies is not to be found in extending the distributional basis for linguistic theory to include meaning, situational context, etc., nor, apparently in the introduction of probabilistic and statistical conceptions, though these do fall within the bounds of formal, distributional analysis. Instead, a new level of transformational analysis is proposed as a higher level of linguistic structure. It is shown that the theory of transformational analysis can be formulated in the same completely distributional terms that are required anyway for lower levels, and that a large and important class of problems that appear in the rigorous application of familiar linguistic theory (as well as certain fairly fundamental descriptive inadequacies of this theory) disappear under the extension of linguistic theory to include transformational analysis.

An argument of this sort is impossible without a careful construction of the theories being investigated. Hence the first task is to construct the theory of linguistic structure that is implicit in current syntactic work. Chapters I-IV are
devoted to some general questions concerning the nature and scope of linguistic theory. In chapter V, we sketch the formal development of the lower levels of phonemes, morphemes, and words, emphasizing the logical status of these constructions, and adding nothing new in the way of analytic procedure. In chapters IV and V we also discuss the level of syntactic categories in greater detail. Chapter VI contains a considerably more detailed account of the level of phrase structure, and chapter VII is concerned with the application of the theory of phrase structure (constituent analysis) to English. In chapter VIII the theory of transformations is developed, and in chapter IX, it is applied to English.

115.2. In chapter I we discuss three major areas of linguistic interest: the construction of grammars of particular languages, the development of an abstract theory of linguistic structure, and the problem of justification of grammars. These three tasks are interdependent. Each grammar can be regarded as essentially the science of a particular subject matter (a given language). For the grammar of each language we can set up certain fairly vague 'external criteria of adequacy', of the type (4), (5), above, that the grammar must meet. Within these broad limits, a grammar is justified by showing that it follows from a given abstract theory of linguistic structure. This abstract theory must provide a practical and mechanical evaluation procedure for grammars. The abstract theory must have the property that for each language, the highest valued grammar for that language meets the external criteria of adequacy set up for the given language. The fact that certain 'clear cases' must be adequately and simply described in the case of many languages is a fairly
heavy condition on the general theory (though it is a weak
condition on the grammar of each particular language). In fact,
we are far from having an abstract theory which is not hopelessly
ad hoc, and from which an adequate grammar of even one language can
be shown to follow. Our interest is in constructing an abstract
theory which is not ad hoc (i.e., which does develop in a simple
and internally motivated way) and which leads to a revealing and
intuitively adequate English grammar, with a relatively simple
structure, as its highest valued interpretation for English,
under an evaluation procedure to be developed as part of the
theory.

Since we wish to apply the general theory to the
evaluation of particular grammars, it must have a basis of
primitive notions for which we can supply cross-culturally
valid operational tests. In the present state of our knowledge,
this requirement already excludes meaning. But a closer study
shows that if we grant the availability of all semantic
knowledge, then it becomes even more apparent that a semantic
basis for this theory must be excluded. In particular, it does
not appear to be the case that differential meaning (i.e.,
synonymy, to use the more familiar term) can be utilized in
determining phonemic distinctness, at least in any simple and
direct way; and it is not the case that the set of sentences
which is to be generated by the grammar is characterized by
a semantic property of significance. If we take meaning seriously
enough to assign meaning correctly to utterances, then the
proposed criterion for phonemic distinctness in terms of difference
in meaning is invalidated by the existence of homonyms and
synonyms. That is, reliance on this criterion would simply lead
to the wrong results in a vast number of cases. E.g., "latter" and "ladder" are phonemically identical in many dialects, even though obviously quite distinct in meaning, and /eˌkənəmɪkə/ and /iˌkənəmɪkə/, "adult" and "adult", etc. are phonemically distinct though identical in meaning. There is an effective operational test for phonemic distinctness, namely, the pair test, which is apparently what every linguist uses, in one form or another, when faced with a real problem of determining phonemic distinctness (as in the case of "latter" and "ladder", "adult" and "adult", "I saw him by the bank" and "I saw him buy the bank", etc., where meaning is clearly irrelevant). More devious suggestions for the use of meaning to determine phonemic distinctness either seem to special variations on the pair test, with an unnecessary and ultimately unacceptable explanation for its results thrown in, or they turn out to lead into a complex maze of involved and obscure operations whose adequacy can scarcely be evaluated, often doing considerable violence to the really important notion of 'meaning' in the process. Further investigation shows even more fundamental objections. Thus it appears that unless the notion of 'repetition' is assumed in semantic analysis, all the problems of determining phonemic identity reappear (in a far more difficult form) in the determination of sameness or difference of meaning. And if the notion of 'repetition' is assumed in the determination of synonymity, then clearly synonymity cannot be used without circularity in determining phonemic distinctness.

Similarly, if we take meaning seriously, the identification of 'grammatical' with 'significant' cannot be accepted, for one thing, because of the existence of grammatical nonsense. Thus
(6) (a) this is a round square
(b) colorless\textcolor{green}{^\wedge} ideas sleep furiously
(c) this fact seems interesting

all appear to be grammatical sentences, while

(7) (a) this are a round square
(b) furiously sleep ideas green colorless
(c) the child seems sleeping

do not, though it scarcely seems reasonable to propose a
semantic basis for the distinction in these cases. (6a,b) and
(7a,b) are all nonsense, and there is no independent semantic
sense in which (6c) is more 'meaningful' than (7c). In the case
of (6b), (7b) it is reasonable furthermore to assume that no
part of either of these has ever occurred before, from which
fact it follows that "grammatical" can \textcolor{green}{^\wedge} be analyzed in
terms of statistical order of approximation than in
terms of meaning. There are certain ways to give a partial,
operational characterization of the notion "grammatical." E.g.,
(7b) will be assigned the normal 'declarative' sentence intonation
by an English speaker, but (6b), though equally new, will be
spoken with the normal intonation pattern of a sequence of
disconnected words, with falling intonation on each word. Much
of the remaining part of this study is devoted to the development
of a systematic account of the notion of grammaticalness.
Cf. in particular chapter IV. In \$102.3 we suggest a deeper
transformational reason for the ungrammaticalness of (7c).
We discover that its exclusion is an automatic consequence of
the transformational analysis of adjectives -- i.e., its exclusion
contributes to the overall simplicity of the grammar, when
transformational analysis is incorporated into grammatical theory,
though in non-transformational terms, its exclusion would have required a special complicating statement.

In short, the challenge "how can you construct a grammar without appeal to meaning" is unfortunately phrased, because the implication that naturally one can construct a grammar with appeal to meaning is apparently false. When proposals for the use of meaning in grammar are carefully formulated and scrutinized they seem invariably to collapse. There seems to be little basis for the common assumption that meaning (in particular, synonymy or 'differential meaning') is required (or useful) in grammatical analysis, and that the problem is to find ways of avoiding recourse to it. It is even doubtful that meaning can supply useful hints for grammatical analysis. The widespread assumption that recourse to meaning is necessary or useful can perhaps be traced to a failure to distinguish "meaning" clearly from "intuition about linguistic form." Obviously, our intuitions about linguistic form (but not our intuitions about meaning) can be useful in the actual process of organizing data. Equally obviously, the whole point of linguistic theory is to replace this obscure reliance on intuition with an explicit account. But neither in the presystematic statement of the problem, nor in the systematic reconstruction, does meaning enter into consideration.

115.4. General linguistic theory provides a set of levels in terms of which the corpus can be described. Chapter II is devoted to the abstract characterization of the notion of linguistic level. Each level is essentially a non-instriptional system of concatenation, (with perhaps further algebraic properties) and related to other levels by conditions of compatibility. We can formulate a level as an algebra $\mathbb{L}$, where
(8) \( L = [L, \land, \ldots, R_1, \ldots, R_m, \mu, \Phi, \varphi_1, \ldots, \varphi_n] \)

where: 
- \( L \) is the set of primes (indecomposable elements) of \( L \)
- \( R_1, \ldots, R_m \) are relations in \( L \)
- \( \mu \) is a set of objects constructed from elements of \( L \) and denoted "L-markers"
- \( \Phi \) is a mapping of \( L \)-markers into grammatical utterances
- \( \varphi_1, \ldots, \varphi_n \) are relations of compatibility between \( L \) and other levels.

We assume that each level has an identity (unit) element \( U \) such that \( UX = XU = X \), for any string \( X \). The set \( \mu \) can be composed of strings in \( L \), classes of strings, sequences of strings, etc. The \( L \)-marker of an utterance \( X \) is the element of \( \mu \) which is mapped into \( X \) by \( \Phi \). It must contain within it a complete description of \( X \) on the level \( L \). Thus on the level \( P_m \) of phonemes, \( P_m \)-markers are strings of phonemes, but on the level \( P \) of phrase structure, it turns out to be necessary to take \( P \)-markers as certain sets of strings in \( P \) which give the full constituent structure of the utterance correlated with the given \( P \)-marker. When several levels are involved, we indicate by superscripts the level to which a given element belongs (e.g., \( \Phi^P \) denotes the mapping of \( P \)-markers into grammatical utterances). It is not necessary to define \( \Phi^L \) in general as a mapping from \( L \) to grammatical utterances. If \( \Phi^{L'} \) has been defined from \( L'-\)markers to grammatical utterances, then \( \Phi^L \) can be defined from \( L \)-markers to \( L'-\)markers -- e.g., \( P \)-markers can be mapped into strings of words (\( W \)-markers). Thus levels can be arranged in a hierarchy.

Suppose that an element of \( L \) is mapped into the unit
element of a lower level. Then this element is called a "zero element" and is designated \( \varnothing_1 \) (e.g., zero morphemes). Suppose that two \( \star \)-markers are mapped into a single element of a lower level. Then this is a case of constructional homonymity, and in an adequate theory, it should be a case of structural ambiguity. With each new level, we have the possibility for new cases of constructional homonymity. E.g., given the level of phonemes, there is only one way to represent the (homophonous) utterances "an aim"="a name" -- but given the higher level of morphemes, there are two ways. Similarly, on the level of phrase structure we find that "they are flying planes" is an instance of the construction Noun – be – Predicate, with the predicate "flying planes", and the conflicting construction Noun-Verb-Noun, with the verb "are flying" and the object "planes" (cf. (3b,d), above). An important way to investigate the adequacy of a general theory of linguistic structure is to determine whether cases of structural ambiguity are automatically assigned two markers on some level in the highest valued grammar resulting from this theory for a given language. One of the most important successes of transformational analysis is in the extended analysis of structural ambiguity that results in this way.

Each linguistic level presents a certain 'potentiality of description', a certain point of view from which the grammatical utterances can be described and in terms of which the boundaries of the set of grammatical utterances can be redrawn. An important part of the characterization of each level is to show exactly how this descriptive potential is utilized to give a closer approximation to the set of grammatical utterances.
We discuss this question for the level of syntactic categories in detail in chapter IV, developing a notion of degree of grammaticalness. A set $\text{Gr}(W)$ of highest degree grammatical sentences (strings of words) is constructed in chapter IV, basically, in terms of that classification of words into categories that provides a minimal projection of the corpus, in a sense there defined. This set offers a first approximation to the set $\mathcal{W}$ of grammatical strings of words (i.e., $W$-markers). $\text{Gr}(W)$ is finite. The higher levels of phrase structure ($P$) and transformational structure ($T$) permit recursive generation of sentences. If we choose $P$ and $T$ so as to describe $\text{Gr}(W)$ and then allow the process of generation to run on freely, we derive the infinite set $\mathcal{W}$. In chapter IX (cf., in particular, §106), various reasons are brought forward for limiting recursive generation to $T$.

115.5. Every factor relevant to the choice among grammars must be built into linguistic theory. The simplicity of a grammar is one such factor, and in chapter III we discuss the possibility of defining simplicity of grammar within linguistic theory. We can approach such a conception by providing notations for grammatical description, convert considerations of simplicity into considerations of length by permitting coalescence of similar grammatical statements. This favors grammars that contain generalizations. We have a generalization when we can replace a set of statements, each about one element, by a single statement about the whole set of elements. More generally, we have a partial generalization when we have a set of similar (not identical) statements about distinct elements. If we can
device notations that permit coalescence of similar statements, then we can measure the amount of generalization given in a grammar by length. Other features of simplicity can also be measured in a natural way in terms of length. For this conception to be meaningful, it is necessary to develop a fixed set of notations in linguistic theory, and a fixed form of grammatical statement. Certain notations are developed in chapter III which appear to be natural and effective. The definition of these notations (essentially, the construction of a 'language of grammar') constitutes the basic part of the definition of simplicity.

It remains to characterize the form of grammars explicitly. A grammar is a device for generating sentences. We can take a grammar to be a sequence (not a set -- order of statement can be used to effect major simplifications in the grammar) of statements of the form

\[(9) \alpha_i \rightarrow \beta_i \quad (i=1, \ldots, N)\]

interpreted as the instruction "rewrite \(\alpha_i\) as \(\beta_i\)"., where \(\alpha_i\) and \(\beta_i\) are strings. Suppose that we have such a sequence, and suppose that \(\alpha_1\) is the element Sentence (one of the primes of the level \(P\) of phrase structure). Call each statement of the form (9) a conversion. Then certain conversions are obligatory and certain conversions are merely permitted, and we can construct a derivation of any sentence by running through the list of conversions, applying each obligatory conversion and certain permitted ones, until the result is a string of phones. In the first tentative formulation of the theory we provide for the possibility of running through the grammar indefinitely many
times on the level $P$ to allow for infinite generation. This possibility is eliminated when transformational analysis provides alternative means for sentence generation.

A derivation is roughly analogous to a proof, with Sentences playing the role of the single axiom, and the conversions corresponding roughly to rules of inference. We can measure the simplicity of a grammar by measuring the length in symbols of the sequence of conversions, under the "translational transformations" that have been designed to convert considerations of complexity (generalization, etc.) into considerations of length. For each linguistic level, we must show how the information about utterances provided on this level can be presented as a sequence of conversions, and how the underlying algebra (i.e., the structure of the level) can be reconstructed from the sequence of conversions. Suppose now that we have given

(10) (i) the abstract structure of each level, and a statement of the relations of compatibility between levels
(ii) an abstract characterization of a bi-unique relation between a set of levels and a sequence of conversions
(iii) a measure of simplicity for a sequence of conversions (i.e., for a grammatical statement).

Then, given a corpus, we can construct a set of compatible levels, each with the proper internal structure, and such that the correlated sequence of conversions produces the corpus (along with much else). The 'structure of the language' is given by that set of levels which is associated (by (10ii)) with the simplest sequence of conversions. We guard against triviality (e.g., simple grammars that generate all sequences) by imposing formal conditions that a level must meet, given a corpus, as part of the abstract definition of this level. E.g., the level of
syntactic categories must give a minimal projection of the corpus (in a sense defined in chapter IV), and other levels can project further only in a specified way. The grammatical sentences of the language are those that are generated by the associated sequence of conversions. Thus each level makes a certain contribution to defining this set. We can state methods by which parts of the corpus (e.g., mistakes, semi-grammatical but occurring sentences, certain metaphors, etc.) can be excluded from the set of grammatical sentences. In other words, we utilize the descriptive apparatus available on each higher level to draw more precisely the boundaries of the set of grammatical sentences, introducing into \( \mathcal{W} \) the sentences that fit the patterns established when this level is designed to account for the already given grammatical sentences, and dropping sentences of the corpus which are instances of inadequately represented patterns (i.e., 'exceptions').

This in brief, is the conception of linguistic theory presented in the early chapters. In the appendix to chapter V we present an essentially complete analysis of a complex morphology (Modern Hebrew) in these terms. It turns out that a partial ordering is determined among the statements of the grammar by the criterion of simplicity, thus giving a hierarchy of morphological processes. It appears that the grammar meets certain desirable formal conditions of optimality (cf. (4)). E.g., all grammatical forms (in this case, words in their phonemic shape) can be derived by running through the grammar once; it is not necessary for any rule to appear more than once in the sequence of conversions; by and large, every rule applies just at the point where the strings to
which it applies are developed (by earlier rules) into just
the forms relevant for the correct application of the rule
(i.e., in the formulation of the given rule, it is not necessary
to list contexts which are overspecified from the point of view
of this rule alone), etc. The fact that a complex morphology
can be adequately handled in an essentially 'optimal' grammatical
statement, makes it seem likely that the conceptions which have
been advanced are adequate for lower levels of grammar, though
of course, much elaboration is necessary.

It is important to note that despite the very great
amount of important grammatical work done in the last few
decades, there are unfortunately no published grammars which
we can use as they stand as data for determining the adequacy
of these theoretical constructions concerning simplicity and
the form of grammars. The reason is that none of the
existing grammars provides a literally mechanical method for
generating sentences. Even though it may be intuitively
evident to every reader how to use these grammars to generate
utterances, we cannot know how much the formulation of the
missing steps will add to the complexity of the grammar. The
difficulty of presenting a really mechanical grammar can be
easily underestimated. But it is absolutely crucial to develop
such grammars if we wish to investigate seriously the
adequacy of a proposed linguistic theory in which such notions
as 'simplicity of grammar' play a part.

115.6. In chapter VI we develop the level $P$ of phrase structure.
This is a system

\[(11) \quad P = \left[ P, \wedge, =, \forall, Gr(P), \mu, \Pi \right] \]
where \((i)\) \(P\) is the set of primes. For English, these will be such elements as Sentence, Noun Phrase (NP), Verb Phrase (VP), Noun (N), John, ing, etc. 
\((ii)\) \(\mathcal{F}\) is the relation read "represents" that holds, e.g., between Sentence and NP\(\triangleright\)VP; between Sentence and John\(\triangleright\)came\(\triangleright\)home; between N and John, etc. \(\mathcal{F}\) is a partial ordering. It is carried over under concatenation. Furthermore, if \(\mathcal{F}(X,Y)\), then \(\mathcal{F}(\ldots X\ldots,Y\ldots)\). There is a unique prime Sentence such that if \(\mathcal{F}(Z, X\triangleright\text{Sentence}\triangleright Y)\), then \(Z\triangleright X\triangleright\text{Sentence}\triangleright Y\), where \(\mathcal{F}(X\triangleright Y, X\triangleright Y)\). There is a set \(\mathcal{F}\) of strings that bear \(\mathcal{F}\) to no string. These are the major characteristics of \(\mathcal{F}\). There are several other axioms to ensure significance of the system. 
\((iii)\) \(\text{Gr}(P)\) is a certain subset of \(\mathcal{F}\), defined as above. It is, essentially, the set of strings in \(P\) that correspond to grammatical strings of words and morphemes (but this is a loose correspondance). If phrase structure is our highest level, there must be a string in \(\text{Gr}(P)\) corresponding to each grammatical string of words, but with transformational analysis we can limit the correspondence to a certain kernel of grammatical strings of words. 
\((iv)\) \(\mathcal{H}_P\) is the set of \(P\)-markers. We will see directly that a \(P\)-marker is a certain set of strings containing exactly one string in \(\mathcal{F}\), which may or may not be in \(\text{Gr}(P)\). As on every level, \(P\)-markers contain within them all relevant information as to the constituent structure of the associated utterance.
(v) $\phi^p$ is the mapping of $p$-markers into strings of words. If (but not only if) the $p$-marker $K$ contains a string of $Gr(p)$, then $\phi^p$ carries $K$ into a grammatical string of words (i.e., a member of $\mu^w$). More generally, $\phi^p$ applies to any set of strings containing exactly one string of $P$, whether or not this set is a $p$-marker.

The major problem on the level $P$ is to develop the notion of "constituent", i.e., the notion of a 'significant' occurrence of a string $X$ in a containing string $Z$. For each constituent, we must also state what sort of a constituent it is. E.g., John is a Noun Phrase (NP) in "John came home". The relation "is a" is in a sense the converse of $\phi$, since John is a NP only if $\phi (NP, John)$, but it is much more complex. It must first of all be relativized to a given occurrence in a given sentence. Thus "called up" is a constituent (a Verb) in "I called up my friend", but not in "I called up the stairs." Secondly, it must be relativized to a given 'interpretation' of a sentence. Thus we can interpret the sentence

(12) they are flying planes

in such a way that "flying planes" is an NP, with adjective "flying" and noun "planes", or in such a way that "are flying" is the verb and "planes" the object. These 'interpretations' give different constituent analyses of (12). This is the important and widespread phenomenon of constructional homonymy on the level $P$, and the theory of phrase structure must be broad enough to admit it. An 'interpretation' of a sentence will be given by a $p$-marker, and if the abstract theory of phrase structure is
adequate, it will be an automatic consequence that its
highest valued (i.e., simplest) realization for an adequate
English corpus will contain two conflicting constituent
analyses (\(P\)-marker assignments) for such sentences as (12).

We approach the notion of constituent in the following
steps.

1. Define \(S_1\) as the relation "directly represents" of which
   \(S\) is the ancestral. I.e., if \(X\) bears \(S_1\) to \(Y\), then it
   bears \(S\) to \(Y\) and there is no \(Z\) bearing \(S\) to \(Y\) such that
   \(X\) bears \(S\) to \(Z\).

2. Given a binary relation \(Q\), we define a "\(Q\)-derivation" as
   a sequence of strings \(A_1, \ldots, A_n\) such that
   (i) \(A_1=\text{Sentence}\)
   (ii) \(Q(A_i; A_{i+1})\)
   (iii) there is no \(Y\) such that \(Q(A_n; Y)\)

   \(A_n\) is the product of the \(Q\)-derivation.
A \(Q\)-derivation is restricted if its product is in \(G_r(P)\),
which is the set of products of restricted \(S\)-derivations.
\(P\) is restricted if all \(S_1\)-derivations in \(P\) are restricted.
It is easily shown that we cannot give a satisfactory
account of English phrase structure if we require that \(P\)
be restricted.

3. We represent a \(S_1\)-derivation diagrammatically in an obvious
manner. Thus the derivation \(D=\text{Sentence, } a^b, c^d^e^f, b^g^h^i\)
can be represented by

(13) \[
\text{Sentence} \\
\quad a \quad b \\
\quad \ \\
\quad c \quad d \quad b \\
\quad \ \\
\quad c \quad d \quad e \quad f
\]
4. Consider now the derivations $D'$ and $D''$ diagrammed as follows

Clearly, there is a good sense in which $D$ and $D'$ differ inessentially from the point of view of the constituent structure of the product $Z = \text{a} \bowtie \text{d} \bowtie \text{f}$, whereas $D$ and $D''$ differ essentially. We make this sense of inessential difference as a definition of "equivalence" of derivations. We can then construct equivalence classes of derivations that assign the same constituent structure to the product.

The derivations that differ only in the order of application of the rules $S_1$. Each equivalence class of $S_1$-derivations clearly contains all constituent information about the common product, which is the unique string of $F$ that appears as a line (in fact, the final line) in each of these derivations. For reasons which appear directly, we define "$K$ is the $P$-marker of $Z$" as:

$K$ is the set of strings which appear as a line of one of the members of $E$, where $E$ is an equivalence class of $S_1$-derivations of $Z$.

E.g., if $\{D, D'\}$ (as in (134)) is an equivalence class of $S_1$-derivations of $Z$, then the $P$-marker of $Z$ is the class $\{\text{Sentence, a}^\bowtie \text{b}, \text{c} \bowtie \text{d} \bowtie \text{b}, \text{e} \bowtie \text{f}, Z\}$. 

5. To know whether $X$ is a $Y$ of $Z$ we must know what strings
represent $Z$. If $I^\text{\textit{Verb}}^\text{\textit{John}}$ bears $\varphi$ to $I^\text{\textit{called\_up\_John}}$, then we can obviously say that $\text{\textit{called\_up}}$ is a $\text{\textit{Verb}}$ of $I^\text{\textit{called\_up\_John}}$. In general, then, we define the relation "is a" as follows:

(15) $X$ is a $Y$ of $Z$ with respect to $K$ if and only if $K$ is a class of strings containing both $X^Z = ...X...$, and $Z' = ...Y'...$ (i.e., $Z'$ is the result of substituting $Y$ for $X$ in $Z$).

This notion is doubly relativized, as required above, to a given sentence in which $X$ occurs, and a given interpretation of this sentence. (Actually, in the careful development, it must be further relativized to a given occurrence of $X$ in this sentence.) Any class $K$ containing $Z$ gives an interpretation of $Z$. In particular, $P$-markers give interpretations of their products.

6. We define "$X$ is a constituent of $Z$ with respect to $K$" as

"$X$ is a $Y$ of $Z$ with respect to $K$, for some $Y$ which is a prime".

With respect to a given $P$-marker of $Z$, two constituents of $Z$ overlap only if one is included in the other. Thus the analysis given by a $P$-marker is essentially that of a properly parenthesized expression, where each parenthesized segment is represented by some prime. This is clearly a presystematic requirement for constituent analysis.

The bulk of the remainder of chapter VI is devoted to the problem of developing a bi-unique relation between $P$ and a sequence of conversions (as required in (10ii), above). Actually, a slightly weaker, but still strong enough condition is met. Basically, the problem is to construct a restricted
relation $\mathcal{F}^r$ corresponding to $\rightarrow$ (as in (9)) on the level $\mathcal{P}$, and such that

(16)(i) $\mathcal{F}^r$ is properly included in $\mathcal{F}$ (if $\mathcal{P}$ is non-restricted)

(ii) $\mathcal{F}$ can be completely and uniquely reconstructed from $\mathcal{F}^r$

(iii) all $\mathcal{F}^r$-derivations are restricted.

If these conditions are met by $\mathcal{F}^r$, then $\mathcal{F}^r$-derivations in the sense of step 2 in the definition of "constituent" above, will be derivations on the level $\mathcal{P}$ in the sense of §115.2. This construction is shown to be possible, if $\mathcal{P}$ is properly axiomatized.

We then develop a theory of grammatical relations in outline, and investigate in more detail just how criteria of simplicity apply in the evaluation of a system of phrase structure for a given language and how constructional homonymity appears on the level $\mathcal{P}$.

115.2. In chapter VII we apply this theory of phrase structure to English syntax. We find that if $\mathfrak{Gr}(P)$ is arbitrarily limited to a certain subset of simple declarative sentences, then the highest valued grammar is both intuitively and systematically adequate --- it correctly accounts for structural ambiguity, etc., and it has desirable formal properties, as did the morphological study commented on above in §115.2. But if we go on to include sentences like "I saw him coming" or "I know the boy studying in the library" in the coverage of the grammar (and at this point there is no justification for not doing so), then the highest valued grammar gives absurd results, and fails to meet the formal conditions of optimality, in the sense that it is impossible to order the rules $\alpha \rightarrow \beta$ in a sequence without repetitions and extensive complication in formulation. Hence a treatment that
is possible for even a very complex morphology is not possible for English phrase structure. There are many other formal deficiencies of the grammar (e.g., inability to state the rule for conjunction, conflict of criteria for constituent analysis, etc.). From this we conclude that our theoretical framework is only partially adequate on this higher level.

115.8 In the introductory sections to chapter VIII (§72-4), we summarize the intuitive and systematic inadequacies of the theory of linguistic structure developed up to the level of phrase structure. Briefly, we find that when this theory is applied to English, there are cases of structural ambiguity with no formal correlate, cases of formal constructional homonymy with no intuitive support, and analyses that are obviously intuitively incorrect. Furthermore, there is a whole class of notions which are not accounted for in this theory, including the notions of sentence type (declarative, interrogative, etc.), relations between sentences (active-passive, question-answer, etc.), and centrality of structure (actives more central than passives, declaratives more central than questions, etc.). Finally, it is impossible to account for all grammatical sentences without an overwhelmingly complex grammar. Thus the theory fails both criteria (4) and (5) of §115.2. We note that these are just the kinds of difficulties that would have been faced by a theory that broke off at the level of words. The fact that they still arise, though in a diminished form, when the level of phrase structure is added, suggests that a still higher level of syntactic analysis should be developed as a part of linguistic theory. Transformational analysis is proposed as a higher level.

In the abstract development of the level of transformational
analysis, we construct a set of 'grammatical transformations' each of which converts a sentence with phrase structure into a sentence with derived phrase structure. That is, each transformation operates on an ordered pair consisting of a sentence $Z$ and an interpretation $K$ (cf. step 5, §112.3), where $K$ is a $P$-marker of $Z$ if (but not only if) $Z$ is in $\text{Gr}(P)$. We consider a set of formal conditions that grammatical transformations must meet (e.g., transformations must be single valued, each transform $T(Z,K)$ of $(Z,K)$ under $T$ must differ from its pre-image $Z$ in a fixed structural property in a specified sense, etc.). A system of grammatical transformations meeting these conditions is then developed. If transformations are to appear in a grammar which is evaluated in terms of simplicity, there must be a fixed way to state transformations (just as the form of grammar must be fixed on every other level). We find that each grammatical transformation $T$ is determined by a finitely Statable elementary transformation and a finite restricting class, where the latter is a class of sequences of strings that determines the domain of applicability of $T$, and the former determines the structural revisions that $T$ effects. We go on to investigate the problem of assignment of constituent structure to transforms, and we develop a procedure for compounding transformations. This enables us, e.g., to form a question from a passive (which is itself derived from a corresponding active). Grammatical transformations are then generalized so that a transformation may apply to a sequence of strings with phrase structure. This extension provides the means for generating such sentences as "proving that theorem was difficult", "I saw him coming", etc., which cause considerable difficulty
when incorporated directly into the grammar of lower levels, and which are intuitively clearly derivative from a set of simple sentences.

In the transformational analysis of a language, we select a kernel of basic sentences for which a simple system of phrase structure can be provided, and in which all grammatical and selectional relations are found. Every grammatical sentence not in the kernel must be derived by means of the transformations set up for the language in question. A syntactic description then consists of a system of phrase structure for the kernel and a system of transformations. These systems jointly exhaust grammatical sentences. Each grammatical sentence $Z$ has an associated class $K$ providing its constituent interpretation. If $Z$ is in the kernel, $K$ is a $P$-marker. If $Z$ is not a kernel sentence, then $K$ is the class that provides its derived constituent structure. $K$ may still turn out to be a $P$-marker. In fact, we find that passives, though transformationally derived, still have $P$-markers assigned to them. We revise (161i) so that $\delta$ is completely recoverable from $\delta^T$ (i.e., from the sequence of conversions that constitute the grammar of phrase structure for the kernel) and from a relation $\delta^T$ such that $Y$ bears $\delta^T$ to $X$ if $X$ is a $Y$ in its derived constituent structure in some transform. For each set of sentences, we determine whether or not to assign them to the kernel by considering the complexity of the alternative syntactic descriptions. The kernel of the language is the set chosen as the kernel in the simplest overall grammatical description.

Transformational analysis can be formulated as a linguistic level. In this formulation, a compound transformation is
represented as a string of the grammatical transformations of which it is a compound, in the proper order. A generalized transformation can be represented in a natural manner as a string with substrings each providing for the transformational development of one of the component kernel strings. Just as on lower levels we can represent a sentence as a sequence (really a string) of phonemes, morphemes, words, syntactic categories, and (in various ways) phrases, we can, on the transformational level, represent each sentence as a sequence of operations by which it is derived from the kernel. Each such sequence of operations, interpreted as a string in a concatenation algebra and originating ultimately from a fixed sequence of kernel sentences, is a \( T \)-marker. We have a case of constructional homonymity when, in the simplest syntactic description, a given sentence can be derived from the kernel by several different routes, i.e., when it is assigned two different \( T \)-markers. Hence structural ambiguity on the level \( T \) has the same character as on other levels. We say that a \textit{syntactic relation} holds between two sets of sentences if some transformation set up for the language carries one set into the other. Such relations as that of active-passive can receive a formal grounding in this way. The notion of 'centrality of structure' can be explained wherever we can show that transformations are irreversible. Sentence types and subtypes can be identified with particular transformations and subsidiary transformations of transforms. The complexity of the grammatical statement is substantially reduced, since sentences with complex phrases and involved structure can be constructed transformationally out of already formed simpler sentences.
An incidental gain resulting from the development of transformational analysis is that it provides the means for characterizing the mapping $\Phi^P$ of $P$-markers into strings of words. This mapping can be characterized as a compound transformation. In chapter VI it was noted that the process of reducing the set of linguistic levels to a sequence of conversions of the type $\alpha \rightarrow \beta$ is actually incomplete. We can reduce the algebra $P$ to a sequence of conversions leading from the element Sentence to strings of $Gr(P)$. And we can reduce lower levels to a sequence of conversions leading from strings of words to strings of phones. But we cannot represent the mapping of a string $\alpha$ of $Gr(P)$ into a string $\beta$ of words by a sequence of conversions, since this mapping must take into account not only the given form of $\alpha$, but the history of derivation of $\alpha$ from Sentence (or equivalently, the $P$-marker of $\alpha$). Hence the program stated in (10), §115.5, has not been completely realized. Transformations, however, have exactly this property that they take into account the history of derivation of the transformed string from the element Sentence. That is, a transformation is defined on a pair consisting of a string $Z$ and an interpretation $K$ giving the phrase structure of $Z$. Hence we can bridge this gap by a transformational analysis of the mapping $\Phi^P$. Since $\Phi^P$ is now a special case of a compound transformation, it can appear in a $T$-marker. In fact, we require that each $T$-marker must contain the component transformations of $\Phi^P$, these providing a skeletal structure for $T$-markers. The minimal $T$-marker contains only $\Phi^P$. This is the $T$-marker of a kernel string. Thus every grammatical string $\alpha$ has a $T$-marker containing $\Phi^P$ (or its equivalent, in a sense defined);
the $T$-marker contains no other transformation if and only if the string in question is in the kernel. The process of generation of sentences by the grammar is thus slightly more complicated than as outlined in §115.5, above. A grammar consists of a sequence of conversions and a set of $T$-markers, (2) with their component transformations characterized in the manner indicated above. One of these $T$-markers is simply $P^p$, the $T$-marker of kernel sentences. The sequence of conversions is divided into two parts, from Sentence to $Gr(P)$ (i.e., the grammar based on $S^r$ and corresponding to the level $P^p$) and from strings of words to strings of phonemes. We generate a sentence by the following sequence of steps:

(17)(i) Derive a string $Z_1$ of $Gr(P)$ from the element Sentence by running through the first part of the sequence of conversions. From this derivation, we can reconstruct uniquely the $F$-marker $K$ of $Z_1$ (cf. step 4, §115.5).

(ii) Select a $T$-marker and apply it to $Z_1$ with the interpretation $K$. If the $T$-marker is just $P^p$, the derived string $Z_2$ is a kernel string. Otherwise, $Z_2$ is a transform. In either case, since $P^p$ is a part of each $T$-marker, $Z_2$ is a string of words.

(iii) Derive a string $Z_3$ of phones from $Z_2$, running through the second part of the sequence of conversions (thus, at the same time, reconstructing the lower level representations of $Z_3$).

Actually, (17) must be generalized. In step (i), we may derive a sequence of strings, and in step (ii), we may apply a generalized transformation to these giving a string with a
complex transformational origin. \( T \)-markers are formulated in such a way that they can apply to transforms as well as kernel sentences. Thus the process of transformational generation is recursive -- infinitely many sentences can be generated. Transformations are chosen in such a way as to exhaust the set of sentences of length \( \leq n \) that have been shown to be grammatical on lower levels (and to produce no new sentences of length \( \leq n \) -- cf. fn. 5, p. 192, chapter VI). By utilizing fully the descriptive potential of the level \( T \) and allowing transformations to run on freely, we complete the characterization of grammatical sentences.

Transformational analysis was motivated by the inadequacies of lower levels surveyed in part in 72-4. However, from the fact that \( \Phi^p \) must be transformationally characterized if the gap in developing grammars is to be closed, it follows that the notions of transformation, compound transformation, derived constituent structure, etc., would have had to be developed anyway, if only to complete the characterization of the level of phrase structure.

From the fact that the components \( \Phi_1^p \) of \( \Phi^p \) form an irreducible skeleton for \( T \)-markers, we see that the fundamental feature distinguishing the \( \Phi_1^p \)'s from the other transformations set up for the language is that the \( \Phi_1^p \)'s are obligatory, while the other transformations are optional. E.g., consider the strings of Gr(P) (18) and (19)

(18) I - called up - him
(19) I - called up - John

and the very similar transformations \( T_1 \) and \( T_2 \), where \( T_1 \) carries (13) into (20), and \( T_2 \) carries (19) into (21).
(20) I called him up
(21) I called John up

$T_1$ and $T_2$ are transformations with the same underlying elementary transformation and different restricting classes. Since (19), (20), and (21) are grammatical, but not (18), we see that $T_1$ must be a component of $\Phi$, while $T_2$ is an ordinary transformation.

Transformational analysis adds a new dimension to the description of linguistic structure. On all lower levels there is some relation between left-to-right order of representation and temporal order in the represented utterance (though discontinuous elements complicate this relationship). But on the transformational level an utterance is represented by a sequence of operations (or, in the case of a generalized transformation, a sequence of such sequences). Here, of course, there is no relation between order of representation and temporal order. The order of representation on the transformational level (i.e., in $T$-markers) represents the history of derivation of a string; the longer the $T$-marker, the farther removed is the represented string from the kernel. That is, on the transformational level, we do not represent a sentence even in a loose sense as a sequence of temporal events of some sort, but rather as a sequence of operations by which this string is derived from more basic sentences of a simpler structure. Transformational representation is thus much more abstract, in a sense, than representation on lower levels. This suggests that perhaps transformational analysis should be given some distinct status, and not be treated as simply a higher linguistic level. Further research may show that there are good reasons for this, but our
Investigation has so far led to the conclusion that transformational analysis is best developed as another linguistic level, with the same basic formal structure as lower levels. This approach has in its favor the fact that it contributes to the unity of linguistic theory and apparently provides adequate means for the development of generalized transformations. Perhaps the most convincing evidence in its favor is the direct and far-reaching generalization of the notion of 'constructional homonymity' that results automatically from this conception of the status of transformational analysis.

In the introductory chapter to this study (§ 0) we mentioned that the original motivation for these investigations was in certain problems that arose in the attempt to extend linguistic techniques to the analysis of discourse. We cannot claim that the problems of discourse analysis are solved by this theoretical study of linguistic structure, but it does appear to be the case that one of the basic operations required for discourse analysis is the inverse of transformation, as this notion has been developed here. Transformational analysis, as a part of linguistic theory, enables us to construct derivative and complex sentences out of central simple ones. Reversing the process, we can use the same transformational development to recover from an ordinary discourse the sequence of simple sentences of which the given text is a 'transform', in an extended sense of the term. (3) That is, we can reconstruct from the text a sequence of kernel sentences (with their phrase structure marked) from which the text can be derived. This reconstruction will be unique except in cases of constructional homonymity on the transformational level. The given text can be recovered from
the sequence of reconstructed kernel sentences and the sequence of associated transformations. It is fairly clear from the examples given in chapter IX that by this process we manage, by and large, to factor out the elementary content elements of the text as underlying kernel sentences. This question deserves more serious investigation, as it may prove to be of some independent significance.

115.2. In chapter IX we turn to the transformational analysis of English. The results achieved in this application of the theory developed in chapter VIII seem to me to lend convincing support to the position taken in the introductory sections to chapter VIII (§ 72-5) that a new level of linguistic structure should be added to linguistic theory. The level \( T \) is established in order to remedy inadequacies of the level \( P \) just as the level \( P \) is motivated in principle by deficiencies of a theory that does not go beyond the level of words. Investigating English structure in terms of the apparatus provided on this new level, we find that the scope and efficiency of grammatical description are considerably increased, and that the specific deficiencies surveyed in § 72-4 receive natural and generally revealing solutions. The cases of structural ambiguity cited in § 72.1 (e.g., "the shooting of the hunters", etc.) are now explained as resulting from constructional homonymity on the level \( T \) (but cf. § 95.5 for a different solution for one of these cases). Similarly we find that such pairs as

(a) this picture was painted by a real artist
(b) this picture was painted by a new technique

(a) I know the boy studying in the library
(b) I found the boy studying in the library
(a) the growling of lions
(b) the reading of good literature

and others cited in §72.1 are distinguished in terms of transformational history (i.e., in terms of their $T$-markers) in a manner which correlates perfectly with the intuitive conception of these sentences, though the relevant distinctions are not marked and are not statable on lower levels. The case of constructional homonymy unmatched by intuition mentioned in §72.2 ("the dog is barking", etc.) has disappeared with transformational analysis of adjective phrases, as have the intuitively incorrect analyses discussed in §72.3. We have also developed a more satisfactory approach to the problems of sentence type, etc., outlined in §72. We have a basic class of declarative sentences subdivided into the simple active declaratives of the kernel and the passives, which, though transformationally introduced, have the derived phrase structure of kernel sentences. Interrogatives are marked by the fact that the transformation $T_q$ (which converts "he came" into "did he come", "he will come" into "will he come", etc.) is the initial element in their $T$-markers. The subclass of yes-or-no questions consists of those which contain no other subsidiary transformations following $T_q$ in their $T$-markers. A second subclass of interrogatives consists of sentences whose $T$-markers contain $T_q T_w$ (for some $i$), where the $T_w$'s constitute a single family of transformations (i.e., they have the same underlying elementary transformation and they have restricting classes all meeting a fixed condition). This subclass contains, e.g.,

(a) whom did he see
(b) who saw him

These are interrogatives, since their $T$-markers contain $T_q$. Despite
the fact that (25a) (but not (25b)) contains an inversion, the sentences (25) have the same transformational histories, and hence belong to the same 'structural type' in transformational terms, though there would be little reason to consider them as constituting a single subclass of interrogatives (or, for that matter, to consider them to be interrogatives at all) on any lower level. Going on in this way, we find that we can apparently develop a systematic and intuitively satisfying typology of sentence structures. Similarly, the problem of syntactic relations between sentences, touched on in §23.3, receives a simple and apparently effective solution. Thus we find that there are very firm grounds for establishing a relationship of transformational derivation between such intuitively related classes of sentences as actives and passives, questions and answers, etc. The notion of 'centrality of structure' discussed in §23.4 is explained by the irreversibility that has been demonstrated case by case for the major transformations. Thus we find that a kernel of basic sentences is established in which all the fundamental selectional and grammatical relations are found, and from which all sentences are derived, and we see that the choice of the kernel is dictated by considerations of simplicity. The complexities and recursions which make it necessary to exclude large classes of sentences from the grammar corresponding to the level of phrase structure have been largely separated out and eliminated. The kernel grammar is simple, and is based on a few rudimentary grammatical relations. Furthermore, it is apparently finite. We can build up the rest of the language step by step, introducing new classes of sentences transformationally, and thus carrying the selectional and grammatical relations of the kernel into new forms and
arrangements. We have seen that this process not only to develop very complex sentence forms by a sequence of very simple operations, thus reducing the tremendous complexity of linguistic structure to much more manageable proportions, but it also supplies formal grounds in a great many cases for strong intuitions about linguistic structure.\textsuperscript{(4)} Furthermore, we have found that many phenomena which appear to be irregular and exceptional when viewed in terms of lower levels of linguistic structure (e.g., the behavior of "be", "have" in questions, negatives,\textsuperscript{and} so-clauses,\textsuperscript{and so forth}, from the class of the exclusion of (7c), etc., from the class of consequences of the simplest transformational description of the 'regular' cases, and thus are shown to be higher level regularities. We have also found that the behavior of sentences under transformation can throw a good deal of light on their phrase structure. In fact, we have found that many of the major criteria for constituent analysis are transformational in character.

Finally, we have seen that no new notions are needed to introduce the level $T_a$, or to validate its application. This new level is established on the basis of the same notions that were required for the establishment of lower levels. This is not to say that we have actually provided a literal demonstration that the given transformational analysis is the correct one, any more than we can provide a real demonstration of validity at this time on any other level of linguistic structure. But we have been able to show that if the lower levels can be established in anything approaching a satisfactory manner (and if our choice of examples in the cases we have considered is somewhere near representative), then the transformational level in English will take the form we have described. We have seen that the major
transformations discussed in chapter IX can be established on
the most rudimentary grammaticalness assumptions, (e.g., for the
passive transformation, the distinction between singular and plural
will largely suffice), and we have seen that as the analysis
of grammaticalness becomes more refined, transformational
analysis becomes continually more revealing.

115.10. There are serious gaps and defects in the constructions
that have been outlined and applied in these chapters. The
delimitation of the basic units, the analysis of simplicity and
grammaticalness, and no doubt, much of the internal structure
of the algebras \( P \) and \( T \), will have to be reformulated and
considerably deepened. Within the general limits of formal
distributional analysis, there are many avenues of
investigation beyond those that have been followed here,
notably, the whole study of the statistical structure of
language. There are also large areas of linguistic analysis that
have been extensively developed in recent years and
that we have not managed to incorporate into this outline
of a linguistic theory. In particular, no theory of linguistic
structure can approach adequacy if it does not cover such
phenomena as pitch, stress, and juncture. Integration of these
various approaches to the study of linguistic form may well
lead to the resolution of many of the difficulties in the
construction of linguistic theory and particular grammars. In
short, there seems to be considerable unexplored territory within the boundaries of formal distributional analysis, and the possibilities and potentialities of such analysis show no sign of having been exhausted. It is surely premature to insist that the basis of linguistic theory be extended to include obscure and intuition-bound concepts, on the grounds that the clear notions of formal analysis are too week and restricted to lead to interesting and illuminating results.
(1) (p. 717) Cf. last paragraph of chapter III, for a brief characterization of the sense in which we use this term, borrowed from Harris (cf. Methods, for an account of distributional procedures in linguistic analysis).

(2) (p. 743) The markers of all other levels are automatically recoverable from the sequence of conversions.

(3) (p. 746) In fact, in the terminology of chapter VIII, we can regard a text as a complex string represented on the level $T$ by a generalized normal string $S_1 \ldots S_n$, where $S_1 = Z_1^{\wedge} K_1^{\wedge} T_1$, and $S_1 = T_1$ or $Z_1^{\wedge} K_1^{\wedge} T_1$, and where (as distinct from the case of a $T$-marker) this normal string contains no generalized transformation which converts this complex string into a single sentence. Thus # will appear in the transform at sentence breaks, and the $P$-basis of $S_1 \ldots S_n$ will be the sequence of kernel sentences (and interpretations) from which the text is derived. It would be possible to extend slightly our transformation notation in such a way that this would moreover be a revealing and efficient way to represent a discourse, with the underlying kernel sentences and the associated transformations separated out.

(4) (p. 750) Intuitions which are often (and in my opinion, mistakenly) called 'semantic'. I think that the results of this transformational investigation give strong support to the position taken in chapter I (and summarized in [115.2], above) that the words 'meaning' and 'semantic' have been used incorrectly to refer to intuitions about linguistic form, intuitions which can be substantiated and grounded by careful and systematic investigation of linguistic form.